Self-shielding superconducting solenoid systems

G. Gabrielse and J. Tan
Department of Physics, Harvard University, Cambridge, Massachusetts 02138

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Superconducting solenoid systems which produce large magnetic fields can be designed to utilize flux conservation to cancel fluctuations in the ambient magnetic field in which they are located. Such self-shielding solenoids could be very useful for mass spectroscopy of trapped particles, nuclear magnetic resonance experiments, and magnetic resonance imaging.

I. INTRODUCTION

Many applications require high magnetic fields which are produced by superconducting solenoids. Most common are NMR and MRI (nuclear magnetic resonances and magnetic resonance imaging) applications. Of special importance to us is the mass spectroscopy of particles in an ion trap for which we use a 6-T field. For these and other high-field applications, it is often desirable that these fields be very stable in time. For example, to compare the masses of a proton and antiproton to a desired precision of 1 part in 10^9 in a 6-T magnet field requires a time stability better than 6 nT per hour. Unfortunately, the fluctuations in the ambient field in which the superconducting solenoid is placed varies in this time period from 10 nT to 10 μT depending upon ionospheric conditions, solar activity, the proximity to subways and elevators, etc. These fluctuations limit the time stability which can be realized in a high-field region, even though the high-field solenoid system itself produces a more stable field.

Many techniques are available for shielding out such fluctuations in the presence of small magnetic fields, but it is much more difficult to shield them out of a region of high magnetic field. One reason is that highly permeable materials like iron and "mu metal" are severely saturated and hence useless for shielding within the high-field region. Another reason is that shields made of type-I superconducting materials like lead and niobium cannot be used because the large field is above the critical field for type-I superconductors. Finally, a type-II superconductor has been used to screen external fluctuations from a very small high-field region,[1] but there was trouble with flux jumps associated with the shield.

We show here how to screen the external fluctuations using superconducting circuits. As is well known, magnetic flux through a closed superconducting circuit is conserved. We discuss how to configure coupled superconducting circuits so that this flux conservation ensures that external field fluctuations are screened from a region of interest. In particular, the solenoid systems used to provide the high field can be designed so that they themselves screen out the fluctuations in the ambient field. For simplicity, the focus here is on superconducting circuits composed of solenoids which are axially symmetric about a z axis. The z component of the external field B_e is reduced by a shielding factor S to B_e/S and the objective is to make S as large as possible.

A self-shielding solenoid system (a system for which S is large) can be constructed using a wide variety of circuit configurations. Therefore, self-shielding systems can be designed to preserve a variety of other properties. For example, a high degree of spatial homogeneity is often also required in the high-field region in order that very narrow resonance linewidths can be obtained. Time stability is then required to allow measurement of the narrow lines, several hours being required for some mass spectroscopy experiments of interest to us. We thus choose our examples of self-shielding solenoid systems to suggest ways that these can be designed with minimal distortions of the field homogeneity. Real solenoid systems are more complicated than our examples, but may be analyzed in the same way. A self-shielding solenoid system which maintains an extremely high degree of spatial homogeneity is now under construction for us by a commerical company.

The magnetic shielding described in this paper applies in principle to external field fluctuations which are arbitrarily fast. High-field solenoids, however, are typically wound on copper or aluminum cylinders which readily support eddy currents, especially when cold. External field fluctuations more rapid than 1 Hz typically are already severely screened by the cylinder.

II. SINGLE SUPERCONDUCTING SOLENOID CIRCUIT

To illustrate the basic shielding scheme, consider a single, axially symmetric solenoid i. The solenoid shown in Fig. 1 is made of superconducting wire and its ends are connected to make a closed circuit. The potential difference around the shorted solenoid is zero. By Faraday's law, an externally applied field B_e induces a current I_i in the solenoid which in turn produces a magnetic field B_i sufficient to keep the flux through the solenoid from changing:

$$\int_i (B_e + B_i) dA = 0. \tag{1}$$

We take the conserved value of the flux to be 0 so that we can focus on fluctuations from some steady state. The subscript on the integral indicates integration over the area of the sole-

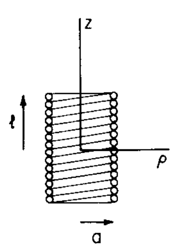

FIG. 1. Simple, single-layer solenoid. Example calculations are for the limit that the solenoid wires are vanishingly small.

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noid. The induced current persists since the resistance around the superconducting circuit is zero.

In what follows, we use cylindrical coordinates ρ and z, so that $B_i = B_i(\rho,z)$. The net field at the center of the solenoid $B_e(0,0) + B_i(0,0)$ can be written in terms of the shielding factor S as $B_e(0,0)/S$ so that

$$S^{-1} = 1 + [B_i(0,0)/B_e(0,0)]. \tag{2}$$

In light of the flux conservation criterion Eq. (1), this can be written

$$S^{-1} = 1 - \frac{\int_i B_e dA / B_e(0,0)}{\int_i B_i(\rho,z) dA / B_i(0,0)}. \tag{3}$$

To aid intuitive interpretation, we note that S^{-1} is linear in the ratio of two averaged fields

$$S^{-1} = 1 - (\bar{b}_e / \bar{b}_i) \tag{4}$$

defined by

$$\bar{b}_e = \frac{\int_i B_e dA}{B_e(0,0) \int_i dA}, \tag{5}$$

$$\bar{b}_i = \frac{\int_i B_i(\rho,z) dA}{B_i(0,0) \int_i dA}. \tag{6}$$

Here $\int_i dA$ is the total area involved in the flux integration for circuit i. Perfect shielding requires a solenoid for which the normalized average values of the external field and solenoid field are equal, $\bar{b}_e = \bar{b}_i$.

Without explicit calculation, one can immediately see that complete shielding of spatially uniform fields is possible with a single superconducting solenoid circuit, even if the solenoid has many layers of windings. For a spatially uniform external field B_e we have $\bar{b}_e = 1$ and the shielding is given by

$$S^{-1} = 1 - (1/\bar{b}_i). \tag{7}$$

For a short solenoid, the magnetic field near the windings is larger than the magnetic field near the center. The average value in the bore $\bar{b}_i$ is thus greater than 1 so that S^{-1} is positive. For a long solenoid, the volume average of the magnetic field produced by the solenoid within the bore is slightly less than the field at the center because of the fringing field at its ends. Thus, $\bar{b}_i$ increases to a value of 1 with increasing length. This corresponds to S^{-1} increasing to a limit of 0. Since S^{-1} must cross zero between these two limits, complete shielding is obtained with an appropriate choice of dimensions.

To facilitate explicit calculation, we eliminate the induced current from the expression for the shielding factor using factors g_i and L_{ii} which depend only upon the geometry of the solenoid circuit. The field at the center is proportional to the current

$$B_i(0,0) = g_i I_i, \tag{8}$$

as is the flux through the solenoid

$$\int_i B_i(\rho,z) dA = L_{ii} I_i. \tag{9}$$

The latter proportionality factor L_{ii} is the self-inductance for solenoid i. Substituting these two expressions in Eq. (3) yields

$$S^{-1} = 1 - (g_i A_i / L_{ii}). \tag{10}$$

For a spatially uniform external field, A_i is the total area $\int_i dA$ used to calculate the flux through circuit i. More generally, A_i is an effective area

$$A_i = \int_i B_e dA / [B_e(0,0)], \tag{11}$$

which depends on the spatial distribution of B_e.

In Fig. 2 we plot S^{-1} as a function of the solenoid aspect ratio l/a for a single-layer, densely wound solenoid in a uniform, external field. The necessary techniques for calculating inductances are well known[2] and efficient calculation techniques have been discussed.[3] The qualitative features discussed above are readily apparent. The self-shielding is complete (i.e., $S^{-1} = 0$) at the aspect ratio[4]

$$l/a = 0.88 \tag{12}$$

for a densely wound solenoid in the limiting case of vanishing wire diameter.

In general, the shielding produced by a persistent superconducting solenoid is far from complete. To illustrate, we use a solenoid represented in Fig. 3, which is not unlike many high-field solenoids which are commercially available. The large solenoid is wound uniformly with n_1 turns and its dimensions and characteristics are given in Table I. This solenoid would produce a field of 6 T at its center for a reasonable current of approximately 40 A. By itself, we calculate that this solenoid will screen external field fluctuations by a factor of $S = -2.9$, which is typical for commercial superconducting solenoid systems. Improving the self-shielding requires more than a simple reshaping of the solenoid. A self-shielding solenoid of the same radial dimensions, for example, would be reduced in length by more than a factor of 9. Such a squat solenoid would have properties very different from the solenoid in Fig. 3. More practical modifications will be discussed next, involving more than one superconducting circuit.

III. COUPLED SUPERCONDUCTING CIRCUITS

Practical solenoid systems typically contain several circuits, one to produce the large field and the others as shims to make the field near the center as homogeneous as possible.

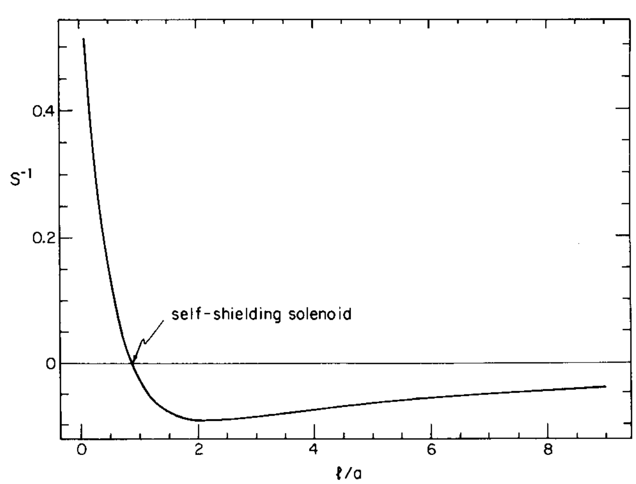

FIG. 2. Shielding of a densely wound, single-layer solenoid as a function of its aspect ratio, the ratio of its half length l to its radius a.

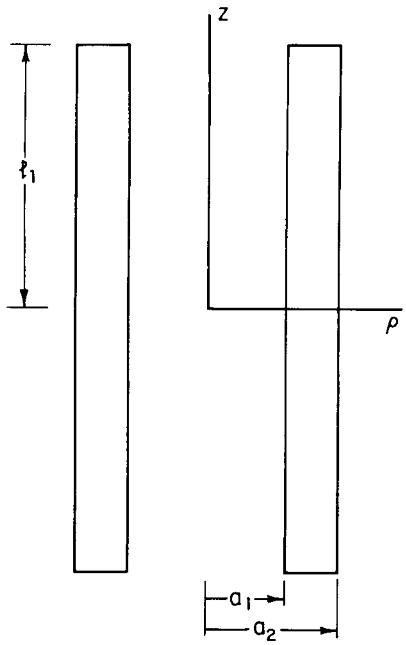

FIG. 3. Large solenoid to illustrate typical properties of high-field persistent solenoids.

We therefore generalize to a system of N closed superconducting circuits, each of which is axially symmetric. The subscript i now becomes an index $i = 1,...,N$ which labels the N circuits. A current I_i in circuit i produces the field $B_i(\rho,z)$. The currents can be represented by a column vector **I** and a related column vector **g** relates the field at the center to the currents with components defined by

$$B_i(0,0) = g_i I_i. \tag{13}$$

The areas of the circuits are represented by column vector **A** with components

$$A_i = \int_i dA, \tag{14}$$

which may be generalized for the case of nonuniform B_e as was done in Eq. (11). The familiar symmetric inductance matrix **L** has components given by

$$\int_i B_j(\rho,z)dA = L_{ij} I_j. \tag{15}$$

A diagonal element L_{ii} is the self-inductance associated with circuit i and off-diagonal elements are the mutual inductances between circuits. The shielding factor is

$$S^{-1} = 1 - \mathbf{g}^T \mathbf{L}^{-1} \mathbf{A}, \tag{16}$$

with the superscript T indicating transposition so that $\mathbf{g}^T$ is a row vector. For a single circuit Eq. (16) reduces immediately to Eq. (10). Complete shielding occurs when

TABLE I. Basic solenoid.

Dimensions	Calculated parameters
$a_1 = 7.62$ cm	$L_1 = 232.3$ H
$a_2 = 12.70$ cm	$A_1 = 2219$ m^2
$l_1 = 25.40$ cm	$g_1 = 0.1469$ T/A
$n_1 = 64\,000$	$S = -2.95$

$$\mathbf{g}^T \mathbf{L}^{-1} \mathbf{A} = 1. \tag{17}$$

This is the condition for a self-shielding solenoid system.

As an illustration, consider a system of two superconducting circuits. One solenoid circuit is characterized by L_1, A_1, and g_1 and the other by L_2, A_2, and g_2. The mutual inductance between the two circuits is M. One circuit could be a commercially constructed NMR solenoid to produce a 6-T magnetic field, for example, and the other circuit could be a solenoid added to make a self-shielding system. From Eq. (16), the shielding factor is

$$S^{-1} = 1 - \left(\frac{g_1 A_1}{L_1} + \frac{g_2 A_2}{L_2} - \frac{M}{L_1 L_2}(g_2 A_1 + g_1 A_2)\right) \times \left(1 - \frac{M^2}{L_1 L_2}\right)^{-1}. \tag{18}$$

For $M \to 0$, comparison with Eq. (10) shows that each coil contributes independently to the shielding. In general, however, the mutual inductance significantly modifies the shielding.

Computing S^{-1} is rather involved and lengthy, even in this simple two-circuit system. Many of the needed quantities, however, can be measured. This may be useful when modifications or additions to commercially constructed solenoid systems are contemplated, since their internal designs are often difficult to obtain. The self-inductance L_2 can be measured in conventional ways, most easily for a large solenoid by measuring the increase of current with time for an applied charging potential V_2

$$V_2 = -L_2 \frac{dI_2}{dt}. \tag{19}$$

For two coupled superconducting circuits, the mutual inductance can be measured by introducing a current I_1 in circuit 1. A current I_2 is induced in the second circuit to conserve flux through circuit 2. Thus M may be determined from

$$MI_1 + L_2 I_2 = 0 \tag{20}$$

when L_2 is already known. Circuit areas A_1 and A_2 can be determined by measuring the shielding factor S for each coil individually.

For a specific example of a two-circuit system, consider in Fig. 4 the addition of a second superconducting solenoid circuit inside the one shown in Fig. 3. Each solenoid is connected as a separate closed circuit. The added inner solenoid is uniformly wound with the same wire as the large solenoid and its radial dimensions are shown in Table II(a). Its inductance, area, and g values will change as a function of its length in a calculable way, as will the mutual inductance between the two solenoid circuits. Correspondingly, S^{-1} for the composite system changes with the length of the inner solenoid as shown in Fig. 5. Ideal self-shielding of uniform external fields occurs at $2l_2 = 9.8$ cm in this example. Characteristics of the inner solenoid for complete self-shielding are shown in Table II(b). This shielding configuration may be used even inside existing solenoid systems whose internal geometries are not well known. Probably the easiest approach in practice is to measure the shielding for inner sole-

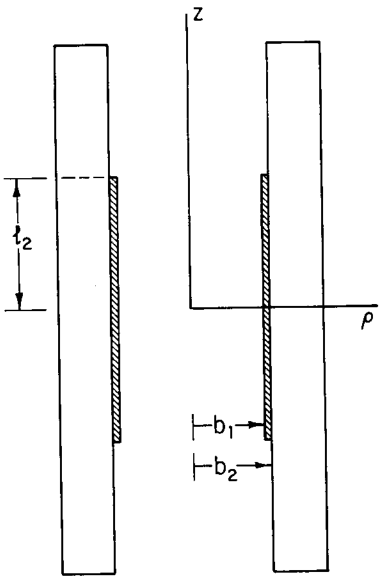

FIG. 4. Addition of a second superconducting solenoid circuit (cross-hatched) inside the large solenoid shown in Fig. 3 to make a self-shielding system.

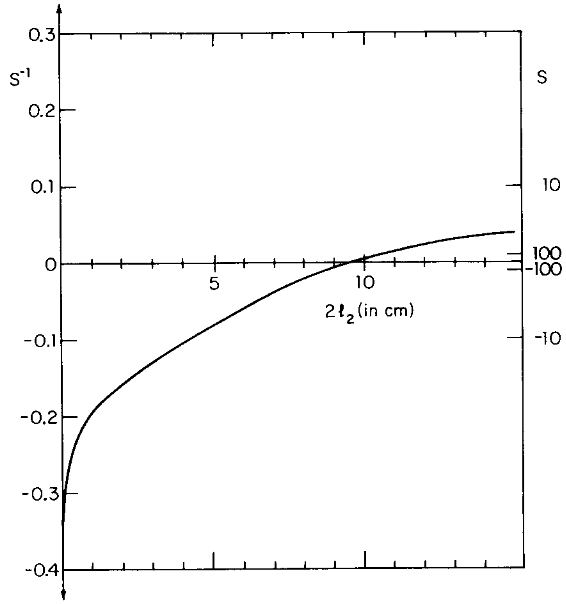

FIG. 5. Shielding in the illustrative solenoid system of Fig. 4 as a function of the length of the inner solenoid.

noids of various lengths and then interpolate to determine the appropriate length for complete self-shielding.

An alternative way of constructing a two-circuit self-shielding system is shown in Fig. 6. Here a second solenoid circuit is added on the outside of the basic solenoid of Fig. 3. Since only small correction fields must be produced by the additional solenoid, it is located relatively far from the central, high-field region. An advantage of this second configuration is that the bore of the magnet remains open for experimental apparatus. The properties of the added exterior solenoid are listed in Table III, with parameters such that a uniform external field is completely screened.

IV. SOLENOID CIRCUITS

In practical solenoid systems, each of N closed superconducting circuits is formed by connecting a subset of $\widetilde{N}$ solenoids in series, with $\widetilde{N} \geqslant N$. Since each solenoid can have a different geometry and current density, we have found it very convenient to first calculate column vectors $(\tilde{\mathbf{g}}, \widetilde{\mathbf{A}})$ and an inductance matrix $(\mathbf{L})$ for the solenoids. These are defined analogously to their circuit counterparts $(\mathbf{g}, \mathbf{A}$, and $\mathbf{L})$, which in turn can be obtained by a simple contraction. To accomplish this, an $N \times \widetilde{N}$ matrix Ω is defined such that the currents in the solenoids and circuits, $\tilde{\mathbf{I}}$ and $\mathbf{I}$, are related by

$$\tilde{\mathbf{I}}^T = \mathbf{I}^T \Omega. \tag{21}$$

TABLE II. Inner solenoid of Fig. 4.

	Dimensions	Calculated parameters
(a) Inner solenoid	$b_1 = 6.99$ cm $b_2 = 7.62$ cm.	
(b) Inner solenoid for complete shielding	$l_2 = 4.92$ cm $n_2 = 1550$	$L_2 = 0.2931$ H $M = 3.839$ H $A_2 = 25.98\ \mathrm{m}^2$ $g_2 = 1.106 \times 10^{-2}$ T/A

In simple cases wherein solenoids are connected in series with their currents flowing in the same rotational sense about the z axis, we have $\Omega_{ik} = 1$ if circuit i includes solenoid k and $\Omega_{ik} = 0$ otherwise. Negative elements may be used to represent currents flowing with opposite helicity with respect to the z axis. The resulting transformation rules are

$$\mathbf{g} = \Omega \tilde{\mathbf{g}}, \tag{22}$$

$$\mathbf{A} = \Omega \widetilde{\mathbf{A}}, \tag{23}$$

and

$$\mathbf{L} = \Omega \widetilde{\mathbf{L}} \Omega^T. \tag{24}$$

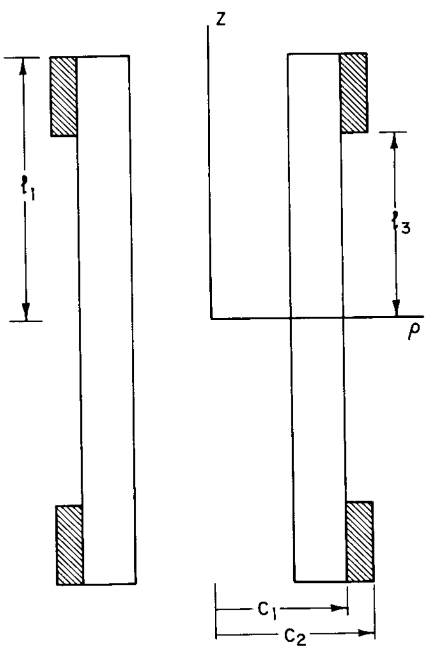

FIG. 6. Addition of a second superconducting solenoid circuit (cross-hatched) outside the large solenoid shown in Fig. 3 to make a self-shielding system.

TABLE III. Outer solenoid of Fig. 6.

	Dimensions	Calculated parameters
(a) Outer solenoid	$c_1 = 12.70$ cm $c_2 = 15.24$ cm	
(b) Outer solenoid for complete shielding	$l_3 = 17.87$ cm $n_3 = 9520$	$L_3 = 15.58$ H $M = 31.59$ H $A_3 = 582.8\ \text{m}^2$ $g_3 = 6.978 \times 10^{-3}$ T/A

The screening is determined by using the contracted values of Eqs. (22)–(24) in Eq. (16).

We have analyzed in detail a commercial NMR solenoid system (Nalorac 6.0/100/118) which involves two superconducting circuits with several solenoids making up each circuit. This system has a calculated shielding factor of

$$S = -4.45 \pm 0.10 \tag{25}$$

which agrees with the measured value. (This comparison is discussed further towards the end of the next section.) The uncertainty reflects some imprecision in our knowledge of the location of the windings and inaccuracy in our inductance calculation. We have studied, moreover, the possibility of adding a simple inner solenoid circuit, in the spirit of the example shown in Fig. 4, to make the solenoid system self-shielding. Such a system seems feasible and is now under construction by Nalorac.

Finally, we note that this approach is related to a technique wherein two concentric, coplanar superconducting loops were used to make a tunable gradient in a large magnetic field.[5] The two loops were connected in series such that the current flowed in the same direction. The radii of the loops were chosen to minimize the shift of the magnetic field at the center of the loops which occurred when the gradient was tuned. Accordingly, external field fluctuations were expected to cancel by perhaps a factor of 10 at the center of the loops, albeit at the expense of changing the field gradient. This configuration is not generally useful for shielding because of the gradients introduced. Still, it could be analyzed by treating each loop as a "solenoid," with the two loops connected in series to form a circuit. A complete analysis would also include the mutual inductances between these loops and the superconducting solenoid used to produce the large magnetic field being stabilized.

V. SPATIAL FIELD HOMOGENEITY

It is extremely important that modifications to make a high-homogeneity solenoid system self-shielding do not spoil the spatial homogeneity. Fortunately, the condition for a self-shielding system in Eq. (17) allows for many possible self-shielding configurations. The approach taken in Figs. 4 and 6 has minimal effect on field homogeneity. The basic solenoid, optimized to provide the desired level of homogeneity, is left unchanged. A separate additional solenoid is added which would carry no current if the ambient field was stable. Since it only carries the very small current required to cancel out the changes in the external field, it produces only a small field gradient. Suppose, for example, that fluctuations of the ambient field B_e as large as 6 μT are encountered. This means that the added solenoid at most must produce a field which is 10^{-6} of the 6-T field produced by the system used for mass spectroscopy. The fractional homogeneity requirement on the center solenoid is thus reduced by this factor. For the inner solenoid in Fig. 4, the field at a distance $d \approx 0.5$ cm from the center varies from the field at the center by $(d/l)^2 \approx 10^{-2}$, so that a homogeneity of 10^{-8} over a sphere 1 cm in diameter would not be compromised by the addition of such a coil. This is the approach we are presently taking with systems under construction, since this homogeneity is comparable to that produced by the unmodified solenoid system. Either the inner solenoid of Fig. 4 or the outer solenoid of Fig. 6 could be shaped to improve the homogeneity further, if this were required.

Spatial homogeneity is also important when shielding factors are being measured. For example, we applied an external field to a Nalorac Solenoid system to measure the shielding using a pair of square solenoids with side length of 2 m. The coils were separated by 2 m rather than being in a Helmholtz configuration. Even with these large coils, we calculate that the spatial inhomogeneity in the applied external field over the volume of the solenoid system reduces the shielding from $S = -4.45$ in Eq. (25) to $S = -4.10$ which agrees with our measurements. To calculate this reduction it is necessary to use the generalized definition of effective areas in Eq. (11), taking B_e to be the nonuniform field of the external coils.

Spatial uniformity of the fluctuating ambient field over the solenoid system is assumed for the configurations used here as illustrations. The high degree of symmetry of these configurations (and many practical systems) means that an external field with a linear gradient across the system is shielded as well. There are also cases wherein it is possible to shield the high-field region from a changing magnetic field which is not uniform over the shielding coil. For example, the highest magnetic fields are produced using multistrand superconducting wire. Solenoids so constructed often are not completely persistent but have a field which decays in time very slowly. If the spatial distribution of this decaying field is known near the center, it may be taken as B_e and used to calculate the effective area of a small, persistent superconducting coil located near the center. The dimensions of this interior coil are suitably chosen to compensate the drift in the field.

VI. SUMMARY AND CONCLUSIONS

Fluctuations in the ambient magnetic field must be shielded out or otherwise compensated to obtain a region with a strong magnetic field which is stable in time. Flux conservation in superconducting circuits makes it possible to design superconducting solenoid systems which produce large magnetic fields and also react to shield the high-field region from ambient fluctuations. This may be realized in many specific solenoid geometries and circuit configurations; the choices depend upon the desired field properties for particular applications. For precision mass spectroscopy, we are particularly concerned about spatially uniform fluctuations in the ambient field. We have thus illustrated how to add either an inner or outer solenoid circuit to a simple high-field solenoid in order to make the system self-shielding, without significant alternation of the spatial homogeneity of the field produced by the basic solenoid.

ACKNOWLEDGMENTS

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