Spin-Flip Resolution Achieved with a One-Proton Self-Excited Oscillator

A thesis presented by

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 to

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Abstract

In a Penning trap with an extremely large magnetic gradient, the axial frequency of a one-proton self-excited oscillator is resolved at the level of the shift from a proton spin flip. This sensitivity opens a possible path towards detection of single-proton spin flips, novel measurements of the proton and antiproton g-factors, and a stringent test of CPT invariance by comparing proton and antiproton magnetic moments at precision likely to be a million times higher than achieved to date.

The central challenge of extending similar electron magnetic moment measurements to one proton is overcoming the substantially larger mass and weaker magnetic moment, which conspire to greatly reduce the frequency shift that signals a spin flip. Within a magnetic bottle gradient 50 times larger than used in the recent electron g-factor measurements, the proton spin-flip shift is still only 60 mHz out of a 553 kHz axial frequency. In such a large gradient, standard application of sideband cooling to reduce the magnetron radius changes the axial frequency by an amount greater than this spin-flip shift on average.

Proton axial frequency resolution at the 60 mHz level is enabled by feedback techniques realized previously only with one electron. Self-excitation produces a narrow feature with large signal-to-noise, ideal for rapid frequency measurements at high precision. Unwanted effects of the strong magnetic gradient are minimized by axial and radial cooling. Feedback cooling is used to reduce the proton axial motion below the temperature of a damping resistor. Axial-magnetron sideband cooling of the undamped radial motion is then demonstrated to reach a 14 mK theoretical limit.

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Chapter 1

Introduction

A common roadblock to improving the precision of fundamental constants is the complexity of systems available for measurement. Unless all theoretical aspects of a system are well understood, uncertainties in the model will ultimately limit the precision of a measurement on that system, even after the development of high-resolution experimental methods. This interplay between theory and experiment provides valuable positive feedback, as advances in experimental technologies motivate refinements in theory, and vice versa.

At the high precision to which most fundamental constants are currently known, even a hydrogen atom contains complexity, e.g. due to finite nuclear size, that introduces uncertainty in the theoretical model. To obtain better measurements of nature, we must sometimes resort to less-natural systems, such as the artificial atom of a single electron in a Penning trap, where particle motions and electromagnetic fields are well controlled and understood [1]. The recent measurement of the electron g-factor (essentially the magnetic moment in Bohr magnetons) [2] is arguably the most precise of all comparisons of theory and experiment [3].

Following the success of the electron experiment, it is natural to consider a similar effort to extract the proton g-factor from precision measurements of a single trapped proton. In terms of risks and rewards, an initial comparison with the electron case appears discouraging. First, the signature of a proton spin-flip transition is reduced by the 2000 times smaller size of the nuclear magneton compared to the Bohr magneton, presenting a considerable challenge for spin-flip detection. Second, the electron experiment obtains its sub-ppt (part-per-trillion) precision by combining a spin-flip and a cyclotron transition to measure the anomaly $(g_e - 2)/2 \approx 0.001$ instead of g_e directly; since $g_p \approx 5.6$, this "free" precision improvement factor of 1000 is unavailable for the proton measurement, and our best hope is fractional precision around the ppb (part-per-billion) level. Third, while the electron experiment gives rise to a fruitful comparison with similarly precise calculations from QED theory [4], QCD theory for the proton g-factor already trails existing measurements by many orders of magnitude (a representative summary of attempts to calculate baryon magnetic moments is found in Table I of reference [5]).

However, there are three overriding motivations for pursuing a proton g-factor experiment. The latter motivations are especially compelling:

- The possibility of a novel and direct measurement of the proton magnetic moment.
- The chance for a spectacular improvement in measured precision of the antiproton magnetic moment, by a factor of a million or more.
- Realization of one of two high-precision CPT tests in the baryon sector, a com-

parison of proton and antiproton magnetic moments.

This thesis reports the critical first steps towards a measurement of $g_{\bar{p}}/g_p$. A single proton is trapped and cooled within an extremely strong magnetic bottle gradient. Resolution of the proton axial frequency is demonstrated to reach the level of the small shift caused by a single-proton spin flip in this gradient, opening a path to spin-flip detection and g-factor measurement.

1.1 The Proton Magnetic Moment

The proton magnetic moment is typically expressed in terms of the dimensionless g-factor g_p ,

$$\mu_p = \frac{g_p}{2} \left(\frac{e\hbar}{2m_p} \right) . \tag{1.1}$$

All previous determinations of g_p have relied on measurements of protons bound in matter ([6], [7], [8]). The proton g-factor is currently obtained by backing out the free proton value from the bound proton/electron measurement using

$$g_p = g_e \frac{\mu_p(H)}{\mu_e(H)} \frac{g_e(H)}{g_e} \frac{g_p}{g_p(H)} \frac{m_p}{m_e} .$$
(1.2)

This calculation requires three high-precision experimental inputs $(g_e, \frac{\mu_p(H)}{\mu_e(H)}, \text{ and } \frac{m_p}{m_e})$ and two correction terms from theory $(\frac{g_e(H)}{g_e} \text{ and } \frac{g_p}{g_p(H)})$. The bound/free g-factor corrections, summarized in [9] and [10], have been calculated to < 1 ppb. The electron g-factor g_e is known to < 0.001 ppb [2]. The proton-electron mass ratio $\frac{m_p}{m_e}$ is known to < 1 ppb [11]. The proton-electron magnetic moment ratio, $\frac{\mu_p(H)}{\mu_e(H)}$, is the weakest link in Eq. 1.2. Currently known to 10 ppb, this quantity determines the overall precision of $g_p = 5.585694713(46)$. The historical progression of proton magnetic moment measurements, expressed in terms of g_p , is shown in Fig. 1.1. Initially measured with molecular beams [12, 13] and later from the NMR signal of protons in water [14], the current best precision for $\frac{\mu_p(H)}{\mu_e(H)}$ is obtained using a hydrogen maser [7] to measure the energy levels of hydrogen in a strong \vec{B} field, where the Breit-Rabi Hamiltonian includes terms for both electron and proton interactions with \vec{B} .



Figure 1.1: Proton g-factor history and projected improvement. Values calculated from bound measurements of μ_p/μ_e ([6], [7], [8]), with 2009 values of g_e , m_p/m_e , and theory corrections.

Our proposed experiment using a single trapped proton would be the first direct measurement of g_p , greatly simplifying the scheme of Eq. 1.2. Measuring g_p requires measuring principally two frequencies, of the cyclotron motion and spin precession. The possibility to measure the former to 9×10^{-11} has already been demonstrated by our research group [15, 16]. This work focuses upon measuring the latter. We believe that an overall fractional precision of 1 ppb is attainable for the measurement of g_p , which would represent an order of magnitude improvement over current measurements.

1.2 The Antiproton Magnetic Moment

In principle, the magnetic moment of the antiproton can be measured in the same way as that of the proton. However, such a measurement would require an equivalent antimatter system, e.g. anti-water for NMR. While trapped antihydrogen might someday enable a measurement of $g_{\bar{p}}$ via precision spectroscopy, there exists today no antimatter system in which to make antiproton measurements corresponding to any of those in Fig. 1.1.

Current comparisons of g_p and $g_{\bar{p}}$ are limited to 3 ppt (parts-per-thousand) [17]. For several decades, the best measurements of $g_{\bar{p}}$ have come from "exotic atom" experiments. In these experiments, high-energy antiprotons from an accelerator collide with a matter target, replacing some electrons to form excited states of exotic atoms. X-ray decays are then observed to measure fine-structure splittings that depend on $\mu_{\bar{p}}$. An early series of exotic-atom measurements was performed in the 1970s at Brookhaven [18, 19]. The longstanding value of $g_{\bar{p}} = 5.601(18)$ was obtained in 1988 from a measurement at CERN, with exotic atoms of Pb [20].

Measurements with exotic atoms must overcome theory uncertainties due to complex internal structure, along with experimental difficulties such as short lifetimes of the exotic states and noisy environment of the particle accelerator. The gradual precision progression for $g_{\bar{p}}$ is shown in Fig. 1.2. The most recent such measurement, using laser spectroscopy of antiprotonic helium [21], obtains $g_{\bar{p}} = 5.572(17)$, with essentially the same uncertainty as the 1988 measurement [20].



Figure 1.2: Projected improvement in the antiproton g-factor. Measurements of $g_{\bar{p}}$ are from (in chronological order) references [22, 23, 19, 18, 20, 21].

If successful with a single proton, our new measurement will translate with relative ease to the case of a single trapped antiproton, allowing for the first time the possibility of a measurement of $g_{\bar{p}}$ at precision similar to g_p . Fractional precision approaching 1 ppb would represent improvement by a factor of nearly one million in the known value of $g_{\bar{p}}$ (Fig. 1.2).

1.3 Testing CPT

Precision comparison of the antiproton and proton magnetic moments would allow a new test of CPT invariance. CPT invariance refers to the symmetry of physical laws under the simultaneous transformations of charge-conjugation (C), parity (P), and time-reversal (T). While violations of the individual symmetries C, P, T, and CP have been observed, no breaking of CPT symmetry has yet been detected. In fact, CPT invariance is a consequence of a locally Lorentz-invariant quantum field theory, such as the Standard Model.

Despite this foundation, there are compelling reasons to put CPT invariance to the test. Symmetry is invoked frequently in theoretical physics, allowing the assumption that some quantity is conserved in all interactions. Violation of symmetry thus indicates a fundamental oversimplification in the existing theory, opening the possibility of new phenomena. Parity, for example, was long believed to be a perfect symmetry of nature. Proof of its violation [24, 25] revolutionized the study of weak interactions and was rewarded almost instantly with the 1957 Nobel Prize. CP was then assumed to be a perfect symmetry, only to be rejected when CP violation was observed in neutral kaon decays [26], a discovery honored with the 1980 Nobel Prize.

Current theories that require CPT invariance provide only an incomplete description of nature. Discovery of CPT violation would guide the development of physics beyond the Standard Model. For example, the possibility of CPT symmetry breaking is allowed in string theory and other proposals to unify gravity with the strong nuclear and electroweak forces [27]. CPT violation has also been invoked in proposed mechanisms for baryogenesis, the yet-unexplained process that resulted in the dominance of matter over antimatter in our universe [28].

If CPT is a perfect symmetry, matter and antimatter must have equivalent physical properties; in particular, a particle and its antiparticle must have (1) equal masses, (2) charges equal in magnitude but opposite in sign, and (3) magnetic moments equal in magnitude but opposite in direction. Precision comparisons of matter and antimatter provide the most stringent experimental tests of CPT. Penning trap experiments are particularly well-suited to this role, providing access to quantities such as q/mand μ_p in terms of ratios of trapped-particle frequencies which can be measured to high precision. The current bounds on CPT violation in lepton and baryon systems are obtained in Penning trap experiments. The lepton bound is at the 10^{-12} level, obtained by comparing g - 2 for electrons and positrons [29], and likely soon will be improved in our lab. The baryon bound is roughly 10^{-10} , obtained by comparing q/mfor protons and antiprotons [16].

It is critical to test CPT in a variety of experiments, as our knowledge of CPT violation is sufficiently limited that we cannot predict which physical systems may be most sensitive to CPT-violating effects. Accordingly, despite a longstanding 10^{-18} bound set using mesons [30], efforts continue to push CPT tests in various lepton, baryon, and combined lepton-baryon systems. Comparison of the proton/antiproton magnetic moment would be a new test of CPT for the baryon sector, with an initial precision goal of 1 ppb. While not immediately challenging the overall CPT bound for baryons, this test would serve as the strongest probe to date of the Lorentz and CPT-violating b_3^p term in the Standard Model Extension, a parameterization of possible CPT and Lorentz-violating interactions [31].

1.4 Outline of Work Presented

This thesis reports several advances enroute to a measurement of $g_{\bar{p}}/g_p$:

- Chapter 2 and Chapter 3 describe the underlying theory, design, and construction of a first-generation apparatus for detection of a single-proton spin-flip transition, which would be proof-of-principle for a g-factor measurement.
- Chapter 4 describes construction of the tuned-circuit amplifiers used for proton detection, including improvements essential to optimize detection signal/noise.
- Chapter 5 describes loading of protons into a Penning trap, the identification and removal of contaminant ions, and transfer into a secondary trap with the strong magnetic field inhomogeneity required for spin-flip detection.
- Chapter 6 and Chapter 7 describe the measurement of trapped-proton frequencies in our Penning traps, including complications introduced by the strong inhomogeneous magnetic field.
- Chapter 8 describes the first realization of techniques for single-proton selfexcitation and feedback cooling—powerful tools for controlling the axial oscillation that must be measured precisely to detect a spin flip.
- Chapter 9 reports the achievement of axial frequency resolution at the level required for proton spin-flip detection, and describes the prospects for driving and detecting spin-flip transitions.
- Chapter 10 summarizes the current status of the experiment and prospects for a next-generation apparatus designed for the eventual antiproton measurement.

Chapter 2

Measuring g-factors in a Penning Trap

2.1 The Open-Endcap Penning Trap

A Penning trap combines a quadrupole electric potential and a strong magnetic field to trap a charged particle in space [1]. The magnetic field provides radial confinement while the electric field provides confinement along the axis of the B field. Given that Laplace's equation permits no global minimum in three dimensions, this is the simplest stable "trap" that can be obtained with static fields.

A variety of electrode geometries are available to produce the quadrupole potential. The first single-electron Penning traps were designed with the trap electrodes machined along hyperboloids of revolution [32]. A similar trapping potential can also be produced using a cylindrical geometry [33], which offers advantages in terms of ease of machining and identifiable radiation modes of the trap cavity. The recent electron g-2 measurements [2] were performed in a closed-endcap cylindrical trap that utilizes cylindrical ring electrodes and flat endcaps. For our proton experiment, in order to allow transfer between spatially separated "precision" and "analysis" traps as discussed in Section 2.3, we use an open-endcap variation [34] in which the endcaps are also cylinders through which particles can be transferred.

The basic structure of our open-endcap proton trap is shown in Fig. 2.1. A potential difference between the ring and endcaps produces the trapping well, while the compensation electrodes are biased to adjust trap anharmonicity.



Figure 2.1: Coordinate system and trap dimensions for the open-endcap Penning trap.

2.1.1 Fields

Penning trap theory is described in detail in reference [1]. For the case of the open-endcap trap, we use the coordinate system shown in Fig. 2.1. The z-axis lies along the magnetic field, which is assumed for simplicity to be coaxial with the cylin-

ρ_0	$3.00 \mathrm{~mm}$
z_0	$2.93 \mathrm{~mm}$
z_c	$2.45 \mathrm{~mm}$
z_e	13.28 mm
C_2	0.545

Table 2.1: Dimensions of the proton Penning trap

drical electrodes. The case of a slight misalignment is discussed in reference [1] and Chapter 6. Table 2.1 lists values of the trap dimensions for our proton trap (identical for the precision and analysis traps), along with the value of the C_2 coefficient described below.

The potential of an ideal quadrupole is given by

$$V = V_0 \, \frac{z^2 - \rho^2/2}{2d^2} \,. \tag{2.1}$$

To approximate this potential in our cylindrical trap, we apply a voltage $-V_0/2$ on the ring, $V_0/2$ on the endcaps, and V_c on the compensation electrodes. Due to cylindrical symmetry, the total potential can be written in terms of Legendre polynomials:

$$V(\vec{r}) = \frac{1}{2} \sum \left(V_0 C_k^{(0)} + V_c D_k \right) \left(\frac{r}{d}\right)^k P_k(\cos\theta) , \qquad (2.2)$$

where

$$d^{2} = \frac{1}{2} \left(z_{0}^{2} + \frac{1}{2} \rho_{0}^{2} \right)$$
(2.3)

is the characteristic trap distance, and the coefficients $C_k^{(0)}$ and D_k are determined by relative trap geometry. Note that d is the only distance scale. For simplicity, we can rewrite the total potential as

$$V(\vec{r}) = \frac{V_0}{2} \sum C_k \left(\frac{r}{d}\right)^k P_k(\cos\theta) , \qquad (2.4)$$

where the net coefficient C_k in the expansion of the total potential is now dependent on the ratio of applied compensation and trapping voltages,

$$C_k = C_k^{(0)} + \frac{V_c}{V_0} D_k . (2.5)$$

All terms of odd k vanish because of reflection symmetry under $z \to -z$, and the k = 0term produces an overall constant offset. To best approximate the ideal trapping potential, we minimize C_4 and C_6 by careful choice of trap dimensions and voltages. The voltage applied to compensation electrodes is set to approximately $V_c/V_0 =$ $-C_4^{(0)}/D_4$, such that $C_4 \approx 0$. The compensation electrode size z_c/z_0 is chosen to set $C_6 = 0$ at this same value of V_c/V_0 . The lowest two anharmonic terms can thus be suppressed by careful tuning of the compensation potential. Finally, the aspect ratio ρ_0/z_0 is chosen to "orthogonalize" the trap [34], setting $D_2 = 0$ such that changes in V_c have a relatively weak effect on the overall potential and the proton axial frequency.

For experimental purposes, we use a potential scheme slightly different than the one described above. Instead of $-V_0/2$ on the ring and $V_0/2$ on the endcaps, we ground the endcaps and apply $-V_0$ on the ring, which is entirely equivalent but reduces the number of precision voltage supplies that are required. With this transformation,

$$V_0 = -V_{ring} \tag{2.6}$$

$$V_c = V_{comp} - V_{ring}/2. (2.7)$$

So, for example, the $V_c/V_0 = -0.381$ needed to tune out the leading order anharmonicity [34] corresponds to $V_{comp}/V_{ring} = 0.881$.

2.1.2 Trapped Particle Frequencies



Figure 2.2: Eigenmotions in the proton Penning trap: (a) shows the trappedparticle amplitudes and frequencies greatly exaggerated for clarity; (b) illustrates the frequency hierarchy for an actual trapped proton.

The basic motion of a trapped proton is shown in Fig. 2.2a. The axial motion is a nearly harmonic oscillation due to the electrostatic potential in the \hat{z} direction. The radial motion is a superposition of relatively fast cyclotron orbits around the \vec{B} field and slower magnetron orbits due to $\vec{E} \times \vec{B}$ drift. The frequency scale is greatly distorted to clearly show each of the eigenmodes. Fig. 2.2b represents relative frequencies more accurately, illustrating the considerable frequency separation of the three oscillations. This frequency hierarchy makes it possible to separately address the three motions of a trapped proton.

The axial equation of motion depends only on the electrostatic potential. Taking the limits $\rho \to 0$, $C_4 \to 0$, and $C_6 \to 0$, which describe the case where the proton is well-centered in an open-endcap trap carefully tuned to reduce anharmonicity, Eq. 2.4 reduces to

$$V(z) = \frac{C_2 V_0 z^2}{2d^2} \,. \tag{2.8}$$

In this ideal case, we have a simple harmonic oscillator. An ion of mass m and charge q oscillates with axial frequency

$$\omega_z^2 = \frac{qV_0C_2}{md^2} \,. \tag{2.9}$$

In practice, we must often consider the anharmonicity of the trapping potential. The effects of anharmonicity can be expressed as an amplitude-dependent frequency [35], given by

$$\bar{\omega}_z^2(A) = \omega_z^2 \left[1 + \frac{3C_4}{2C_2} \left(\frac{A}{d}\right)^2 + \frac{15C_6}{8C_2} \left(\frac{A}{d}\right)^4 + \dots \right] \,. \tag{2.10}$$

Radial motion of the proton is dominated by the cyclotron frequency, which in free space has the value (in SI units)

$$\omega_c = 2\pi\nu_c = \frac{qB}{m} \,. \tag{2.11}$$

However, in a Penning trap, the ρ^2 term from the electrostatic potential also contributes to the radial Lorentz force. The full solution of the resulting radial equation of motion is a superposition of two radial motions [1]. A circular magnetron orbit in the direction of $\vec{E} \times \vec{B}$ occurs with frequency

$$\omega_m = \frac{\omega_z^2}{2\omega_c'} \,, \tag{2.12}$$

while the cyclotron orbit remains, but at the "trap-modified" frequency

	2006	proton	proton
	electron trap	precision trap	analysis trap
Magnetic Field	5.36 T	5.68 T	$5.22 \mathrm{~T}$
Magnetic Bottle B_2	$1540~\mathrm{T/m^2}$	$< 10 \text{ T/m}^2$	$78000~\mathrm{T/m^2}$
Trapping Voltage (V_0)	+101.4 V	-4.4 V	-1.6 V
Trap Radius (ρ_0)	$4.56 \mathrm{~mm}$	$3.00 \mathrm{~mm}$	$3.00 \mathrm{~mm}$
Trap Size (d)	$3.5 \mathrm{~mm}$	$2.56 \mathrm{~mm}$	$2.56 \mathrm{~mm}$
Magnetron Frequency	133 kHz	$5.1 \mathrm{~kHz}$	$2 \mathrm{~kHz}$
Axial Frequency	200 MHz	940 kHz	$553 \mathrm{~kHz}$
Cyclotron Frequency	$150~\mathrm{GHz}$	86.5 MHz	79.6 MHz
Spin-flip Frequency	150.2 GHz	241 MHz	222 MHz

Table 2.2: Proton and electron experiment trap parameters

$$\omega_c' = \omega_c - \omega_m \,. \tag{2.13}$$

Though less pronounced than for an electron, the frequency hierarchy

$$\omega_c^{\prime} << \omega_z << \omega_m \tag{2.14}$$

holds in our proton trap, as summarized in Table 2.2.

2.1.3 Quantum Mechanical Description

Though in general our trapped-proton eigenmotions are in the classical limit of large quantum numbers, it is often convenient to rely on a quantum-mechanical description. In terms of the frequencies defined in the previous section, we can write

$$E_c = \hbar \omega_c' \left(n + \frac{1}{2} \right) \tag{2.15a}$$

$$E_z = \hbar\omega_z \left(k + \frac{1}{2}\right) \tag{2.15b}$$

$$E_{\ell} = -\hbar\omega_m \left(\ell + \frac{1}{2}\right) \tag{2.15c}$$

for the cyclotron, axial, and magnetron motions in a single quantum state.

A relevant experimental parameter obtained from this quantum picture is the temperature T for which $k_B T$ corresponds to a single-quantum increase in the energies above. A comparison with the recent electron experiment is given in Table 2.3.

	2006-2008 electron trap		2009 proton trap	
	frequency ν	$h u/k_B$	frequency ν	$h u/k_B$
Magnetron	$133 \mathrm{~kHz}$	$6 \ \mu K$	$5.1 \mathrm{~kHz}$	$0.25 \ \mu K$
Axial	$200 \mathrm{~MHz}$	$10 \mathrm{mK}$	$940 \mathrm{~kHz}$	$45 \ \mu K$
Cyclotron	$150~\mathrm{GHz}$	7.2 K	86.5 MHz	$4 \mathrm{mK}$

Table 2.3: Comparison of proton and electron thermal levels

Considering the thermal levels in Table 2.3, it is possible to cool a trapped electron to the cyclotron ground state, but the proton will remain in an excited thermal state even at dilution-refrigerator temperatures. We return to a classical picture to calculate the energy in each motion in terms of the oscillation amplitude. E_c is primarily kinetic; the energy in a cyclotron orbit of radius ρ_c is given by

$$E_c = m\omega_c^2 \rho_c^2 / 2 \,. \tag{2.16}$$

 E_z is likewise calculated from the amplitude of the axial motion; the energy in an axial oscillation of amplitude A is given by

$$E_z = m\omega_z^2 A^2 / 2 \,. \tag{2.17}$$

 E_ℓ is primarily a potential energy; the energy in a magnetron orbit of radius ρ_m is given by

$$E_{\ell} = \frac{m}{2} \left(\omega_m^2 - \frac{\omega_z^2}{2} \right) \rho_m^2 \approx \frac{-m\omega_z^2 \rho_m^2}{4} \,. \tag{2.18}$$

Note there is a difference in sign for the magnetron motion. While the cyclotron and axial motions are stable, allowing us to damp these motions as described in Chapter 4, the magnetron motion is unstable, such that reducing the magnetron energy actually increases the radius ρ_m . Fortunately the magnetron motion is effectively stable owing to a negligible radiation rate. To control the magnetron motion, we rely on sideband cooling techniques described in Chapter 6.

	2006-2008 electron trap	2009 proton trap
Magnetron	$\langle\ell\rangle=1.6\times10^4$	$\langle \ell \rangle = 1.7 \times 10^7$
Axial	$\langle k \rangle = 10$	$\langle k \rangle = 9.3 \times 10^4$
Cyclotron	$\langle n \rangle = 1$	$\langle n \rangle = 1.0 \times 10^3$

Table 2.4: Thermal average quantum numbers for the electron trap at 0.1 K and the proton precision trap at 4.2 K

Comparison of these classical energies with the quantum picture (Eq. 2.15) yields the average quantum number corresponding to a given oscillation amplitude for any of the trapped-proton motions. Table 2.4 presents this average quantum number for a thermal amplitude, assuming equilibrium with the cryogenic trap environment, which (in the absence of additional cooling) sets a lower bound on the proton temperature. Effective proton temperatures realized in the experiment are discussed in Chapter 7.

2.1.4 Measuring the g-factor

In addition to the axial, cyclotron, and magnetron frequencies, which correspond to physical motions of the proton in the trap, there is also a spin-flip frequency,

$$\omega_s = 2\pi\nu_s = \frac{g}{2}\frac{qB}{m} \,, \tag{2.19}$$

where $\hbar \omega_s$ is the energy difference between spin states s = 1 and s = -1, aligned or anti-aligned with the Penning trap magnetic field that sets the quantization axis. For a proton, this is a relatively accessible RF frequency ($\nu_s \approx 240$ MHz in the precision trap), but the spin-flip describes a single transition between quantum states, not a classical oscillation like the axial or cyclotron motion. Hence we do not observe spinflips using direct detection of an image current; instead we rely on the magnetic bottle technique described in the next section.

The proton g-factor is related to the ratio of the spin-flip frequency and the freespace cyclotron frequency, $g/2 = \omega_s/\omega_c$. To obtain our goal of measuring g_p to a ppb, we would need to measure both ω_s and ω_c to better than a ppb.

However, the cyclotron frequency measured in our experiment is actually the trap-modified ω'_c , shifted from its free-space value by the presence of the Penning trap electric field. To obtain the true cyclotron frequency with high precision, we rely on the Brown-Gabrielse Invariance Theorem for Penning traps [36], which states:

$$(\omega_c)^2 = (\omega_c')^2 + (\omega_z)^2 + (\omega_m)^2, \qquad (2.20)$$

valid even for a misalignment of the electric and magnetic axes and for harmonic distortions of the trapping potentials—the leading imperfections of a laboratory trap. The Invariance Theorem enables precision measurement of ω_c despite the complications of a Penning trap, but it also makes our experimental task more difficult, since now we must measure all three of the trapped-proton frequencies. Fortunately the frequency hierarchy works to our advantage, in terms of relative precision. For a ppb measurement of ω_c , the magnetron frequency ω_m must only be known to about 25%. Requirements for ω'_c (1 ppb) and ω_z (10 ppm) are stricter, but within range of our standard measurement techniques (Chapter 6). Previous work in our group has demonstrated measurement of ω_c with a precision of 9 parts in 10¹¹ [16].

2.2 The Magnetic Bottle

2.2.1 Theory and Application

A magnetic bottle [37, 1] is a substantial perturbation introduced in our otherwiseuniform magnetic field \vec{B} , of the form

$$\Delta \vec{B} = B_2 [(z^2 - \rho^2/2)\hat{B} - (\hat{B} \cdot \vec{z})\vec{\rho}].$$
(2.21)

As for electron measurements [38, 2], the purpose of the bottle term is to couple the proton spin state to its axial motion. The $-\vec{\mu}_p \cdot \Delta \vec{B}$ interaction now enters the axial Hamiltonian with the same z^2 dependence as qV(z), where V(z) is the harmonic oscillator potential given in Eq. 2.8. The axial frequency (Eq. 2.9) in the presence of the bottle thus becomes

$$\omega_z^2 = \frac{qV_0C_2}{md^2} - \frac{2\mu B_2}{m} , \qquad (2.22)$$

where μ is the total magnetic moment of the proton in the $+\hat{z}$ direction. While the proton spin is not the only source of this magnetic moment (see Section 2.2.2), a change from spin-up (aligned with \vec{B}) to spin-down (anti-aligned with \vec{B}) will modify the second term in Eq. 2.22 and produce a small shift in ω_z , given by

$$\delta\omega_z = \frac{2\mu_p B_2}{m\omega_z} , \qquad (2.23)$$

where μ_p is the magnitude of the spin magnetic moment of the proton. In the presence of a strong enough bottle, a proton spin-flip could thus be detected by carefully monitoring the axial frequency for a jump of the characteristic size $\delta \omega_z$.

To generate a magnetic bottle, we introduce ferromagnetic material into the trapping region. The ferromagnetic material saturates in the strong field of the solenoid, producing a bottle field as described below. In past electron experiments, nickel rings concentric with the trap axis were used for this purpose. In the proton experiment, to obtain the largest possible B_2 , we build the bottle directly into the trap by machining our ring electrode from high-purity iron.

The geometry of the iron electrode determines the strength and profile of the magnetic bottle field. The magnetic field of a small ring can be derived from a magnetostatic potential,

$$\Psi(\vec{r}) = -\sum_{l=1}^{\infty} l^{-1} B_{l-1} r^l P_l(\cos \theta) = -\sum_{l=1}^{\infty} l^{-1} B_{l-1} z^l \text{ on axis.}$$
(2.24)

The corresponding magnetic field is

$$\Delta \vec{B} = -\vec{\nabla}\Psi = \sum_{l=1}^{\infty} B_{l-1}r^{l-1}P_l(\cos\theta) \,\hat{r} + \hat{\theta} \,\operatorname{term} = \sum_{l=0}^{\infty} B_l z^l \,\hat{z} \,\operatorname{on}\,\operatorname{axis.} \tag{2.25}$$

The bottle field $\Delta \vec{B}$ is thus expressed in terms of bottle coefficients B_l ; in particular, B_2 indicates the strength of the desired z^2 perturbation.

The saturated ferromagnetic material that produces our bottle can be viewed as a superposition of rings of magnetic dipoles, aligned in the \hat{z} direction. The scalar potential in cgs units due to a magnetic dipole $\vec{p} = |p|\hat{z}$ at a point (ρ, z) in cylindrical coordinates is given by

$$\Psi(\rho, z) = \frac{\vec{p} \cdot (\vec{r} - \vec{r'})}{(r - r')^3} = |p| \frac{z - z'}{((z' - z)^2 + (\rho - \rho')^2)^{\frac{3}{2}}}, \qquad (2.26)$$

where (ρ', z') is the position of the dipole. A series expansion of $\Psi(\rho, z)$ in z can be written conveniently in terms of Legendre polynomials,

$$\Psi(\rho, z) = -|p| \sum_{n=0}^{\infty} P_{n+1}(z'/r') \left(\frac{1+n}{(r')^{2+n}}\right) = -|p| \sum_{n=0}^{\infty} P_{n+1}(\cos\theta) \left(\frac{1+n}{(r')^{2+n}}\right) . \quad (2.27)$$

Comparing on-axis with Eq. 2.24, we have

$$B_{l} = |p| \frac{(l+1)(l+2)}{(r')^{l+3}} P_{l+2}(\cos \theta')$$
(2.28)

for the bottle coefficients due to a single dipole at (r', θ') .

Our actual bottle consists of rings of ferromagnetic material. The dipole strength of a ring at (ρ', z') with magnetization (magnetic moment per unit volume) M is given by $|p| = M \cdot 2\pi \rho' d\rho' dz'$. The total bottle coefficients are found by integrating over all such rings in our bottle, to obtain [1]
$$B_{l} = (l+1)(l+2)M \int 2\pi \rho' d\rho' dz' (r')^{-l-3} P_{l+2}(\cos\theta') . \qquad (2.29)$$

The form of this expression aids in construction of the magnetic bottle. If the bottle is symmetric in z around trap center, B_l will vanish for odd l. We are thus left primarily with B_2 , the desired bottle term, and B_0 , an constant offset which does not influence the bottle coupling (but does significantly change the overall field in our analysis trap).

To maximize B_2 , we must place ferromagnetic material as close to trap center as possible, and within regions of $P_4(\cos \theta') > 0$ (Fig. 2.3). It suffices to use the ring electrode itself as the bottle. Our final design is represented in Fig. 2.3. Further increasing the radial extent of the bottle yields diminishing returns in B_2 , while producing an undesirable increase in the force required to insert or remove the experiment from the superconducting solenoid.



Figure 2.3: Lines of $P_4(\cos(\theta)) = 0$ superimposed on our spin-flip analysis trap.

The coefficients B_0 and B_2 for our bottle are determined by numerically integrating Eq. 2.29. Comparison with the total magnetic field calculated using a boundary element method (RADIA code) demonstrates that the bottle profile is well-described by the B_2 gradient, within 1 mm of trap center (Fig. 2.4). The calculated B_2 is 78000 gauss/cm², using a saturation value of M = 1714 emu/cm³ for high-purity iron. The maximum force on the iron ring as the trap is removed from the solenoid field is estimated to be 24 N (\approx 5 lbs), an easily managed force.



Figure 2.4: Magnetic field generated by the proton experiment magnetic bottle. The analytic solution (Eq. 2.29) up to the B_2 term of interest is numerically integrated and compared to a full numerical model calculated using RADIA.

2.2.2 Complications of the Magnetic Bottle

Though essential for spin-flip detection, the strong magnetic bottle perturbation introduces several unwanted effects. First, the magnetic bottle couples indiscriminately to the total (spin + orbital) magnetic moment. Along with the s dependence of interest, there are also contributions from the cyclotron and magnetron orbits:

$$\Delta\omega_z = \delta_s \left[\frac{gs}{4} + n + \frac{1}{2} + \frac{\omega_m}{\omega'_c} \left(\ell + \frac{1}{2} \right) \right] , \qquad (2.30)$$

where

$$\delta_s = \frac{\hbar\omega_z}{2m\omega_m} \frac{B_2}{|B(0)|} \frac{\omega_c}{\omega'_c - \omega_m} \approx \frac{4\mu_p B_2}{g_p m_p \omega_z} \,. \tag{2.31}$$

A spin-flip (s = 1 \rightarrow s = -1) produces the axial frequency shift given in Eq. 2.23, which is 60 mHz in our proton analysis trap (Chapter 7). However, note that an even larger shift can result from a cyclotron excitation, e.g. $\Delta \omega_z = 2\pi \times 65$ mHz for $\Delta n = 3$, a change of as little as $\Delta \rho_c = 1$ nm at our typical cyclotron radius $\rho_c = 500$ nm. The cyclotron temperature must be stable to $\Delta T_c < 12$ mK to avoid such a shift. Axial frequency shifts due to magnetron state ℓ , though greatly reduced by the ratio ω_m/ω'_c , are also visible in our experiment. In the magnetron case, a shift $\Delta \omega_z = 2\pi \times 60$ mHz results from $\Delta \ell \approx 10^5$, a change of $\Delta \rho_m \approx 1 \ \mu m$ at our typical magnetron radius $\rho_m = 12 \ \mu m$. Observations of cyclotron and magnetron effects in the presence of our strong magnetic bottle are discussed in detail in Chapter 7.

The second undesirable effect of the bottle is a substantial broadening of the cyclotron and spin lines. The bottle adds a z^2 dependence to the magnetic field in the Penning trap, such that $B(z) = B(0) + B_2 z^2$ on-axis. A proton oscillating in z thus samples regions of different magnetic field. While we do not explicitly excite the axial motion during a cyclotron or spin measurement, the unavoidable thermal amplitude gives rise to a "bottle broadening" parameter

$$\Delta\omega = \omega(0) \frac{B_2}{B(0)} \frac{k_B T_z}{m\omega_z^2} , \qquad (2.32)$$

$ ho_0$	B_2	$\nu_z/\sqrt{V_0}$	$\delta_s \sqrt{V_0}$
(mm)	$(gauss/cm^2)$	(Hz/\sqrt{V})	$(\text{Hz} \cdot \sqrt{V})$
6	28900	224783	0.055
4.5	51400	299711	0.073
3	115600	449566	0.110
2	260100	674349	0.165
1.5	462400	899133	0.220
1.2	722600	1124000	0.275
1	1040500	1349000	0.330

Table 2.5: Upper limit on magnetic bottle strength and spin-flip shift attainable with various trap sizes

valid for any trap frequency (ω_s , ω'_c) that depends on the magnetic field. In our proton bottle, assuming an axial temperature of 4 K, this broadening effect is roughly 40 ppm for the cyclotron and spin-flip lines, which greatly exceeds our target precision of 1 ppb and motivates the double-trap scheme described in Section 2.3.

2.2.3 Choice of Trap Size

From the form of Eq. 2.29, the strength of the magnetic bottle will increase as trap size decreases and the ferromagnetic ring moves closer to trap center, approximately as $1/\rho^2$. Table 2.5 shows the bottle strength obtainable with various trap diameters, for an ideal bottle configuration that fills the entire region shown in Fig. 2.3. In practice, space constraints due to neighboring electrodes and spacers reduce the bottle strength by 30-40% in the final design. For example, our proton trap has radius 3 mm, but B_2 of only 78000 gauss/cm². One proposed alternative trap (Chapter 10) has radius 1.5 mm, but B_2 of only 286000 gauss/cm². Table 2.5 suggests using a small trap for the proton experiment. However, we expect various unwanted effects to increase with diminishing trap size. Machining tolerance does not improve as we scale down, magnifying the perturbations introduced in our trapping potential by gaps, imperfections, and misalignments of the electrodes. Patch effects on the inner surface of the electrodes become a greater concern. And a stronger bottle is hardly an unmitigated benefit, as described in Section 2.2.2. We thus desire the smallest possible bottle for which a spin flip can be observed. For our first-generation proton experiment, we selected an inner diameter of 6 mm ($\rho_0 = 3 \text{ mm}$). This trap size, half the previous lab standard, seemed large enough to keep the Penning trap electrostatics well-behaved, while still small enough to give some chance of detecting a spin flip. The optimal trap size for proton g-factor measurement remains an open question (Chapter 10).

2.3 Double-Trap Sequence for the Proton Measurement

As described in Section 2.2.2, the strong magnetic bottle needed for spin-flip detection produces an unwanted broadening effect in the spin and cyclotron lines. This broadening would severely limit the precision of g-factor measurements performed in the field of the bottle. To avoid this problem, we rely on a double-trap scheme as used for measurements of the g-factor of electrons bound in heavier ions [39], in which magnetic bottles about 10 times smaller than our B_2 have been utilized. The double-trap scheme requires two spatially separated Penning traps. The "analysis" trap contains a strong magnetic bottle as described above. The "precision" trap has a standard copper ring electrode, no added bottle perturbation, and hence no bottle-broadening effect.

Measurement of the g-factor could proceed in the following sequence:

- 1. Detect the spin state in the analysis trap. By driving a spin flip and observing the direction of the change in axial frequency in the presence of the bottle, we can determine the spin state $s = \pm 1$ of the proton.
- 2. Transfer the proton to the precision trap. Transferring adiabatically from the analysis trap to the precision trap will not affect the spin state.
- 3. Flip the spin by driving at the expected transition frequency in the precision trap.
- 4. Transfer back to the analysis trap.
- 5. Detect the spin state in the analysis trap. By again observing the direction of axial frequency shift when we drive a spin-flip transition, we can determine the spin state $s = \pm 1$ of the proton and see if it has changed since the initial detection step. A change in *s* would indicate a successful spin flip in the precision trap.
- 6. Repeat the process to build up a histogram of spin-flip successes vs. drive frequency in the precision trap, thus measuring the spin-flip lineshape.

Note that the spin-flip "measurement" takes place in the precision trap, and we only use the strong magnetic bottle to analyze the results in the analysis trap. For a g-factor measurement, we would also measure the proton cyclotron frequency in the precision trap, using techniques described in Chapter 6.

A side benefit of this technique is that it allows us to avoid having a cyclotron amplifier in the analysis trap. In the presence of the bottle, a change of 2-3 cyclotron quanta produces an axial frequency shift equivalent to a spin-flip (Section 7.3.1). During a spin-flip measurement, then, the proton cannot be allowed any means of exchanging cyclotron energy with the 4.2 K environment of the trap. With a tunedcircuit cyclotron amplifier in the analysis trap, we would need a way to reliably decouple the effective damping resistance from the proton during a spin-flip measurement. (A mechanical switch would be ideal but is difficult to realize at cryogenic temperatures.)

As a necessary condition for our goal of ppb measurement, any magnetic bottle in our precision trap must be small enough $(B_2 < 25 \text{ T/m}^2)$ to keep the bottlebroadening effect (Eq. 2.32) to a few ppb, or $\Delta \omega_s/2\pi \approx 1 \text{ Hz}$. The field of the analysistrap iron ring produces an unavoidable bottle at the location of the precision trap. Increasing the distance between traps greatly suppresses this bottle in the precision trap, but the separation is limited in practice. Both traps must be kept near the center of the solenoid field, and larger distances also necessitate additional transfer electrodes for moving the proton between traps. Also, to null out possible unwanted effects of the B_1 linear gradient, we place a second iron ring symmetrically below the precision trap, which cancels B_1 but doubles B_2 . Our electrodes are designed for $B_2 \approx 1 \text{ T/m}^2$ in the precision trap, though this depends on a reduction due to negative contributions to B_2 from the macor spacers, and magnetization data for macor at 4.2 K is not well known. The value from the iron rings alone, $B_2 = 7.5$ T/m², sets an upper bound on the precision trap bottle. Though still sufficient to keep bottle broadening at a ppb, this B_2 would produce a bottle shift due to cyclotron energy [1] that is only smaller by a factor of two than the relativistic shift (Eq. 6.1), potentially complicating the measurement of ω'_c .

Chapter 3

Experimental Apparatus

Though the proton in our g-factor experiment is confined to a cylindrical region of diameter 6 mm, the full experimental apparatus (without supporting roomtemperature electronics) requires a volume roughly 1 million times larger. An overview schematic of the cryogenic apparatus is shown in Fig. 3.1.

The Penning trap electrodes are housed in a UHV vacuum enclosure, called the "trap can." DC and RF connections to the trap are made via vacuum feedthrough pins in the "pinbase." On the other side of the pinbase is our cold electronics "tripod" region, containing filters and tuned circuit amplifiers. The LHe dewar cools the tripod and trap regions to 4 K by conduction. DC and RF lines run up through the dewar to the experiment "hat," where we make electrical connections at room temperature. The entire assembly fits inside the 4" bore of a superconducting solenoid. Its main solenoid, secondary solenoid, shim coils, and added shielding coil [40] provide a uniform 5.6 Tesla field at trap center.



Figure 3.1: Cross-sectional view of the experimental apparatus.

3.1 Trap Electrodes

The electrostatic portion of our Penning trap is generated by applying voltages to a series of cylindrically symmetric electrodes, the electrode "stack." The stack for our proton g-factor experiment (Fig. 3.2) consists of two open-endcap Penning traps, sandwiched between the field emission point (FEP) and "PLATE" electrodes used for loading protons. Transfer electrodes allow us to move the proton between the two traps. Macor spacers between the electrodes provide electrical isolation. In the case of half-split electrodes, precision sapphire balls are used as standoffs between the two halves. Gaps between electrodes are made as small as possible to minimize deviation from the electrostatic potential of perfect cylinders.

The stack is bolted against the bottom of the pinbase. BeCu springs maintain compression as the trap dimensions change during thermal cycling.

3.1.1 Electrode Machining and Processing

The strong magnetic bottle in the analysis trap is generated by a ring electrode (ARING) made from high purity iron. Iron has the highest saturation magnetization among pure metals. Alloys such as Permendur offer saturation magnetization roughly 10% higher still, but unlike a pure metal, the conductivity of an alloy does not improve dramatically at cryogenic temperatures. Although we prefer the higher conductivity, resistive losses may actually be manageable in either case, given that tuned-circuit detectors have now been demonstrated to reach $Q \approx 5000$ while attached to the endcap of a trap with an alloy ring electrode [41].

All other electrodes in the stack are made from OFE grade OFHC (oxygen-free



Figure 3.2: The electrode stack for the proton g-factor experiment.

high conductivity) copper. Critical dimensions are machined to a tolerance of 5 microns (0.0002"). We then polish the inner surface of each electrode (the surface exposed to the proton) to roughness of less than 1 micron (0.00004"). To avoid distorting our small electrodes during the polishing process, we slip-fit each electrode into a custom teflon ring, then clamp the teflon into the collet of a lathe. We polish each electrode to a mirror finish by using lapping paper of decreasing grit size and finishing with Simichrome paste. This polishing process requires roughly 1.5 hours per electrode.

The splitting cut for half-split electrodes (the precision trap ring electrode, plus compensation electrodes in both traps) is performed following this polishing step. The electrodes are then cleaned carefully, and bias wires are attached via brazing in a hydrogen oven.

3.1.2 Gold Evaporation

To improve conductivity and prevent oxidation, our research group has often electroplated the polished inner electrode surfaces with a thin layer of gold. For this experiment, we instead developed a new technique to deposit a thin film of gold via thermal evaporation. Our main concern was the possibility of patch effects, which are DC and/or RF potential fluctuations that arise because the surface of a real metal has some grain structure and is not a true equipotential. The proton g-factor experiment requires an unusually small trap and unusually stable trapping potentials, making us especially sensitive to patch effects.

In our experiment and many others, patch potentials are considered a noise source to be minimized and then ignored. An unfortunate consequence is that the patch effects are often not particularly well understood. Patch potentials have been directly investigated in several systems, e.g. a capacitive Kelvin probe near conducting surfaces [42, 43], ions in an RF (Paul) trap [44], and neutral atoms in a cavity [45]. In general, the size of a patch potential voltage scales directly with the size of the feature responsible for the patch. Compared to physical deposition methods such as thermal evaporation, electroplating tends to produces surfaces of larger surface roughness and grain size. Electroplating is also conformal, reinforcing whatever grain boundaries are present in the base material; in our case, copper electrodes which have been annealed at high temperature (a process that increases grain size) during our hydrogen-brazing step. While advanced techniques such as pulsed plating have been shown to reduce grain size of an electroplated surface [46], thermal evaporation is generally preferred to electroplating for applications with stringent surface-quality requirements, for example in atom chips [47] or surface-electrode ion traps [48].

Careful characterization of patch potentials would be a thesis project unto itself; in fact, a recent effort has been launched at MIT with the goal of quantifying patch effects as a source of decoherence in ion traps for quantum information processing [49]. While we are unable to know for certain if patch potentials are a limiting noise source in our experiment, we have selected thermal evaporation for our electrode processing, based on the above concerns and following initial tests in which we obtained qualitatively better surface finishes with evaporation compared to electroplating.

The standard configuration for thermal evaporation involves a tungsten boat that holds a pellet of gold inside an evacuated jar. Current is sourced through the boat to produce resistive heating, which melts the gold and sends gold vapor outward in all directions. A sample mounted high enough above the boat then acquires a gold film that is basically uniform.

Evaporating gold on the 6 mm inner diameter of an electrode, however, requires a somewhat different approach. The standard boat is too large to fit inside an electrode, and the electrode walls would be too close to the gold pellet for an even coating. After several trials, we developed the setup shown in Fig. 3.3. Evaporation takes place inside a Sharon Thermal Evaporator in the Harvard CNS Cleanroom. Instead of the tungsten boat and gold pellet, we use a tungsten filament (0.040" welding rod). A controlled amount of gold is electroplated onto the filament, using a platinized screen anode in a bath of 3 parts deionized water and 1 part TG-25 RTU gold plating



Figure 3.3: Setup for evaporating gold on the inner surface of trap electrodes. Electrodes are shown mounted on the alignment jigs inside the thermal evaporator. The gold-plated tungsten filaments are loosely clamped at each end to the evaporator bus bars.

solution (3.17% Sodium Gold Sulfite, 8.4% Sodium Sulfite). The filaments are plated for 40 minutes at a current of 0.90 mA and a bath temperature of 40-50° C.

We designed a high-temperature mounting jig, made from steel and high temperature alumina ceramic beads, to hold the electrodes and filaments during evaporation. The ceramic beads serve to insulate the plated tungsten filament electrically and thermally, and also to maintain alignment of the filament along the axis of the cylindrical electrodes. The entire assembly is carefully positioned in the thermal evaporator and the filament is clamped lightly to the evaporator bus bars. The loose connection here is necessary to avoid a kink observed to form in the filament due to thermal expansion during the evaporation process. As the filaments are heated with roughly 20 A, the gold boils off radially outward in all directions, producing an even coating on the inside of the surrounding electrodes. The boiloff process lasts one minute, and total time to ramp the evaporator current up and back down is roughly 2 minutes. The heating time is kept short to avoid overheating the trap electrodes, which sit only a few millimeters from the filament.

We estimate the amount of gold deposited on the electrodes by weighing the filaments and electrodes at all stages of the process, using a precision balance with 0.1 milligram resolution. The target thickness of the evaporated layer is 100 nm, chosen to limit the possible grain size in the gold film. On this scale (small compared to machining/polishing tolerances), slight variations in the thickness are relatively noncritical.

The most common failure mechanism of the thermal evaporation process was the formation of oxide spots on the electrode surface, which would rub off to reveal patches of bare copper not coated with gold. Though not investigated in detail (electrodes with such spots could be repolished, and the evaporation step repeated until a good surface was obtained), we attempted to minimize this oxidation by keeping the electrodes and filaments as clean as possible, re-cleaning all evaporation materials with isopropanol and air duster in the Cleanroom immediately before evaporation. In later trials with silver electrodes [50], a heat-treating step was also added to the evaporation process. Before positioning the trap electrodes and performing the actual gold evaporation, the gold-plated filaments were heated in an otherwise-empty thermal evaporator, to some temperature below the melting point of gold but high enough to boil off possible impurities from the tungsten.

3.1.3 The Field Emission Point

A field emission point (FEP) is mounted in a collet at the top of our electrode stack (Fig. 3.2). The FEP is made by etching a 0.020" tungsten welding rod in NaOH solution, following the basic procedure outlined in Appendix A of reference [35], to produce a tip that is sharp on the atomic scale. A sharp FEP will "fire" at high voltage, as electrons tunnel out of the tip, at a rate of order 1 nA with -400 V on the FEP relative to the surrounding electrodes. In the strong magnetic field of our experiment, this produces a collimated electron beam used for loading protons as described in Chapter 5.

To prepare our FEP, we first etch a batch of several candidate tips. Each tip is tested in vacuum by biasing to high voltage and watching for field-emission current on a grounded copper surface nearby. Without the guiding magnetic field that is present in the actual experiment, we must position this copper target extremely close to the FEP in order to effectively collect current; separation of order 1 cm is typical. Ionization in the relatively poor vacuum of our test setup can damage the tips, so care must be taken to stop the test at the first sign of field-emission. The characteristic I-V curves shown in Chapter 5 are traced out only later, once the FEP is operating in the better vacuum of the experiment trap can. While testing, we typically stop after observing some consistent current on the order of 10-100 nA. A satisfactory FEP will fire at voltage below 1 kV. To check that we are seeing field-emission rather than vacuum breakdown, we reverse polarity of the high voltage to confirm that no current is collected at voltage of equal magnitude but opposite sign.

3.2 Cryostat and Trap Can Vacuum

3.2.1 Maintaining Low Temperature

As is typical for a Penning trap experiment, we require cryogenic temperatures for reasons of signal/noise and vacuum quality. Unique to our proton experiment is the effect of temperature on transfer between our two traps; as described further in Chapter 7, the thermal distribution of cyclotron states in PRING determines the range of axial frequencies over which we must search for the proton after each transfer to ARING. At room temperatures or even at 77 K, this spread would be prohibitively large for the repeated transfer and detection sequence necessary for an eventual gfactor measurement.

In the electron g-factor experiment, a dilution refrigerator is utilized to operate at temperatures around 100 mK. The primary benefit of this low temperature is quantum control of the electron cyclotron motion, which drops essentially to its ground state. For a proton, however, cooling to 100 mK is less critical, as the proton cyclotron motion still remains in a highly excited state (Table 2.3). Moreover, the proton cyclotron state is expected to remain fixed during attempts at spin-flip detection, due to the absence of any cyclotron amplifier in the analysis trap. Thus, for simplicity of the apparatus, we elect to avoid a dilution refrigerator and to operate instead at 4 K, which can be obtained (relatively) easily with liquid helium.

To maintain temperature of 4 K in the experiment region, we fill the 4-liter experiment dewar with liquid helium, which cools the tripod region and electrode stack by conduction. To reduce heat loads from the room (300 K), we allow no significant conductive paths from the experiment to the hat. Before cooling to 4 K, the magnet bore region surrounding the experiment is pumped down to 10^{-6} torr. The bore tube is cooled to 77 K by contact with an auxiliary liquid nitrogen dewar. Wiring between the hat and the experiment is limited to approximately 100 constantan wires (0.002" diameter) and 8 stainless steel microcoax lines (UT-34-SS, 0.008" center conductor with 0.034" diameter sheath). The experiment dewar is attached to the hat via thin G-10 rods and stainless steel bellows. Three flat copper radiation baffles, cooled by the exhausting helium gas, reduce heat load between the hat and the experiment dewar. The uppermost plate is thermally anchored with BeCu fingers to the 77 K magnet bore. The lower plates are floating, thermally connected only to each other. Similarly, a baffle placed in the bottom of the magnet is anchored to the 77 K bore in order to reduce heat load from the magnet bottom flange (300 K) to the trap can (4 K). Finally, a radiation shield, wrapped in aluminized mylar superinsulation, surrounds the 4 K region of the experiment (dewar, tripod, and trap can) in order to reduce radiative heating from the 77 K magnet bore.

To cool down the experiment in a cost-effective way, we precool overnight by filling the auxiliary magnet dewar and the experiment dewar with liquid nitrogen, then cool to 4 K the following day with repeated slow LHe fills. A typical cooldown from room temperature takes roughly 24 hours and 50 liters of liquid helium. Temperatures of the copper baffles I (top) and II (middle) are monitored using platinum RTDs (resistance temperature detectors) with a four-wire technique. Temperatures of copper baffle III (bottom) and the tripod region are measured with carbon-glass RTDs.

The hold time of the experiment dewar is approximately 50 hours; in practice,

we refill the LHe once every two days. This hold time, however, depends critically on the cryostat functioning properly. In one iteration of the experiment, an added hat spacer prevented the BeCu fingers from contacting the 77 K bore tube. The top copper baffle floated up as high as 150 K rather than its usual 120 K, and the helium hold time subsequently dropped to a mere 20 hours. In another case, a leak in one of the hat feedthrough flanges softened the bore vacuum by two orders of magnitude, and hold time dropped to roughly 40 hours.

3.2.2 The Trap Can Vacuum

In order to avoid unwanted interactions with background gas, good vacuum in the trapping region is essential. The trap can is constructed using indium seals to maintain UHV in a 4 K environment. The pinbase serves as the top flange of the trap can. It contains three unused ports, 3 high voltage (5 kV) vacuum feedthrough pins, and 51 standard (500 V) vacuum feedthrough pins, all attached via hydrogen brazing with Ag-Cu eutectic. The trap can bottom flange contains two unused ports and a central pump-out/pinch-off port. We pump out the trap can to the 10^{-7} torr level at room temperature, then seal off the copper pump-out tube with a pinch-off tool that creates a permanent cold-welded seal. Once lowered into the magnet and cooled to 4 K, cryopumping improves the vacuum in the trap can by several orders of magnitude (5 x 10⁻¹⁷ torr based on antiproton lifetime in a similar apparatus [51]).

Even a small vacuum leak in the trap can causes problems with loading and detection of protons. Signatures of such vacuum problems are described in Chapter 5.

3.3 Electrical Connections

3.3.1 DC wiring

To produce the potentials needed for trapping and transferring protons, a separate DC bias is applied to each electrode in the stack. Depending on the electrode, this bias is supplied by a Fluke 5440 series precision voltage calibrator, a BiasDAC channel (range +10 V to -10 V), or a "SuperElvis" high voltage amplifier (range -1 kV to +1 kV). The DC line is heavily shielded and filtered to avoid transmitting noise down to the trap. The signal is first carried from the electronics rack to the experiment hat via shielded 2-pin Lemo cables, which are further bundled inside electrically grounded aluminum tubes (adapted from dryer vent ducts). At the hat, each line passes through an RF choke, then a feedthrough into the bore vacuum space. Inside the bore vacuum space, twisted-pair constantan wires run from the hat down to the tripod region. At the tripod, each line passes through an LC-RC low-pass filter board, and finally another RC low-pass filter directly above the pinbase feedthrough. Inside the trap can, gold plated OFHC copper straps are used for the final connection to the electrodes.

To minimize ground loops, the pinbase ground plane serves as the common ground for all lines. Rather than referencing to the chassis ground of our voltage sources or equipment rack, the "high" and "low" of each DC line are separately filtered, until the "low" is finally grounded at the pinbase. The ground plane of each filter board is soldered through thick copper straps to the pinbase ground.

Stability of the ring and endcap voltages is critical for the trapping potential. A

shift of as little as 350 nV on the ring-endcap trapping potential, roughly 200 ppb, would produce an axial frequency shift of 60 mHz, enough to obscure a spin-flip in our analysis trap. In normal operation, the endcaps are tied directly to the pinbase ground via a 10 M Ω resistor. We can apply nonzero voltage as needed during transfer, but after removing the voltage source, the endcap is pulled down to pinbase ground. With the endcaps grounded, the ring voltage thus determines stability of the trapping potential.

Our starting point for obtaining an ultra-stable ring voltage is to mimic the setup of the electron g-2 experiment. We use a cold 10 μ F capacitor (Vishay MKP-1840 series metallized polypropylene), charge-pumped by a Fluke 5440 precision voltage calibrator, to provide a stable ring bias with a long RC time constant. Great care is taken with the DC bias path in order to minimize leakage resistances to ground. A large leakage resistance R_L is desirable because R_L is not guaranteed to be stable. The presence of finite R_L forms an unavoidable voltage divider with our resistors (typically 2 M Ω) in the DC bias line, causing a slight reduction in the voltage applied to the ring electrode compared to the value specified at the hat. By maximizing R_L we minimize this voltage divider effect, in particular the shift in trapping potential that results from a given fluctuation in R_L . At the scale of leakage resistance obtained in our proton experiment, fluctuations in R_L should produce axial frequency shifts below the 1 mHz level.

DC lines for the two ring electrodes are treated specially to produce the highest possible R_L to ground. Unlike the other DC lines, which are bundled into 32-pin feedthroughs at the hat, the ring electrode bias lines use a separate hat feedthrough and travel down to the tripod through isolated constantan bundles. In the tripod filter boards for the ring lines, we replace the 1812 NPO capacitors with ATC microstrip capacitors that have smaller capacitance but higher leakage resistance. Bad filter board capacitors were observed to limit leakage resistance to $10^{10} \Omega$, while use of a standard 32-pin connector instead of dedicated feedthrough pins at the hat limited leakage to $10^{13} \Omega$. The increase in LC cutoff frequency from using smaller ATC capacitors is more than compensated for by the pinbase RC filter, which on the ring and endcap lines is increased to give $\tau_{RC} \approx 10$ sec.

Leakages much above $10^{12} \Omega$ are difficult to measure at room temperature due to the range of our electrometer, so identifying and removing all the low-leakage components is somewhat iterative, requiring several cooldown cycles. Once the experiment is cold, the proton provides the most sensitive available test of the leakage. We can unplug the ring electrode from the Fluke supply, and watch for a decay in the proton axial frequency caused by the ring voltage decay $V(t) = V_0 e^{-t/R_L C}$, where R_L is the leakage resistance and C is the filter capacitance to ground (10 μ F for our ARING electrode). With the ring line fully optimized for high leakage, we observe $R_L \approx 10^{16} \Omega$, comparable to the $6 \times 10^{15} \Omega$ leakage obtained in the electron experiment [50].

The full wiring diagram as of late 2009 is shown in Fig. 3.4 and Fig. 3.5. The 1 $M\Omega$ resistor between top and bottom compensation electrodes in each trap allows us to use a single DC line to bias both compensation electrodes (which are typically held at the same potential), and also serves as a continuity check.



Figure 3.4: Trap wiring diagram, upper stack. Amplifier detail for the large axial amp is shown in Fig. 4.6.



Figure 3.5: Trap wiring diagram, lower stack. Amplifier detail is shown in Fig. 4.7 (small axial amp) and Fig. 4.8 (cyclotron amp).

3.3.2 RF wiring

To excite and detect the various oscillations of the trapped proton, RF drives are applied to several of the electrodes. PTS-250 and SRS DS345 frequency synthesizers supply the drives, which travel through double-shielded coaxial cables (inside grounded dryer vent duct) to the hat. From the hat, the axial and sideband drives are sent down through twisted pair constantan wires, while the cyclotron and spin-flip drives travel through stainless steel microcoax to reduce losses at the higher frequencies.

Drives must be applied to an electrode with the proper geometry for the motion involved. A cyclotron drive is applied to one half of the split ring in the precision trap. Axial drives are applied to an endcap electrode in each trap. Axial-magnetron "sideband" drives are applied to one half of a split compensation electrode in each trap. Spin-flip drives are applied to a compensation electrode in each trap, to generate a component of magnetic field perpendicular to the vertical trap axis and uniform \vec{B} field (Chapter 9).

In the axial and cyclotron cases, we attach tuned-circuit amplifiers to detect the resulting proton motions. These amplifiers, described in detail in Chapter 4, are basically inductors used to tune out the inherent capacitance between trap electrodes, creating a large effective parallel resistance at some resonance frequency. The resonance frequency is chosen to be equal to the proton frequency, such that when the proton oscillation produces small image currents in the electrode walls, the transimpendance amplifier converts these currents into a voltage large enough for detection. This detected signal is subsequently amplified by a field-effect transistor (FET) and sent via microcoax up to the hat for analysis.

In terms of wiring budget, each amplifier requires two additional lines: a DC line for the FET gate bias, and a DC/RF coax for the FET drain bias and signal output. We use three amplifiers: a cyclotron amplifier, connected to the non-drive half of the split precision trap ring, and two axial amplifiers, connected to a compensation electrode in each trap.

RF wiring is complicated by the effective coupling between neighboring electrodes that arises from trap capacitances on the order of a few pF. For both drives and amplifiers, we must generally consider the wiring on all neighboring electrodes, in order to determine whether a signal at a particular frequency will see impedances that allow it to travel along the intended path.

Details of the RF wiring are shown in Fig. 3.4 and Fig. 3.5. The three-inductor series that appears in the filter boards and also between halves of split compensation electrodes is designed to ensure a large inductive reactance at all frequencies. In particular, the 33 μ H Coilcraft inductors have self-resonant frequency around 20 MHz, above which they begin to behave like capacitors.

Physical space in the tripod region presents a constant challenge, as we have the same general wiring requirements as the electron g-factor experiment, but with 3 times as many amplifiers and roughly 4 times as many electrodes. To accommodate the extra components, we start with a tripod that is 50% longer than the 6 inch lab standard. Double-sided DC filter boards are positioned at angles to the tripod legs around the amplifiers. While this configuration is inconvenient for mounting the filter boards and particularly unpleasant for modifying the inward-facing lines, it allows us

to utilize nearly the full cylindrical volume of the tripod region.

Careful checks are essential before any cooldown because of the close proximity of wires in the tripod region. We test the DC bias on each electrode and check that each RF drive reaches the intended pin with minimal feedthrough/crosstalk to neighboring electrodes. The last step before cooldown is to test the leakage resistance on each line, both before and after attaching the radiation shield, to reveal any electrical shorts.

3.4 The Magnetic Field

Besides the electrostatic trapping potential, the other essential component of a Penning trap is a strong axial magnetic field. We use a self-shielding superconducting solenoid [52] to produce a field of 5.6 Tesla. This is close to the maximum field of our solenoid (roughly 6 Tesla), since a large field is preferable to minimize the fractional effect of small fluctuations. We chose our particular operating point to set the proton cyclotron frequency at 86.5 MHz, in a relatively quiet region of the electromagnetic spectrum. (This is below the FM radio band and within the region allocated to television channel 6, which in Cambridge is relatively weak.)

The magnetic field is set and shimmed using NMR with a water sample. The magnet contains large and small main coils, plus a number of superconducting shims used to produce a maximally homogeneous field in a small volume near field center. When setting the magnetic field, we are primarily concerned with the field profile over the 1.86 inches between ARING and PRING, which could affect proton transfer between the traps. The field curvature is controlled by the ratio of large and small coils. After optimizing this ratio, we obtained the field profile shown in Fig. 3.6.

For this prototype phase of the experiment, we have placed the analysis trap in the maximally homogeneous region; accounting for thermal contraction and the travel range of our adjustable hat spacer, ARING is essentially located at field center. The precision trap sits in a somewhat larger field gradient, slightly outside the region where we measure a clear water NMR signal. Note that the magnetic fields measured with NMR differ slightly from the values observed in the experiment, primarily due to the effect of iron in our electrode stack. In the analysis trap, the iron ring electrode reduces the field magnitude by 8%. In the precision trap, the contribution from the two distant iron rings increases the field magnitude by 0.5 ppt.



Figure 3.6: Field map after shimming of magnet 51x, measured using a water NMR probe. The analysis trap is coincident with the superconducting shim center.

We moved magnets in the summer of 2008; the original magnet lacked a selfshielding coil. We did not notice any direct improvement with the addition of the shield coil, but the experiment before this point was not particularly sensitive.

3.5 Alignment Issues

The bulk of our apparatus hangs from the experiment hat, with a long enough moment arm that even a slight tilt can produce a significant misalignment at the location of the trap. Initial shimming is done by wedging pieces of copper foil between the experiment dewar and the tripod top flange, to ensure the trap can is centered beneath the hat. With proper alignment, the radiation shield slides on evenly, while nylon standoffs maintain concentricity of the radiation shield and trap can.

This rough alignment only ensures that the experiment is straight enough to avoid thermal contact shorts. For precision positioning of the trap, we make use of an adjustable hat spacer. This spacer consists of a bellows that can be tilted using three equally spaced screws, and/or translated by as much as 20 mm full scale. The alignment of interest is between the axis of the electrode stack and the direction of the magnetic field. In principle there are several methods to probe this alignment *in situ*. Precision measurement of the trapped proton cyclotron or magnetron frequency can establish limits on misalignment using the Brown-Gabrielse Invariance Theorem [36]. This technique is described further in Chapter 6. We also observe visible shifts in the proton cyclotron frequency while adjusting the hat spacer. These shifts are large enough (hundreds of Hz) to be easily detected, but useful alignment based on the cyclotron frequency would likely require the stack rearrangement proposed in Chapter 10, in order to position the precision trap at superconducting shim center where the field profile is well-characterized.

In practice we rely on a visual technique to align the trap with the magnetic field. A plexiglass window and iris are installed in the magnet bottom flange to provide a viewport. A 1/8" diameter "pinhole" is drilled in the bottom 77 K baffle, and a small target feature, also 1/8" in diameter, is bolted to the bottom of the radiation shield (Fig. 3.7). With the experiment in the magnet, the target sits roughly 1 inch above the pinhole, and we adjust the hat spacer to center the target as viewed from beneath the magnet.



Figure 3.7: Alignment in the magnet bore. The target circle (left) is mounted on the bottom of the radiation shield. After insertion in the magnet, the adjustable hat spacer is used to position this target directly above an equaldiameter "pinhole" (right) at the bottom of the magnet bore. LEDs wired to the pinhole baffle provide illumination.

Using this visual method, we are able to align the experiment to better than 0.1°, where this angle is expressed in terms of a tilt measured from the hat. Since the radiation shield and magnet bottom flange are only proxies for the trap and the magnetic field, some misalignment remains possible even with ideal visual alignment. For experimental purposes, we judge satisfactory alignment based on the success of proton transfers (Chapter 5). In our best configuration, no proton loss is observed during transfer, nor is much sideband cooling required after transfer to reduce the magnetron radius to the point where axial signals are detectable. Assuming the

proton follows field lines as it travels from PRING to ARING, this implies a worst-case misalignment of order 0.1°. The proton would then arrive in ARING in a magnetron orbit of order 100 μ m, a radius from which we have demonstrated that efficient sideband cooling is possible (Chapter 7).



Figure 3.8: Target assembly for improved *in-situ* alignment of electrode stack with magnetic field.

For better alignment of the electrode stack, we designed a second technique (not yet tested in the experiment). The flat PLATE electrode is replaced with an assembly (Fig. 3.8) similar to our FEP collet, containing a small tungsten rod (0.01" diameter) that serves as the target surface for proton/ion loading. A shield electrode with a small hole is positioned such that electrons from the FEP will only strike the target electrode if the stack is well-aligned with respect to the magnetic field, which guides the electron beam. By collecting current on the tungsten rod, we will thus be able to check the stack alignment. While the angular resolution attainable here is still of order 0.1°, this technique has the advantage of directly testing the alignment of interest, rather than relying, as in the visual method above, on flanges that may not perfectly correspond to the electrode stack and magnetic field.

Chapter 4

Tuned-Circuit Amplifiers



Figure 4.1: Circuit for detecting and dissipating image currents induced by the axial oscillation of a trapped proton. A parallel LC circuit is tuned to match the proton frequency, presenting large effective damping resistance R_{eff} on resonance.

To detect the signals and damp the energy of a single proton in the Penning trap (Fig. 4.1), we rely on the fact that a charged particle in motion will generate image currents in the nearby walls of our trap electrodes. However, the image current generated by a single proton is tiny, of order 0.1 pA even with a strongly driven axial motion with amplitude of 1 mm. Homemade cryogenic transimpedance amplifiers are used to convert this image current into a detectable voltage. A figure of merit for these amplifiers is the effective damping resistance presented at the proton oscillation frequency. To maximize this resistance, we use a tuned-circuit amplifier that is resonant at each proton frequency of interest. A high-Q inductor tunes out the trap capacitance at the proton oscillation frequency, presenting a large effective damping resistance on resonance. A low-noise HEMT FET (Fujitsu FHX13LG high electron mobility transistor) operating at 4.2 K provides power gain. Since the tuned-circuit resistance directly affects detection signal/noise, great effort has gone into minimizing losses in these amplifiers and reducing the loading due to various couplings in our complex trap wiring.

4.1 Theory and Model

Some understanding of the amplifier is essential to reduce unwanted loading effects. We use the model of the detection circuit in Fig. 4.2, from the trap electrodes to the coaxial cable used to carry the final signal to room temperature. Standard techniques of AC circuit analysis are applied to calculate various quantities of experimental interest.

The L and C form the "front-end" tuned-circuit, resonant at the proton oscillation frequency. The C arises from a parallel combination of trap capacitance (typically 10-15 pF), distributed capacitance in the inductor (typically 5-10 pF), and extra capacitance added at the pinbase for tuning. Losses in this capacitance are characterized



Figure 4.2: Circuit diagram for proton signal detection and first-stage amplification.

by resistor r_C . Similarly, r_L represents losses in the inductor L.

A capacitive voltage-divider input network formed by C_1 and C_2 couples the frontend signal voltage to the FET for amplification. The ratio of C_1 and C_2 can be adjusted to decrease loading of the front end and to optimize signal/noise. The combined capacitance of C, C_1 , and C_2 is tuned out by the reactance of L.

Elements internal to the FET are enclosed in red. A current I_D flows between drain and source when the FET is biased with V_D on the drain and V_G on the gate. The FET is characterized by transconductance $g_m = \frac{\partial I_D}{\partial V_G}$, gate-drain Miller capacitance C_m , and internal drain-source resistance R_{FET} . We assume no other significant coupling between the gate, drain, and source leads of the FET. The FET output resistance $R_{FET} = \frac{\partial V_D}{\partial I_D}$ must be matched to the microcoax cable Z_{LOAD} that carries the signal up to room temperature. At our typical operating point, $R_{FET} \approx 1 \text{ k}\Omega$. The output matching is done by means of a pi-network, consisting of series inductor L_{π} and capacitors $C_{1\pi}$ and $C_{2\pi}$ to ground. The gate DC bias voltage is applied on a line filtered through components R_{BIAS} and C_{BIAS} . The drain DC bias voltage is applied on the same microcoax line that carries the output signal.

The current produced by a proton axial oscillation is
$$I = \frac{e\kappa}{2z_0} \dot{z} , \qquad (4.1)$$

where κ is a geometrical constant described in Chapter 6. Our tuned circuit converts this signal to a voltage $V = IR_{eff}$, with a noise floor dominated by Johnson noise $\sqrt{4k_BT_{eff}R_{eff}}$ (Volts/ $\sqrt{\text{Hz}}$). For our axial and cyclotron amplifiers at a temperature $T_{eff} = 4.2$ K, this is 90 nV/ $\sqrt{\text{Hz}}$ and 3 nV/ $\sqrt{\text{Hz}}$, respectively.

The effective damping resistance presented to the proton, R_{eff} , arises primarily from the losses r_L and r_C , but improper circuit design can produce an unwanted loading by the FET connection. To calculate this loading effect, we must account for the input impedance of the FET, which depends on the drain load by way of coupling capacitance C_m . Fig. 4.3 shows our model for the FET plus drain load, where Z_D represents the pi-network and output load in Fig. 4.2.



Figure 4.3: Single-gate HEMT model with internal resistance and drain load. (a) FET drain-source modeled as an ideal voltage source $V_S = -R_{FET}g_mV_G$ with internal series resistance R_{FET} , and Thevenin equivalent circuit with FET as an ideal current source $I_S = g_mV_G$ with R_{FET} in parallel. (b) Model with drain load, using current-source description for the FET.

Applying Kirchoff rules, we obtain the following equations:

$$I_G = I_S - I_D \tag{4.2}$$

$$I_G = \frac{V_G - V_D}{Z_m} \tag{4.3}$$

$$I_D = \frac{0 - V_D}{Z'_D} \tag{4.4}$$

$$I_S = g_m V_G \tag{4.5}$$

where $Z'_D = R_{FET}Z_D/(R_{FET}+Z_D)$. This system can be solved to obtain the effective input impedance seen looking into the gate, which is

$$Z_{in} = \frac{Z'_D + Z_m}{1 + g_m Z'_D} \,. \tag{4.6}$$

 Z_m comes from the Miller capacitance C_m , which we take to be 0.1 pF, though this value has not been carefully measured, particularly at cryogenic temperatures. To calculate the effective load on the front-end, we add Z_{in} in parallel with the impedance due to gate bias line components R_{BIAS} and C_{BIAS} . This load is then transformed through the capacitive divider formed by C_1 and C_2 . The transformed load adds in parallel with Z_L (from the coil L and r_L) and Z_C (from the trap C and r_C) to form Z_{eff} , the total impedance of the front end. The total effective damping resistance is given by $R_{eff} = 1/\text{Re}[Z_{eff}^{-1}]$.

Another parameter of interest is the effective temperature of R_{eff} , i.e. the temperature of the thermal reservoir to which the proton oscillation is coupled. This temperature sets the Johnson noise floor in our measurements, as well as the limit of sideband cooling (Section 7.4). T_{eff} is determined by the various resistances that contribute to R_{eff} and by the temperatures of these resistances. In terms of total Johnson noise, the effective temperature of parallel resistors R_1 and R_2 can be written as

$$T_{total} = \left(\frac{T_1}{R_1} + \frac{T_2}{R_2}\right) R_{total} , \qquad (4.7)$$

where $R_{total} = R_1 R_2/(R_1 + R_2)$. Contributions from the output load and the FET in particular may be considerably hotter than the 4.2 K trap environment. In practice, we overcome the effect of a room-temperature load resistance by tapping down considerably with the input divider C_1 and C_2 , keeping T_{eff} below 10 K. A second-stage amplifier, described in Section 4.2.3, is currently being investigated to further reduce the effective damping temperature. Another technique to reduce T_{eff} is feedback cooling, described in Section 8.1.

4.2 Amplifier Construction and Testing

Our tuned-circuit inductors are wound by hand and housed in a grounded "amp can." We maintain as closely as possible the aspect ratios and spacings for empirically optimized shielded helical resonators [53]. The amp can is grounded at the pinbase via copper straps. One end of the coil is connected to the pinbase feedthrough corresponding to the desired trap electrode. The other end is grounded at RF to the can through a large ceramic capacitor (typically 1 nF).

The FET circuit board is copper-clad G10, with the pad layout machined using a CNC mill. The source lead and back ground plane of the FET board are soldered directly to an OFHC copper post that bolts to the amp can. This post is also used to mount the amp assembly above the pinbase, bolting directly to a tripod leg. A strap from the coil carries the proton signal up to the input side of the FET. A strap from the amp can grounds the back plane of the FET board. On the output side of the FET, the final element is a PCB-mount SMA jack, which connects to the microcoax line that carries the output signal up to the hat. At room temperature the signal passes through a Bias-Tee, necessary to provide a DC drain bias on the same line, and the RF portion is sent to our detection electronics.



Figure 4.4: Noise resonance of the large axial amplifier on the experiment, with Lorentzian fit indicating $Q \approx 6000$.

Noise in the amplifier circuit is sufficient to drive a tuned-circuit resonance that is visible on a spectrum analyzer after 60 dB of room-temperature amplification. A sample noise resonance for one of our axial amplifiers is shown in Fig. 4.4. The quality factor Q of this noise resonance is determined by losses in the tuned circuit and is directly related to the damping resistance by

$$Q = \frac{\omega}{\Delta\omega} = \frac{R_{eff}}{\omega L} , \qquad (4.8)$$



Figure 4.5: Network-analyzer measurement of reflection off the large axial amplifier output.

where ω is the center frequency of the noise resonance, $\Delta \omega$ is the width of a Lorentzian fit to its power spectrum, L is the inductance of the coil, and R_{eff} is the effective parallel resistance presented by the tuned circuit. The position of the noise resonance can be used to measure L. Assuming the rest of the amplifier circuit is properly designed to minimize capacitive loading, the noise resonance will appear at a frequency

$$f_C(Hz) = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{L(C_{coil} + \Delta C)}}, \qquad (4.9)$$

where C_{coil} is the distributed capacitance in the coil, and ΔC is an added parallel capacitance to ground that replaces the trap capacitance for testing purposes. We can extract L and C_{coil} by varying this added capacitance and observing the shift in center frequency of the noise resonance; a plot of $(1/(2\pi f_C))^2$ vs ΔC will have slope and intercept given by L and LC_{coil} , respectively. Once L for a given amplifier has been measured in this way, we can obtain the effective damping resistance R_{eff} from a measurement of amplifier Q on the experiment. Comparison of amplifier Q on the experiment and on the test jig is used to estimate the loss resistances r_L and r_C in Fig. 4.2.

To characterize the output matching of the amplifier, we send the RF output signal to a network analyzer. An impedance measurement in reflection mode reveals a broad dip feature at the resonant frequency of the pi-network, shown for the large axial amplifier in Fig. 4.5. The axial amplifier noise resonance is also visible in this scan as the feature with considerably higher Q, just above the center of the pi-net dip.

We fill all available space with three amplifiers. The cyclotron amplifier and a "small" axial amplifier connect to the precision trap. The "large" axial amplifier connects to the analysis trap. Table 4.1 and Table 4.2 summarize the operating parameters for these amplifiers as of late 2009, in the configuration used to obtain the majority of data presented in this thesis. Parameters in Table 4.1 are measured in a 4 K test setup, where a test capacitance ΔC adds to the coil distributed capacitance C_{coil} as described above. In the test setup, loss in the tuned circuit is assumed to be dominated by r_L . Parameters in Table 4.2 are measured in the actual experimental setup. The total trap capacitance C is determined from the observed resonance frequency and the known L of the coil. C includes contributions from the following: inter-electrode capacitance (~ 15 pF if the amp is connected to a single electrode, but higher for the axial amp configuration described below), distributed capacitance in the coil (Table 4.1), and stray or added tuning capacitance to ground (~ 1 pF for the cyclotron and large axial amp, ~ 10 pF for the small axial amp). In the

	ID (in.)	Coil	Dist.	Added		Test	Teat	
Amplifier		L	C_{coil}	ΔC	T_L	Front-end	Q	
		(μH)	(pF)	(pF)	(32)	Freq.		
Cyclotron	1.0	0.175	5	14	0.05	$86.5 \mathrm{~MHz}$	1500	
						(*)	(*)	
Small	1.95	670	8	35	0.27	040 bHz	13000	
Axial	1.20					940 knz		
Large	2.0	2500	13	20	0.38	$553 \mathrm{~kHz}$	20000	
Axial								

Table 4.1: Parameters for the proton experiment tuned-circuit amplifiers in a test setup. For varactor-dependent quantities (*), optimal values used in the experiment are shown.

experiment, loss in the tuned circuit is dominated by the series resistance r_C in this total capacitance.

The axial amplifiers described in Table 4.2 are connected to both a compensation and an endcap electrode, as shown in Fig. 3.4 and Fig. 3.5. We have since improved the large axial amplifier Q to roughly 8000, with the amp attached only to a compensation electrode. This configuration removes losses associated with the coupling capacitance to the endcap; where possible, we have also added thicker grounding straps to any capacitors in parallel with the large axial coil.

4.2.1 Superconducting Axial Amps

The axial amplifiers operate at relatively low frequency, under 1 MHz. With our typical trap capacitance of order 10 pF, the coil needed for an LC resonance at this frequency is of order 1 mH. To reduce losses in such a large inductor, the axial

Amplifier	Coil	Trap		Expt.	Expt.	Errot	Expt.
	L	C	$\begin{bmatrix} T_C \\ (0) \end{bmatrix}$	Pi-net	Front-end	Expt.	R_{eff}
	(μH)	(pF)	(32)	Freq.	Freq.	Q	$(M\Omega)$
Cyclotron	0.175	19	0.25	85 MHz	86.5 MHz	340	0.03
		(*)		(*)	(*)	(*)	(*)
Small	670	43	1.85	025 1-11-	040 1-11-	1800	7
Axial				950 KHZ	940 KHZ		
Large	2500	33	1.80	550 kHz		4000	25
Axial				(*)	OOO KHZ		35

Table 4.2: Parameters for the proton experiment tuned-circuit amplifiers connected to the trap. For varactor-dependent quantities (*), optimal values used in the experiment are shown.

amplifier is made using type-II superconductor, as introduced originally for \bar{p} and p measurements of q/m [15], which can operate in the strong magnetic field of our experiment. The can is machined from NbTi rod stock. The coil is wound from bare NbTi wire, which has Formvar insulation but no copper cladding. Since hundreds of windings are required, we choose a very thin wire: 0.0032" diameter NbTi, 0.0042" with insulation. This superconducting design, though essential for high Q, results in various practical complications. Since we cannot solder to bare NbTi, all connections to the coil and can must be spot-welded. We use OFHC copper straps to ground the can, and copper-clad NbTi for the connection from coil to electrode. Meanwhile, the normal DC resistance of the coil is of order 1 k Ω , making the amplifier impossible to test at room temperature. All testing of the axial amps therefore occurs in a LHe test dewar, and after installation on the experiment we are unable to check the position of the noise resonance without a full cooldown.

The axial coils are wound by hand on teflon forms. A thin layer of polystyrene

"Q-Dope" varnish is used to provide some mechanical support; without this, our first coil became visibly distorted after a single thermal cycle. We experimented with solenoidal and toroidal geometries for the coil, ultimately observing somewhat higher Q values with the toroids. While this behavior is not fully understood, we attribute it to magnetoresistance, since the difference between solenoids and toroids only becomes visible in the strong field of the experiment.



Figure 4.6: Toroid coil construction and amplifier schematic for the large axial amplifier.

The poor thermal conductivity of the axial coil presents additional experimental challenges. The coil is generally the last part of the experiment to cool down, requiring at least 3 hours longer than the rest of the tripod. Until this point, the axial amp noise resonances are not visible. Without proper heat sinking, we found the axial amps had a tendency to quench occasionally during operation, after which we would have to wait several hours for the coil to cool back down. This quenching behavior was generally not a problem for the solenoid amps; with the toroids, a tight-fitting



Figure 4.7: Solenoid coil construction and amplifier schematic for the small axial amplifier.

alumina insert (visible in Fig. 4.6) was used to improve thermal contact between the coil and the can.

Amplifier performance generally improves with physical size. Since strong axial signals in the analysis trap are essential for spin-flip detection, the analysis trap amplifier was designed to be as large as could fit between legs of the tripod. The remaining volume of the tripod region is shared between the precision trap axial amplifier and the cyclotron amplifier. Circuit schematics for the large amp (analysis trap) and small amp (precision trap) are shown in Fig. 4.6 and Fig. 4.7, respectively.

4.2.2 Cyclotron Amp

The cyclotron amplifier operates at much higher frequency, typically 86 MHz, and is built from copper and silver rather than NbTi. Our cyclotron coil is wound from pure silver wire. The amplifier can is gold-plated OFHC copper. Connections to the coil and can are made using OFHC copper straps. The full amplifier schematic is shown in Fig. 4.8. While construction of the cyclotron amplifier is generally easier than the superconducting axial amplifiers, there are two experimental difficulties unique to the cyclotron amp. The first is the need for precise tuning of the resonant frequency. Unlike the axial amplifiers, where the ring voltage can be adjusted to match the proton axial frequency to the resonant frequency of the amplifier, the only way to change the proton cyclotron frequency is to adjust the magnetic field *in situ*. However, adjusting the magnet current disrupts the careful shimming described in Chapter 3, and the small Z0 bore-tube shim can only produce a field change of approximately 250 ppm. In practice, we must construct the cyclotron amplifier such that the tuned circuit resonant frequency matches the expected proton cyclotron frequency almost exactly. The consequence of a mistuning $\Delta \omega$ is a reduction in the effective damping resistance $R_{eff} = Q\omega_0 L$ presented by the tuned circuit,

$$R_{mistuned} = \frac{R_{eff}}{1 + \left(Q\left(\frac{\Delta\omega}{\omega_0}\right)\left(\frac{2+\frac{\Delta\omega}{\omega_0}}{1+\frac{\Delta\omega}{\omega_0}}\right)\right)^2} \approx \frac{R_{eff}}{1 + 4Q^2\left(\frac{\Delta\omega}{\omega_0}\right)^2}, \quad (4.10)$$

with subsequent reduction in the cyclotron damping rate and signal/noise. Exact tuning is complicated by the changes during cooldown from room temperature to 4 K. Amplifier Q generally improves by a factor of 2-3, while effective trap capacitance decreases by roughly 2%, a shift representing many linewidths of the tuned-circuit noise resonance once cold. Furthermore, the size of this cooldown shift changes slightly with each modification to the amplifier circuit or related trap wiring; even the quality of solder joints and the physical arrangement of grounding straps will contribute at the 0.1 pF level.

To compensate for these frequency shifts between room temperature and 4 K, we

experimented with varactors to allow *in situ* tuning of the amplifier noise resonance, as was done for antiproton q/m measurements [16]. Varactors are reverse-biased diodes that act as voltage-variable capacitors, allowing some tunability of the amplifiers when the experiment is in the magnet and otherwise inaccessible.



Figure 4.8: Coil and circuit schematic for the cyclotron amplifier.

Varactors were utilized on the cyclotron amp front end (to tune the noise resonance frequency), in the cyclotron and large axial amp pi-networks (to optimize output matching), and for the large axial amp input capacitor C_2 (to optimize signal/noise). For the cyclotron front-end tuning, we use a GaAs varactor in the manner described in references [54, 55], tuning a series capacitor of order 1 pF. We find a substantial reduction in tuned-circuit Q if we operate the cyclotron front-end varactor in series with C > 1 pF, suggesting a loss mechanism at work in the varactor diode. We are also particularly sensitive to effects of magnetic field on the varactor, as our cyclotron amplifier sits in a region of $|\vec{B}| > 5 T$. We find substantially decreased tuning range for certain varactor models at high frequency in the strong field. (Note that this decrease was still observed after changing the varactor orientation relative to the field.) By comparison, we observe no decrease in tuning for varactors on the large axial amp, operating below 1 MHz. In the experiment, the axial amp varactor sits in a region of lower field, with $|\vec{B}| \approx 1.5 T$, but we have also observed normal tuning of this varactor at 5.6 T in a 4 K test setup.

These experimental varactor limitations restrict our effective tuning range of the cyclotron amp noise resonance to roughly 200 kHz (Fig. 4.9). A few iterations of trial-and-error are generally still required in order to obtain adequate placement of the cold noise resonance, but fine-tuning with the front-end varactor can then improve the cyclotron damping rate by a factor of 3 or better.



Figure 4.9: Cyclotron amp noise resonance on the experiment, with frequency tuning set by the front-end varactor.

A second challenge with the cyclotron amplifier is the presence of unwanted couplings at high frequency. Operating at a frequency 100x higher than the axial amplifiers, the cyclotron amplifier can be loaded by capacitive coupling to neighboring electrodes, in this case the compensation electrodes, or "comps." Since the comps support two drive lines and an axial amp, they cannot simply be grounded at RF, as would be ideal for cyclotron amp performance. We had some success using series LC shorts to ground the comps at the cyclotron frequency; this improved amplifier Q, but introduced a second resonant feature near the cyclotron frequency that complicated tuning of the noise resonance. We ultimately modified our drive scheme such that all splits of the comps could be grounded through large capacitors, though some coupling to the neighboring small axial amp was still observed (Fig. 4.10).



Figure 4.10: Coupling of the small axial amplifier to the cyclotron amplifier. A resonant dip feature, acting like a series LC short, is visible in the cyclotron amplifier noise resonance after the small axial coil goes superconducting during cooldown. The coupling effect, while not fully understood, is avoided for experimental purposes by tuning to higher frequency with the front-end varactor.

4.2.3 Second-Stage Axial Amp

As discussed in Section 4.1, the temperature of elements in our amplifier circuit, particularly the final 50 Ω load, can produce an effective temperature for the tunedcircuit that is higher than the 4.2 K trap environment. Direct measurements of our proton axial temperature (Section 7.4) suggest that this effective temperature may indeed be closer to 10 K in our current apparatus. Since low axial temperatures are particularly critical in the analysis trap (Chapter 9), we later added a second-stage amplifier (Fig. 4.11), to reduce the temperature of the output load seen by the firststage axial FET. The second-stage amp is thermally anchored to 4.2 K by bolting the amp PCB to a flange on the top of the experiment LHe dewar. A copper microcoax carries the output signal from the first-stage (large axial) amplifier to the input of the second stage, and a stainless steel microcoax takes the second-stage output to the hat.



Figure 4.11: Circuit schematic for the second-stage axial amplifier.

The second-stage amplifier was only recently installed on the experiment and was not in use for the majority of work presented in this thesis.

4.2.4 Electron Axial Amp



Figure 4.12: (a) Photo of PCB amp (highlighted) amidst the tripod wiring. (b) Axial "dip" feature in the noise resonance of the PCB amp, caused by 10⁵ electrons.

In the early stages of the experiment, we relied on electrons for initial trap diagnostics. Electrons are easier to load than protons, and since electron cyclotron energy damps quickly via synchrotron cooling, we need only address the axial and magnetron motions. To detect electrons, the cyclotron amp can be converted to an electron axial amp by moving it from the ring to a comp or endcap electrode. We also experimented with PCB axial amps, where the "coil" is a commercial surface-mount inductor, the "can" is a wrap of copper foil, and the entire amplifier fits on a printed circuit board. Though the Q of such amps is limited, small size and ease of construction makes them a useful temporary addition to our crowded tripod region. We obtained Q values of roughly 200 with a PCB amp connected to the trap, sufficient to observe electron axial signals from the center-of-mass motion of 10^5 electrons (Fig. 4.12).

Chapter 5

Loading and Transferring Protons

5.1 Loading Protons

5.1.1 Field Emission Point

Protons are loaded by ionizing hydrogen atoms in the region of our trapping well. The loading process begins with an electron beam that originates at the field emission point (FEP). As described in Chapter 3, the FEP is mounted at the top of the electrode stack and biased through a high-voltage feedthrough in the pinbase. At voltages around -500 V, electrons tunnel out of the FEP tip in a current of order 1 nA. Guided by the magnetic field, they travel down the axis of the trap and strike the PLATE electrode at the bottom of the electrode stack. By measuring current collected on the PLATE electrode using an electrometer, we obtain the characteristic exponential current-voltage curve of the FEP. This characteristic curve changes slightly in voltage with each thermal cycle. On a given cooldown it is generally stable, but the current will occasionally jump by a noticeable amount during the firing sequence. Presumably this behavior is due to the cleaning of cryopumped atoms from the point of the FEP. Characteristic curves from several firings of the FEP are displayed in Fig. 5.1.



Figure 5.1: FEP current-voltage characteristic curves. (a) Exponential increase in current with negative FEP bias, plotted against positive-bias control test to rule out breakdown current. (b) Close-up of a sequence of FEP firings, showing occasional jumps in the current.

To load protons, we fire the FEP with the electrodes biased as shown in Fig. 5.2. A negative well for trapping protons is nested inside a deep positive well for electrons. Since the electrons emerge from the FEP with energies around 600 eV in this example, they will still strike the PLATE electrode despite these 150 V barriers. Gas adsorbed on the PLATE electrode is dislodged, then ionized by the continuing electron beam. If ionization occurs within the potential well of the precision trap, the positive ion can fall into the trap while the ionized electron carries excess energy away. By this method, various species of positive ions, including some protons, are loaded into the negative well that includes PRING. Meanwhile, secondary electrons from the ionization and the initial impact with PLATE are trapped in the 150 V well, increasing the yield of protons since more electrons are available for ionization (too many electrons will inhibit FEP emission, but in practice we did not approach this limit). The FEP collet electrode is also at high negative voltage during loading, but we found it considerably less effective to rely on this as the upper wall of the electron well; without the -150 V on T3, proton yields were lower and a deeper loading well on PRING was required.



We found that currents on the nA scale were necessary to load protons. For fear of damaging the FEP, we rarely exceeded 20 nA, but the point did survive a few accidental increases to current of order 100 nA. Besides firing at higher FEP current, proton yield can be increased by using deeper loading wells, or by loading closer to the PLATE surface. The electron well depth is limited primarily by the voltage rating of filter capacitors. The proton well depth is chosen based on ion-cleaning considerations, discussed in Section 5.1.4.

5.1.2 Mass Scan

A mass scan technique is used as an initial search for protons and heavier ions after loading, as well as to diagnose vacuum problems (Section 5.1.5). To run a mass scan, we fire the FEP at a current of 10-20 nA for 1-2 minutes and turn on the axial amplifier. When loading at high enough current, an ion fly-by signal becomes visible as a swelling and frequency shift of the amplifier noise resonance. After loading is complete, the FEP is turned off, and we ramp the PRING voltage while recording the integrated signal in the amplifier noise resonance. The axial frequency of an ion is given by

$$\omega_z^2(ion) = \frac{q_{ion}V_0C_2}{m_{ion}d^2} = \frac{(q/m)_{ion}}{(q/m)_{proton}}\omega_z^2(proton) .$$
(5.1)

As the ring voltage sweep brings various positive ion species into resonance, the ions transfer their energy into the effective damping resistance of the tuned circuit, producing a visible signal. When not in resonance with the tuned-circuit, the ions have no efficient means of damping, so mass-scan signals are generally visible even several hours after loading.

A sample mass scan is shown in Fig. 5.3, after firing the FEP at roughly 20 nA for 5 minutes. The electrode bias scheme for this load was slightly different than in Fig. 5.2. We used an unusually deep loading well of -50 V on PRING, and we applied -150 V to PLATE but not T3. Without the full nested well, this configuration is less effective for proton loading, but a wide variety of heavier ion species are observed. Since the loading process depends critically on the particular layer of gas adsorbed on the PLATE electrode surface, the mixture of ions varies considerably over time. The

strongest ion peaks in Fig. 5.3 are labeled by m/q relative to the proton. Rearranging Eq. 5.1, we expect ions to come into resonance with the axial amplifier at a voltage given by

$$V_{ion} = \frac{(m/q)_{ion}}{(m/q)_{proton}} \cdot V_{proton} .$$
(5.2)



Figure 5.3: Ion species in a mass scan. The trapping potential is ramped in steps of 20 mV, with 10 seconds averaging between steps.

5.1.3 Ion Cyclotron Heating

If ions of any species are left in resonance with the tuned circuit for sufficiently long, their energy will damp below the noise floor and the signal will no longer be visible. Since various ions in the cloud are coupled by collisions, this serves to damp away the signal from all species, not just the one currently resonant with the amplifier. The ion axial signals can be restored in a number of ways, but one particularly useful technique involves driving the ion cyclotron motion, which transfers energy to the ion axial motion through collisions. This technique can be applied to determine the particular species of ions in the trap. While the mass scan has insufficient resolution to distinguish between species of similar charge/mass, the ion cyclotron resonance is narrow enough to allow exact identification of each ion species.

To measure an ion cyclotron frequency, we complete a mass scan and damp away the ion axial signals as described above. Setting the ring voltage at any value where a signal was observed in the mass scan, we sweep a cyclotron drive slowly through the frequency expected for a given ion species, based on our known magnetic field. At the ion cyclotron resonance frequency, the axial signal is observed to rise sharply as the ions are reheated. The ion axial signal then remains visible until damped back below the noise floor.

Typical settings for the heating drive sweep are -70 dBm, 10 Hz per step, and 10 seconds averaging per step. The sharp edge of the resulting feature can be used to measure the (trap-modified) ion cyclotron frequencies at the ppm level. Fig. 5.4 shows one such measurement of the cyclotron frequency for C^{4+} .

Since the ions are coupled collisionally, the cyclotron heating technique will work for any ion species that exists in sufficient number in the trap, not just the one corresponding to the current ring voltage setting. Fig. 5.5 shows similar cyclotron resonances for other ions, observed via heating of the C^{4+} axial signal. The high resolution of this technique allows us to distinguish between species of similar m/q, for example C^{3+} and O^{4+} , which appear in the same m/q = 4 peak in the mass scan

of Fig. 5.3.



Figure 5.4: Cyclotron heating of axial motion of C^{4+} ions.

The cyclotron heating technique is particularly useful as an initial diagnostic, since even a relatively small population of ions can be detected by observing the axial reheating of a more abundant species. For example, in the early stage of the experiment, after obtaining some indication of protons in a mass scan, our first supporting evidence of trapped protons came from reheating the strong C^{4+} response by driving at the proton cyclotron frequency.

The cyclotron heating resonance is complicated by the presence of magnetron sidebands. The ion axial response can also be excited by driving on a magnetron sideband of the cyclotron frequency, producing visible heating signals at frequencies of $\omega'_c + N\omega_m$, where N is a small integer. These sideband resonances produce heating signatures that are qualitatively similar to those from direct cyclotron heating.



Figure 5.5: Cyclotron heating of several ion species, observed via collisional heating of C^{4+} axial motion.

Fig. 5.6 shows a set of C^{4+} ion axial responses generated by sweeping a strong cyclotron drive over several magnetron sidebands of the C^{4+} cyclotron frequency. To isolate the cyclotron heating response, we can compare the teeth of these sideband "combs" for two different ion species, accounting for the effect of magnetron frequency on ω'_c . Since the sideband frequency $\omega'_c + \omega_m$ is approximately equal to ω_c [56, 57], the true cyclotron responses are revealed as the only combination where the ratio of heating frequencies agrees precisely with the ratio of m/q for the selected ions. In a clean proton cloud, we can also observe the cyclotron signal directly, as described in Chapter 6. At the drive strength typically used for direct cyclotron excitation (-40 dBm at the hat), driving on a magnetron sideband produces no visible signal on the cyclotron amplifier, though an axial heating response is observed for large proton clouds.



Figure 5.6: Heating of C^{4+} ion axial motion by driving on magnetron sidebands of the C^{4+} cyclotron frequency. The drive level is -5 dBm, with a sweep rate of -20 Hz every 2 seconds. The 15 kHz spacing between heating responses corresponds to the ion magnetron frequency.

5.1.4 Notch Filters for Ion Cleaning

As described above, protons are generally loaded along with a wide variety of heavier ions. In order to avoid couplings that would perturb the measured proton eigenfrequencies, we must remove all these contaminant ions. To "clean" the cloud, we rely on the fact that all contaminant ions have smaller q/m than the proton and therefore oscillate at lower axial frequencies in a given trapping potential (Eq. 5.1). To selectively drive at these lower frequencies, we apply a very strong and heavily filtered white noise drive from a DS345 synthesizer (initial strength +7 dBm), to the axial drive line on the precision trap endcap during the loading procedure. The filters used are a low-pass filter with cutoff just below the proton axial frequency [58, 54], plus two bandstop or "notch" filters [59] at the proton axial and sideband heating frequencies.

The low-pass filter (Fig. 5.7) is a 5-branch elliptic low-pass filter [58]. We choose the cutoff frequency to be below the proton axial frequency and above the axial frequency of the lightest contaminant ion He^{2+} , which has relative m/q = 2. This filter design also features two high-attenuation notches, which are tuned to provide additional attenuation at ω_z and $2\omega_z$, to protect against direct or parametric axial excitation.



Figure 5.7: Schematic and measured filter characteristic for the elliptic low-pass filter.

Additional axial and sideband heating notch filters are built with the design in Fig. 5.8. By adjusting the variable resistor to cancel losses in the inductor, we can produce a notch almost 100 dB deep at the resonant frequency. The variable capacitor provides slight tunability of the notch frequency.



Figure 5.8: Schematic and measured filter characteristic for notch filters at the proton axial and magnetron heating frequencies.

The combination of the elliptic low pass with the notch filters provides more than 100 dB protection at the proton axial and magnetron heating frequencies, as shown in Fig. 5.9.



Figure 5.9: Combined filter on the axial noise drive for cleaning ions.

The low-pass and notch filters are built on copper-clad G10 circuit boards. They operate at room temperature; the notch filters in particular are sensitive to tempera-

ture variations, sometimes drifting by a few kHz over several days. Variable capacitors allow for periodic retuning of the notch filters, though in practice we found that a slight mistuning did not noticeably affect performance. Since our 5 kHz magnetron frequency is comparable to the width of the notches, the proton axial and sideband heating notches basically overlap to produce a broader stopband, providing considerable attenuation at the proton heating frequencies unless both notch filters are badly mistuned.

To obtain a clean proton cloud, we prefer to load with the ring voltage set to bring protons into resonance with the axial amplifier, while also applying the filtered noise drive on an endcap. During the load, protons will damp into the axial amplifier, while heavier ions are driven out by the noise. Immediately after loading, we turn off the noise drive and lower the well to -0.25 V, allowing the axially excited ions to spill out of the trap. It is also possible (but harder) to remove ions later in the process, waiting to apply the noise drive until after the load is complete. This could be useful if the proton yield is particularly low, and wells deeper than the proton resonant voltage are needed for effective loading.



Figure 5.10: Effect of filtered noise cleaning drive on ion and proton loading.

As a demonstration of the effectiveness of noise cleaning, Fig. 5.10 displays results from a comparison of two mass scans performed under the same loading conditions, with and without the cleaning drive applied. As shown, ion species present in large quantities without the cleaning drive are entirely absent from the mass scan after cleaning, while the proton peak is unaffected. To confirm the absence of heavier ions, we can use the axial cyclotron heating technique described above, checking that there is no axial heating of protons when driving at various ion cyclotron frequencies.

5.1.5 Diagnosis of the Vacuum

Techniques in this chapter are most useful in the initial phase of the experiment. With a well-characterized trap, we still drive with filtered noise while loading, but we typically run no explicit tests to confirm the absence of contaminant ions, relying instead on the direct cyclotron signals described in Chapter 6, which are visibly destabilized by the presence of ions. However, the mass scan remains as a useful diagnostic, in particular to test for vacuum problems in the trap can. Bad vacuum, generally first noticed if we have trouble obtaining a stable and repeatable direct cyclotron signal, is characterized by a strong He^+ peak in the mass scan. Due to space charge effects, this peak can appear shifted to considerably lower voltage, compared to the normal value for an ion of relative m/q = 4. Several examples of mass scans in bad vacuum are shown in Fig. 5.11.

The vacuum leaks exposed in Fig. 5.11 are relatively small, caused in most cases by slight loosening of an indium seal during thermal cycling. In such a soft vacuum, protons can generally be loaded and trapped normally, but unusual loss is observed



Figure 5.11: Mass scans indicating the vacuum in the trap can. (a) Typical mass scan under ideal conditions, with proton signal stronger than any contaminant ions. (b) Mass scan in bad vacuum, dominated by broad He^+ peak. (c) Mass scan in bad vacuum, with broad He^+ and He^{2+} peaks. (d) Mass scan in bad vacuum, with voltage-shifted and double-peaked He^+ feature.

during cyclotron excitation or attempted transfer along the electrode stack. At one point, due to a bad pinch-off that left a small hole in the knife-edge seal, we observed the effects of a substantially larger vacuum leak. In that case the only indications of protons or ions came during a load; mass scans after the FEP was turned off revealed no ion signals. On another occasion, a suspected vacuum leak smaller than those in Fig. 5.11 produced only a faint ion signature in the mass scan, but caused small jumps in the exponential cyclotron decay (Section 6.1) and frequent loss of protons during transfer.

5.2 Transfer of Protons Between Traps

Protons are loaded into the precision trap, then transferred to the analysis trap for spin-flip detection. The precision trap is closer to the PLATE surface, allowing us to load with smaller FEP currents than if we loaded directly into the analysis trap. We also require the cyclotron amplifier, which exists only in PRING, for quick identification of a single proton and subsequent damping of the cyclotron radius.

Protons are transferred between the traps by applying to the trap electrodes a series of voltages designed to spill the protons from one electrode to the next. Though the trapping potential change follows more of an inchworm motion than a smooth translation, this technique is considered adiabatic, since RC filters on the electrodes ensure than the trapping potential can only change on timescales slow compared to the period of an axial oscillation.

Our typical transfer sequence is shown in Fig. 5.12. Transfer voltages of order 10 V were sufficient to move the proton from PRING to PTEC and back, but we found that further transfers only succeeded reliably with the deeper wells shown. We attribute this to magnetron heating, which was frequently detected via a visible sideband cooling response immediately following transfer. Magnetron heating was particularly common in the transfer steps involving the ring and compensation electrodes, which have long but unequal RC time constants. To avoid magnetron heating, we found it necessary to adjust the ring and compensation voltages in separate steps in the transfer sequence, rather than maintaining the standard comp/ring tuning ratio



Figure 5.12: Sequence of potentials for transferring between traps.

throughout.

To apply these high voltages for transfer, we use SuperElvis amplifiers stacked on our normal BiasDAC supplies. Since the SuperElvis connection is relatively noisy, all electrodes are switched back to BiasDAC voltages after the transfer. Total transfer time between traps is roughly two minutes, limited primarily by the long RC time constants on ring and endcap electrodes in the analysis trap.

We were often unable to transfer large clouds of protons between traps without significant loss of particles. Since our focus was transfer of a well-damped single proton, we never fully investigated the probable loss mechanisms for clouds with a significant radial extent. One likely loss mechanism is misalignment. Assuming the protons follow field lines during transfer, a misalignment between the axis of the electrode stack and the axis of the magnetic field could result in an effective heating of the radial motion. Particularly if starting at a large radius, this misalignment heating, over the 1.9 inches between traps, could carry protons into the electrode walls. A second possible mechanism for loss is "magnetic bounce," since at some point in the transfer the protons must move from regions of lower to higher magnetic field, due to the significant contribution from our iron ring in the analysis trap.

Chapter 6

Proton Motions in the Precision Trap

In the double-trap scheme envisioned for eventual measurement of g_p (Section 2.3), the "precision" trap is a standard Penning trap with copper electrodes and no explicit magnetic bottle. The two frequencies that primarily determine g_p are eventually to be driven in this trap. Without any significant B_2 bottle coupling, spin-flip transitions may be driven but not detected in the precision trap. At the current stage of the experiment, the precision trap is thus utilized primarily for loading a single proton. We use a method that relies upon special relativity to resolve individual protons, a method pioneered for proton q/m measurements [15]. In loading the proton and preparing it for transfer to the analysis trap, we routinely demonstrate measurement of the cyclotron, axial, and magnetron frequencies at the resolution needed for a high precision measurement of g_p .

6.1 Cyclotron Motion

Cyclotron motion in a Penning trap is described in Chapter 2. We drive and detect the cyclotron motion using the scheme in Fig. 6.1. The cyclotron drive is typically swept down in frequency over a range of 150 Hz, in steps of -10 Hz every 5 seconds, with a drive strength of -37 dBm at the hat. Detection relies on a tuned-circuit amplifier as described in Chapter 4. Since the cyclotron drive feeds through directly to the cyclotron amplifier, the FET is turned off while a drive is applied and the drive is turned off during detection.



Figure 6.1: Radiofrequency schematic for excitation and detection of the proton cyclotron motion.

The first evidence of protons after loading is typically the driven cyclotron signal from multiple protons. The broad response from a large cloud of protons can be shifted by adjusting the ring voltage to change ω_z , which modifies ω_m and produces a small shift in ω'_c . With smaller clouds and higher resolution on the spectrum



Figure 6.2: Multiple-proton cyclotron signals. The driven response of a large proton cloud (a) is a broad superposition of cyclotron signals with different relativistic shifts. As we reduce the number of protons by repeatedly lowering the trapping potential, we can resolve the individual cyclotron signals from four protons (b) and two protons (c).

analyzer, the multiple-proton signal shows itself to be a discrete spectrum, as in Fig. 6.2. The peaks are individual protons in the end, separated in frequency due to special relativity. For a proton cyclotron orbit with $\gamma = (1 - v^2/c^2)^{-1/2}$, the cyclotron frequency is modified due to an effective increase in mass, $m \to \gamma m_p$. A proton with kinetic energy E_c in the cyclotron motion thus acquires a "relativistic shift" given by

$$\frac{\Delta\omega_c'}{\omega_c'} = -\frac{E_c}{E_c + m_p c^2} \approx -\frac{E_c}{m_p c^2} \,. \tag{6.1}$$

The protons collide until they separate in radius and energy, whereupon each orbits
with a different cyclotron frequency, γ , and $\Delta \omega'_c$. In practice, protons of similar γ tend to lock together, such that the actual number of protons is greater than the number of cyclotron peaks observed. However, a small number of protons can be reliably counted by driving repeatedly to separate the cyclotron responses and observing the number of distinct peaks.

For the experiment, we require a single proton. Adjusting the FEP current provides some control over the number of protons loaded, and in principle we can fire the FEP at low current until a single proton appears in the trap. In practice, the loading rate is inconsistent enough that we tend to load several protons, then reduce to one. Unwanted protons are removed from the trap by first applying a strong cyclotron drive to split the proton signals as described above, then reducing the trapping potential to spill out the most highly excited protons (Fig. 6.2). During this process, the proton cyclotron signals shift up in frequency due to a reduction in ω_m , which approaches 0 for shallow trapping wells. Magnetron heating provides the mechanism by which protons leave the trap; with particularly clear signals, we observe magnetron sidebands developing on the cyclotron signal shortly before the proton is lost. In our setup, an effective trapping well shallower than -0.25 V is required to remove any protons.

The relativistic shift implies an inverse relation between signal frequency and energy in the cyclotron motion. To effectively drive the cyclotron motion, we sweep a drive down in frequency, exciting the proton into larger and larger cyclotron orbits. With the cyclotron drive off, the observed proton signal then gradually increases in frequency as cyclotron energy is dissipated in the tuned circuit detector. This decay

	Energy	Radius	Relativistic Shift	Quantum State	
	$E_c \; (eV)$	$ ho_c \; (\mu { m m})$	$\frac{\Delta \omega_c'}{2\pi}$ (Hz)	n	
Typical Driven	1000	205	0.2	2.8×10^9	
Excitation	1000	805	-92		
Cyclotron-Axial	0.022	4.6	0.002	0.2×10^{4}	
Cooling Limit	0.055	4.0	-0.005	9.3×10	
4.2 K Thermal	2.6×10^{-4}	0.48	-3.3×10^{-5}	1.0×10^3	
Limit	3.0×10^{-5}				

Table 6.1: Energies of various proton cyclotron orbits, for $\omega_c' = 2\pi \times 86.524$ MHz

is an exponential

$$E_c = E_0 e^{-t/\tau_c} \longrightarrow \omega'_c = \omega'_c(0) - \Delta \omega'_0 e^{-t/\tau_c} , \qquad (6.2)$$

where $\Delta \omega'_0$ is the relativistic shift (Eq. 6.1) corresponding to initial excitation energy E_0 and $\omega'_c(0)$ is the "zero-energy" cyclotron frequency of a proton at $\rho = 0$. The energy decays with time constant $\tau_c = 1/(2\gamma_c)$, where γ_c is the cyclotron amplitude damping rate, given by

$$\gamma_c \approx \left(\frac{e\kappa_c}{2\rho_0}\right)^2 \frac{R_{eff}}{m_p} \,. \tag{6.3}$$

The approximation here neglects a small correction from the magnetron frequency, since the proton radial motion is not purely a cyclotron orbit. R_{eff} is the effective damping resistance on resonance of the cyclotron amplifier tuned circuit; the case of a mistuning is treated in Chapter 4. The geometrical factor κ_c characterizes the amount of current induced by an orbiting proton on the electrode segment used for detection. We detect on one half of a split ring electrode, for which $\kappa_c = 0.356$ [54]. With a single proton, the cyclotron signal frequency can be tracked over time, producing a characteristic decay trace (Fig. 6.3). Fitting this decay to Eq. 6.2, we obtain the energy decay time constant τ_c and extrapolate the zero-energy cyclotron frequency $\omega'_c(0)$.



Figure 6.3: Decay of the single-proton cyclotron signal, with sample traces shown at three points in time.

Additional trapped protons can disrupt the cyclotron decay, such that the tracking plot no longer follows a clean exponential; in fact, we typically use the decay curve to confirm that there is only one proton in the trap. Repeated cyclotron drive and detection sweeps are necessary to rule out the presence of a second proton, since if only one of two protons is strongly excited, the observed decay will resemble the single-proton case. If the two protons acquire large but unequal relativistic shifts, a double decay is observed (Fig. 6.4), as the protons interact only weakly until they decay to similar energies, at which point they tend to lock together.



Figure 6.4: Simultaneous cyclotron decays of two protons.

Strong cyclotron damping is desirable for ease of measurement and for reducing the waiting time required before transfer to the analysis trap (see Chapter 7). As described in Chapter 4, varactors are used to tune the cyclotron amplifier on resonance with ω'_c , optimizing the damping time.

We also experimented with cyclotron-axial sideband cooling to increase the overall damping rate. In an effect analogous to the axial-magnetron sideband cooling described in Section 6.3.1, a drive applied at $\omega'_c - \omega_z$ will couple the cyclotron and axial motions and cool to the limit of equal quantum numbers. To obtain the xzasymmetry required, we apply this cooling drive to one half of a split compensation electrode. Since our sideband drive line (Fig. 3.5) is a twisted-pair intended for low-frequency axial-magnetron cooling, drives as strong as +13 dBm at the hat are required for effective cyclotron-axial cooling.



Figure 6.5: Demonstration of cyclotron-axial sideband cooling during a cyclotron decay. The sideband drive was applied during the 100 seconds indicated, producing a cooling effect equivalent to 640 seconds on the decay curve.

Cyclotron-axial sideband cooling proves useful for quickly reducing the radius of a strongly excited proton, as shown in Fig. 6.5. However, the limit of this cooling process is n = k, where n and k are the cyclotron and axial quantum numbers introduced in Chapter 2. Since $\omega_z \ll \omega'_c$, cyclotron-axial sideband cooling cannot closely approach the 4 K thermal limit of the cyclotron motion (Table 6.1).

In loading and identifying a single proton, we typically drive the cyclotron motion to an energy of 1 keV, producing an easily detectable relativistic shift of roughly 100 Hz (Table 6.1). Before transferring to the analysis trap, where we lack a cyclotron amplifier for further damping, we wait for the cyclotron motion to decay to a thermal energy, such that E_c as predicted from Eq. 6.2 satisfies $E_c < k_B T$ for $T \approx 4$ K. Using only the tuned-circuit detector to damp the cyclotron motion, this requires waiting for $15\tau_c$, where the observed cyclotron energy decay lifetime is $\tau_c \approx 10$ minutes. Starting instead from the cyclotron-axial sideband cooling limit, a thermal energy is reached in $5\tau_c$.

6.2 Axial Motion

We detect the axial motion as described in Chapter 4, via image currents induced on the trap electrodes. The proton axial motion is that of a damped, driven anharmonic oscillator,

$$\ddot{z} + \gamma_z \dot{z} + \omega_z^2(A)z = \frac{1}{m}F_d(t)$$
 (6.4)

The axial damping at rate

$$\gamma_z = \left(\frac{e\kappa}{2z_0}\right)^2 \frac{R_{eff}}{m_p} \tag{6.5}$$

is caused by R_{eff} , the damping resistance presented by our tuned-circuit amplifier. The constant κ is a geometrical factor characterizing the amount of current induced by an oscillating proton on the electrodes used for detection. For the case of a closedendcap trap where the endcaps are infinite flat planes, $\kappa = 1$. Values of κ for our openendcap trap are given in Table 6.2. In an early stage of the experiment, we detected on an endcap for simplicity of trap wiring. We eventually used a comp+endcap for

Electrode	К
Licetrode	
endcap	0.335
compensation	0.90
comp+endcap	1.24

Table 6.2: Values of the geometrical factor κ for axial detection on various electrodes [34]

detection to realize the considerable increase in κ and thus γ_z .

6.2.1 Driven Axial Signals



Figure 6.6: Radiofrequency schematic for excitation and detection of the proton axial motion.

When the relation between ring voltage and axial frequency is not precisely known, as in a new trap where the constant C_2 has not yet been measured, the proton axial signal is most easily observed using a voltage sweep technique. An axial drive, set at the center frequency of the tuned-circuit axial amplifier, is applied to an endcap, and the ring voltage is slowly ramped to adjust the trapping well. As in the mass scan technique described in Chapter 5, a response is observed when the ring voltage reaches the value needed to bring the proton into resonance with the axial drive and amplifier. To avoid direct feedthrough which would otherwise dominate the proton signal, we apply two separate drives to the endcap, at frequencies δ and $\nu_z - \delta$. The proton responds at the sum frequency, while the direct feedthrough signal at the FET (which can also act as a mixer if driven too strongly) is greatly reduced.

To reduce the noise bandwidth for detection, and to enable low-frequency digital signal processing, we use the mixdown chain shown in Fig. 6.6. The axial signal is mixed down from ν_z to an intermediate frequency (f_I) , filtered through a sharp bandpass, and mixed down again to 5 kHz for final detection.



Figure 6.7: Driven axial proton response, observed with fixed drive frequency and ring voltage sweep.

A typical voltage-sweep response is shown in Fig. 6.7. For this plot, the axial drives were set to 807 kHz and 141 kHz, each -17 dBm at the hat. The ring voltage was ramped down (in absolute value) in steps of 0.1 mV, and the axial signal was recorded as the value in a 0.25 Hz frequency bin centered on ν_z =948 kHz, averaged for 25 seconds on an FFT signal analyzer (HP 3561a). The slight anharmonic shape is due to a comp/ring voltage tuning ratio of 0.870, discussed further in Section 6.2.2.

The voltage sweep is particularly useful for an initial proton axial search, since it allows us to cover a frequency range larger than the width of the axial amplifier. However, the RC filters on our trap electrodes limit the voltage sweep rate. Once the proton response has been identified and the trap constant is fairly well known, we can detect the axial signal more rapidly using a similar technique of drive sweeps at fixed voltage. The ring voltage is fixed at a value that places the proton axial frequency within the resonance of the axial amplifier, and the endcap drive is swept in frequency. As the endcap drive sweeps through the proton resonance, a signal is observed at the drive frequency. We use the same mixdown/detection scheme (Fig. 6.6) as for the voltage sweep, but the mixed-down axial signal is typically measured using the computer DAQ card (National Instruments PCI 4474) instead of the HP signal analyzer.

6.2.2 Anharmonicity Tuning

The driven axial signal can be used to tune out the leading-order anharmonicity of our Penning trap. This step is critical for precision measurement of the axial frequency, as the axial response is narrowest when the trap is most harmonic. The effect of trap anharmonicity on the axial frequency is given by Eq. 2.10. By adjusting the ratio of compensation and ring electrode potentials, we can minimize the amplitude dependence of $\omega_z(A)$ by setting $C_4 \approx 0$. This harmonic tuning ratio is determined by trap geometry, but machining errors and gaps between electrodes cause the observed optimal ratio to differ substantially from the value we calculate based on known trap dimensions.



Figure 6.8: Driven proton axial responses as the trap anharmonicity is adjusted to set (a) $C_4 < 0$; (b) $C_4 \approx 0$; (c) $C_4 > 0$.

We determine the optimal comp/ring tuning ratio by observing the shape of the

driven axial response. The equation of motion for a damped, driven anharmonic oscillator admits a multivalued solution, such that the driven response takes on a characteristic skewed shape. The sign of C_4 in our trap is apparent from the shape of this anharmonic response. When the comp/ring ratio is too high ($C_4 > 0$), the response is stronger for a drive sweep up in frequency. When the comp/ring ratio is too low ($C_4 < 0$), the response is stronger for a drive sweep down in frequency. At the comp/ring ratio that sets $C_4 = 0$, the response is narrower, and the sweep-up and sweep-down responses are identical. A demonstration of this tuning is shown in Fig. 6.8.

Due to higher order terms in the anharmonic potential, the optimal tuning ratio has some amplitude dependence and thus changes slightly with axial drive strength; in particular, C_4 is made slightly nonzero to compensate for C_6 . To obtain the narrowest driven axial signals, we follow an iterative procedure, finding the optimal tuning at a particular drive setting, then reducing the drive strength and tuning again.

6.2.3 Proton Axial Dips

The proton axial signal is also visible as a "dip" in the noise resonance of our tuned-circuit axial amplifier. The proton is then driven only by the Johnson noise in the cold circuit. This feature can be understood by modeling the proton as a high-Q series lc circuit [60] that adds to the parallel LC circuit of the amplifier as shown in Fig. 6.9.

The dip is essentially an axial signal in the limit of very weak drive, as the series lc resonance partially shorts out the Johnson noise generated by R at the proton axial



Figure 6.9: Equivalent circuit for proton axial dips, with the trapped proton modeled as a series lc circuit.

frequency. Since the proton only oscillates at a thermal amplitude, careful tuning of the trap anharmonicity is less critical than for the strong driven response, and the width of the dip feature is set by the proton axial damping width γ_z . However, considerable averaging is required to overcome the low signal/noise. Drift in the trapping potential during the averaging can obscure the dip feature or inflate the apparent width.

To measure a proton dip, we use the same axial mixdown chain as for driven detection (Fig. 6.6), but with the RF switches turned off so that no axial drive is applied to the endcap. The dip is observed by averaging the mixed-down amplifier noise resonance viewed on the audio analyzer or with the DAQ card. A sample dip in the precision trap is shown in Fig. 6.10, after 27 minutes of averaging. Quality of the dip is visibly affected by any instability in the trapping potential. For example, averaging time to resolve the dip is reduced if the endcaps are biased through cold 1 M Ω resistors to the common pinbase ground, rather than with BiasDACs set nominally to 0 V.

In the analysis trap, where the larger axial amplifier provides better damping and



Figure 6.10: Proton axial dip feature in the precision trap, averaged for 27 minutes.

signal/noise, the proton dip is considerably easier to observe; examples of proton dips in the analysis trap are shown in Chapter 7.

6.3 Magnetron Motion

Unlike the axial and cyclotron motions, which are detected using tuned-circuit amplifiers, the proton magnetron motion is observed only indirectly. The magnetron orbit generates relatively weak image currents at its low frequency. Moreover, since the magnetron motion is unstable, attaching an effective damping resistance would actually cause a runaway increase of the magnetron radius.

Our treatment of the magnetron motion focuses on the effects of nonzero magnetron radius on the proton axial frequency. Experimentally, we find that the axial signal is only well-behaved when the magnetron radius is small. As the magnetron radius increases, the electric potential deviates more and more from an ideal quadrupole, causing the axial signal to broaden and eventually disappear entirely. Fig. 6.11 shows the effect on the axial signal of a small deliberate magnetron heating, produced with a -47 dBm sideband drive applied for 10 seconds at the heating frequency. Since we require the best possible measurement of the axial frequency, "cooling" to reduce the magnetron radius is an essential technique.



Figure 6.11: Effect of magnetron heating on the proton axial signal in the precision trap. The response to an identical axial drive sweep is broader after a deliberate magnetron heating and narrower after sideband cooling restores the original magnetron radius.

6.3.1 Sideband Cooling

Since we are unable to directly damp the magnetron motion, we reduce the magnetron orbit by using a sideband cooling technique [1, 61, 62]. Sideband cooling in some form is ubiquitous in precision experiments with trapped particles, including the most accurate mass spectroscopy [63], the most stable optical clocks [64], and the manipulation of qubits [65]. The purpose of sideband cooling is to transfer energy from an otherwise-isolated oscillatory motion to a second oscillatory motion that is anchored to a reservoir. For laser sideband cooling of trapped ions [66, 67] and atoms in optical lattices [68, 69], the second oscillation has population essentially only in the internal ground state of the ion or atom. In our case, this second oscillation is the proton axial motion, which is damped by a resistor at our trap temperature of ~ 4.2 K.

For axial-magnetron sideband cooling, a drive is applied on a magnetron sideband of the axial frequency, at $\nu_z + \nu_m$. To couple the magnetron and axial motions, this sideband drive is applied on one half of a split compensation electrode, producing an xz potential gradient. As the drive interacts with a proton in initial axial and magnetron quantum levels (k, ℓ) , there are two possible outcomes (Fig. 6.12a). Absorption of a photon at the drive frequency increases the axial quantum state to k+1 and takes the magnetron state to $\ell - 1$. Stimulated emission causes the axial quantum number to decrease to k - 1 and the magnetron state to increase to $\ell + 1$. Transition rates for these processes depend on the value of k and ℓ (Eq. 6.7-6.8). For $\ell > k$ the absorption process dominates, causing the magnetron motion to decrease in quantum number, increase in energy, and decrease in radius. This cooling process continues, increasing k and decreasing ℓ , until the limit $\ell = k$ is reached. A sideband drive at $\nu_z + \nu_m$ will thus cool the proton to a cooling-limit radius given by $\rho_{\ell=k} = \sqrt{4\hbar\omega_m(k+1/2)/m\omega_z^2}$, where k is set by the proton axial energy (Eq. 2.15b).

The distribution of magnetron states in the sideband cooling limit can be derived by considering the transitions in Fig. 6.12b. The occupation probability $P_{k,\ell}$ satisfies the steady-state rate equation



Figure 6.12: Energy levels and transitions involved in axial sideband cooling and heating of the proton magnetron motion. (a) Absorption and stimulated emission transitions from initial state (k, ℓ) corresponding to magnetron cooling (left) and heating (right) drives. (b) Transitions involved in calculation of the steady-state limit of sideband cooling.

$$0 = -P_{k,\ell}(\Gamma_+ + \Gamma_-) + P_{k-1,\ell+1}\Gamma_- + P_{k+1,\ell-1}\Gamma_+ , \qquad (6.6)$$

where the transition rates are defined in terms of axial and magnetron raising and lowering operators [1]:

$$\Gamma_{+} \sim | < k+1, \ell - 1 |a_{z}^{\dagger} a_{m} | k, \ell > |^{2} \sim (k+1)\ell$$
(6.7)

$$\Gamma_{-} \sim | < k - 1, \ell + 1 | a_z a_m^{\dagger} | k, \ell > |^2 \sim k(\ell + 1) .$$
 (6.8)

The effective damping resistance of our tuned-circuit amplifier serves to couple the axial motion to a reservoir at T_z . The axial state thus satisfies a Boltzmann distribution, and $P_{k,\ell} = p_\ell \exp\left[-k\hbar\omega_z/k_BT_z\right]$. The magnetron distribution that solves Eq. 6.6 is then

$$p_{\ell} \sim \exp\left[-\ell \hbar \omega_m / k_B T_m\right] ,$$
 (6.9)

where T_m is an effective magnetron temperature defined by $T_m = T_z \omega_m / \omega_z$. Our direct measurements of this limiting distribution are discussed in Section 7.4. The theoretical sideband cooling limit [1, 62],

$$k_B T_m = \langle -E_{mag} \rangle = \frac{\omega_m}{\omega_z} \langle E_z \rangle = \frac{\omega_m}{\omega_z} k_B T_z , \qquad (6.10)$$

follows from evaluating the expectation value of the magnetron energy, $\langle E_{mag} \rangle = \sum_{\ell} p_{\ell} E_{\ell}$, where $E_{\ell} = -(\ell + 1/2)\hbar\omega_m$ from Eq. 2.15. Though we have assumed a resonant cooling drive, this limit also holds for drives that are slightly detuned [1].

Magnetron heating results from a sideband drive at $\nu_z - \nu_m$, where the absorption process takes $k \to k+1$ and $\ell \to \ell+1$ and dominates over stimulated emission for all k and ℓ . This produces a heating of the magnetron motion to larger ℓ , lower energy, and larger radius. A drive at $\nu_z - \nu_m$ will heat the proton to larger and larger radius, ultimately limited only by changes in ν_z as $\rho \to \rho_0$ that shift the sideband drive away from the heating resonance.

Sideband heating and cooling is most readily observed by watching for the response at the axial frequency during application of a sideband drive. For a cooling (heating) drive applied at $\nu_d = \nu_z + (-)\nu_m + \delta$, a response is visible at $\nu_d - (+)\nu_m$. Provided δ is small, the drive is close to resonance and this signal appears as a sharp peak on the axial amplifier. Note that the cooling signal is typically not visible if the magnetron motion is already well-cooled. To observe a cooling signal, we first increase the magnetron radius with a weak sideband heating drive, then switch to a cooling drive. A typical cooling signal, observed after deliberate magnetron heating, is shown in Fig. 6.13. The cooling drive strength is -47 dBm at the hat, and the signal remains



visible for several seconds before the cooling limit is reached.

Figure 6.13: Response at the axial frequency to a sideband drive on the magnetron cooling resonance.

The primary role of sideband cooling in the precision trap is to prepare the proton for detection of the axial signal and transfer to the analysis trap. While loading a new proton, a fixed sideband drive at -47 dBm is left on at the cooling frequency $\nu_z + \nu_m$. This fixed-drive cooling is usually sufficient to keep the magnetron radius small enough for our purposes. Occasionally, strong magnetron heating of the proton occurs during cyclotron drive sweeps or transfer between traps. Since the effective trapping well becomes deeper as $\rho \rightarrow \rho_0$, the proton axial frequency increases with magnetron radius, and our usual fixed-drive sideband cooling is ineffective. To cool a strongly heated proton, we use a sideband ramp procedure. The cooling drive is left on at $\nu_z + \nu_m$, and the ring electrode potential is ramped up from lower voltage (in absolute value) to higher voltage. A typical cooling ramp in the precision trap covers 0.2 V in steps of 0.5 mV every 1 second. Alternatively, the ring voltage can be fixed and the sideband drive swept down in frequency. For a strongly heated proton, several such cooling ramps may be required before the axial signal becomes visible. After each ramp, we also check for a cooling signal, to see if the magnetron radius has been sufficiently reduced that the drive at $\nu_z + \nu_m$ is effective.

6.3.2 Measurement of the Magnetron Frequency

As described in Chapter 2, only a rough measurement of magnetron frequency ν_m is required for ppb measurement of the proton g-factor. The required precision is easily obtained from the position of sideband heating and cooling signals (Fig. 6.13), which can be measured to a few Hz out of the 5 kHz magnetron frequency.

A higher precision measurement of ν_m , however, enables a test of trap misalignment via the Brown-Gabrielse Invariance Theorem [36, 56]. The magnetron frequency ν_m measured in our trap is related to the expected magnetron frequency $\tilde{\nu}_m = \nu_z^2/(2\nu_c')$ by

$$\nu_m \approx \tilde{\nu}_m \left(1 + \frac{9}{4} \theta^2 - \frac{1}{2} \epsilon^2 \right) , \qquad (6.11)$$

where ϵ indicates the size of harmonic distortions of the quadratic electric potential and θ characterizes the misalignment of the Penning trap (specifically, the angle between magnetic field \vec{B} and electric field symmetry axis \hat{z}). Except in the unlikely event that the imperfections cancel in Eq. 6.11, a comparison of ν_m and $\tilde{\nu}_m$ will thus establish bounds on θ and ϵ for our trap. The precision of this comparison is limited by the measurement of ν_m .

A variety of techniques exist for precision measurement of the magnetron fre-

quency. An axial-magnetron avoided crossing has been utilized for this purpose in bound-electron g-factor measurements [70]; we observe this behavior clearly in our analysis trap (Section 7.2), but not with the weaker axial amplifier in our precision trap. The magnetron frequency has also been measured using an axial decoupling technique, in which the correction voltage for a locked axial signal is disrupted by a resonant magnetron cooling drive [1, 71]. We have made use of a variation on the latter technique to measure the magnetron frequency to order 0.1 Hz. Instead of locking the axial signal, we excite an axial response that is slightly anharmonic, stopping the drive sweep at a frequency partway up the slope of the anharmonic lineshape (Fig. 6.14a). With the axial frequency fixed by the drive, we then monitor the strength of the axial signal as a weak (-50 dBm at the hat) sideband cooling drive is applied and ramped slowly in frequency. As the sideband drive passes through the magnetron cooling frequency, energy transfer between the magnetron and axial motions changes the axial amplitude, temporarily knocking the anharmonic response off resonance with the fixed-frequency axial drive. We observe a characteristic disruption of the measured axial signal (Fig. 6.14b), identical for a sideband drive sweep up or down in frequency (Fig. 6.14c), with a sharp feature at the sideband cooling resonance that tracks in the expected way with axial frequency (Fig. 6.14d).

For these measurements, the sideband drive is swept in 0.1 Hz steps, with 25 seconds of averaging per step. The trap-modified cyclotron frequency here is $\nu'_c =$ 86531760 Hz. Subtracting the known axial frequency from the sideband drive frequency where the sharp magnetron feature is observed, we obtain a magnetron frequency ν_m that agrees with $\tilde{\nu}_m$ to within 0.3 Hz, where the uncertainty comes from



Figure 6.14: Sideband drive sweep for precision measurement of the magnetron frequency. (a) Positioning of the axial drive frequency on the rising slope of an anharmonic response. (b) Disruption of the driven axial signal as a weak magnetron cooling drive is swept through resonance. (c) Closeup of the sharp magnetron cooling feature, showing identical behavior with sideband drive swept up or down in frequency. (d) Close-up of the sharp magnetron cooling feature at two different settings of the driven axial frequency.

the scatter of repeated measurements using this technique. If we attribute the entire discrepancy to angular misalignment, this sets a bound of $\theta < 0.005$, or roughly 0.3° . Alternately, this deviation sets a bound $\epsilon < 0.01$ on possible harmonic distortion of the electric potential.

Chapter 7

Proton Motions in the Analysis Trap

The analysis trap is designed to enable single-proton spin-flip detection, with a magnetic bottle 51 times stronger than was used in the electron g-factor experiment [72] and 8 times stronger than used in bound-electron g-factor measurements [73]. In order to observe the spin-flip shift, the proton axial frequency in the analysis trap must be resolved to a precision better than 60 mHz. The unusually large magnetic gradient complicates this task, introducing shifts in the axial frequency due to radial motions of the proton.

7.1 Precision Axial Measurement Techniques

The proton axial signal in the analysis trap is initially detected and tuned using voltage and drive sweeps as described in Chapter 6. The drive/detection scheme is

	Cyclotron	Axial	Mag.	I.	Harmonic	Axial	
Trap	Freq.	Freq.	Freq.	V_{ring}	Ratio	Damping	D
	(MHz)	(kHz)	(kHz)	(\mathbf{v})	V_{comp}/V_{ring}	$\frac{\gamma_z}{2\pi}$ (Hz)	
Precision	86.5	940	5.1	-4.33	0.866	0.6	5.68
Analysis	79.6	553	1.9	-1.62	0.780	2.7	5.22

Table 7.1: Comparison of trap parameters and proton frequencies for the precision and analysis traps

identical to that used in the precision trap, aside from a different axial frequency. Table 7.1 compares relevant quantities in the two traps.

In the precision trap, the axial frequency must only be resolved to better than 10 Hz, in order to obtain ω_c to a ppb using the Invariance Theorem. In the analysis trap, we require a measurement of the axial frequency precise enough to detect a 60 mHz spin flip. Two tools described here for high-resolution measurement in the analysis trap are axial dips and locking of the driven axial signal. A third technique, self-excitation, is described in Chapter 8.

7.1.1 Axial Dips

A key difference between the traps is the substantially improved damping rate in the analysis trap. Space constraints in the tripod region limit us to one large (2.0" amp can ID) and one small (1.25" amp can ID) axial amplifier. We reserve the large amp for the analysis trap, where axial frequency resolution is more critical. As a consequence of this larger axial damping, proton axial dips become considerably easier to resolve. A comparison of axial dip features is shown in Fig. 7.1.

We also gain dramatically in averaging time; we can begin to observe a dip after



Figure 7.1: Proton axial dip features averaged for 800 seconds in (a) precision trap and (b) analysis trap.

averaging several seconds in the analysis trap, compared to several minutes in the precision trap. This short time scale, combined with relative insensitivity to the comp/ring tuning ratio, makes the axial dip a useful tool for axial detection in the analysis trap, generally more convenient than drive sweeps. Fig. 7.2 shows the single-proton axial dip clearly visible in a wide frequency window after only one minute of averaging.

7.1.2 Locked Driven Axial Signal

By applying feedback that adjusts the trapping potential and corrects for frequency shifts, the proton axial signal may be locked at a fixed frequency. The feedback voltage is monitored and converted to a change in proton axial frequency; i.e. the shift in frequency that would occur if the trapping potential was not being adjusted. By locking the axial signal, we are thus able to monitor changes in the axial frequency without actually shifting away from the center of our mixdown/detection electronics. This technique proves particularly useful for measuring the proton radius



Figure 7.2: Proton axial dip feature averaged for one minute in the analysis trap. With this frequency span, the shape of our tuned-circuit axial amplifier is visible in the background profile.

via magnetron heating, as described in Section 7.3.2.

A suitable error signal for feedback is generated by separating the in-phase and quadrature components of the driven axial response. The driven proton axial motion is described in Eq. 6.4. If we have tuned the trap to be as harmonic as possible, $\omega_z(A) \approx \omega_0$. For a drive $F_d(t) = F \cos(\omega t)$, we then obtain the standard solution for a damped, driven harmonic oscillator,

$$z(t) = z_{cos}\cos(\omega t) + z_{sin}\sin(\omega t) , \qquad (7.1)$$

where the in-phase (cosine) and quadrature (sine) amplitudes are given by

$$z_{cos} = \frac{F}{m} \frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \gamma_z^2 \omega^2}$$
(7.2)

$$z_{sin} = \frac{F}{m} \frac{-\gamma_z \omega}{(\omega_0^2 - \omega^2)^2 + \gamma_z^2 \omega^2} .$$
 (7.3)



Figure 7.3: Radiofrequency schematic for generating the error signal used for axial frequency lock.

The in-phase response, described by a dispersive Lorentzian lineshape, contains the sharp zero-crossing feature required for an error signal. To separate out this component of the response, we use the setup shown in Fig. 7.3. One additional frequency synthesizer is added to the standard axial drive/detection scheme (Fig. 6.6), allowing us to independently adjust the signal phase before the final mixdown stage. By setting this phase to match the cosine response, only the in-phase component survives the mixdown to DC, and we obtain the desired error signal as a voltage on the Fluke DMM. The cosine and sine components can also be obtained digitally, by programming the DAQ card to perform an equivalent phase-sensitive Fourier transform. The error signal is chosen to have positive slope on the zero-crossing feature (Fig. 7.4),



Figure 7.4: Error signal for locking the proton axial response. (a) Proton axial response separated into in-phase and quadrature components. (b) Analog error signal measured on the Fluke DMM.

which provides the proper sign of voltage feedback for a proton trap.

The feedback voltage is determined by a digital PID controller algorithm, sampling the error signal at a rate of approximately 5 Hz. In principle, the control voltage can be applied to any of the trap electrodes. In practice, we choose to apply feedback corrections to the transfer electrodes T2 and T3, due to long RC time constants on the analysis trap ring and endcap electrodes.

7.2 Avoided Crossing

Though unrelated to the magnetic bottle, our axial frequency resolution in the analysis trap allows a particularly clear demonstration of the avoided-crossing feature that results from sideband coupling, with application to measurement of ω_m . This avoided crossing has been described using a classical dressed-atom formalism [74] and demonstrated experimentally with trapped ions [74, 75].

For sideband cooling of the proton magnetron orbit (Section 6.3.1), the axial and

magnetron motions are coupled by an RF drive at frequency $\omega_z + \omega_m + \delta$, with δ a small detuning from the cooling resonance. In the presence of this coupling, the axial frequency acquires a shift from ω_z to $\omega_z + \epsilon_{\pm}$ [74, 75], where

$$\epsilon_{\pm} = \frac{\delta}{2} \pm \frac{1}{2}\sqrt{\delta^2 + |V|^2}$$
(7.4)

for effective sideband drive strength |V| and detuning δ .

Fig. 7.5 demonstrates experimental observations of the axial-magnetron avoided crossing feature, measured using proton axial dip responses at various sideband drive settings. A standard proton dip with the axial and magnetron motions decoupled is shown in Fig. 7.5a. With a sideband drive sufficiently strong and close to resonance, the axial response splits into two dips (Fig. 7.5b) as suggested by Eq. 7.4. The average axial frequency (midpoint of the split response) shifts by the expected $\delta/2$ as the drive is detuned (Fig. 7.5d, Fig. 7.5e). Since the magnitude of the splitting is observed to increase with drive strength (Fig. 7.5c) but not with detuning (Fig. 7.5d, Fig. 7.5e), we conclude we are in a regime where $|V|^2 >> \delta^2$.

The magnetron frequency ω_m is easily measured at the 1 Hz level by tuning for maximum symmetry of the split-dip feature, which occurs at $\delta = 0$. For $\delta > 0$, a positive detuning of the sideband drive, the lower-frequency axial response, corresponding to ϵ_- , appears stronger than the higher-frequency axial response, corresponding to ϵ_+ (Fig. 7.5d). If the sideband drive frequency is instead too low ($\delta < 0$), the higher-frequency axial response appears stronger (Fig. 7.5e). Additional effort could potentially yield a measurement of ω_m at higher precision [70] and also a calibration of the drive strength V.



Figure 7.5: Avoided crossing axial "double dip" feature observed in the presence of a magnetron cooling drive. (a) Axial dip feature with no sideband drive. (b) Axial dip feature split symmetrically by application of a -72 dBm cooling drive at $\nu_z + \nu_m$. (c) Increased splitting due to stronger (-67 dBm) cooling drive. (d) Asymmetric splitting due to -72 dBm drive at $\nu_z + \nu_m + 5$ Hz. (e) Asymmetric splitting due to -72 dBm drive at $\nu_z + \nu_m - 5$ Hz.

7.3 Shifts due to Radial Motions

7.3.1 Cyclotron Effects

As discussed in Section 2.2, the magnetic bottle couples radial and axial motions, such that the proton axial frequency in the analysis trap depends on both the distribution of cyclotron states n and the magnetron states ℓ . For the cyclotron case, recasting Eq. 2.30 in terms of parameters in the analysis trap, we have

$$\Delta \nu_z = 0.021 \ Hz \cdot \Delta n \ . \tag{7.5}$$

This shift is potentially quite large; for a typical analysis-trap cyclotron radius of 0.5 μ m, as would result from adiabatic transfer from the precision trap, the axial frequency shift due to a 1 μ m increase in ρ_c is almost 200 Hz. Since we require measurement of ν_z to better than 100 mHz, the cyclotron quantum state must essentially remain unchanged during an attempt at spin-flip detection. Equivalently, the cyclotron radius must be stable to better than 1 nm. To prevent thermal fluctuations in the cyclotron state, we deliberately avoid connecting a cyclotron tuned-circuit amplifier in the analysis trap, since any damping resistance would couple the proton cyclotron motion to the 4 K trap environment. Since the cutoff frequency for our trap cavity is well above the 80 MHz cyclotron frequency, the proton cyclotron damping rate in the analysis trap should be effectively zero. With improved confidence in our transfer sequence, this assumption could be tested by transferring an excited cyclotron state to the analysis trap, waiting some time, and then transferring back to the precision trap to check for any change in the cyclotron radius by measuring the size of the relativistic shift.

Without any cyclotron damping, the axial frequency in the analysis trap is determined by whatever cyclotron state n we have "locked in" as the transfer from the precision trap begins. The distribution of n in the precision trap is in turn determined by the amount of damping time we allow the proton after our most recent cyclotron excitation. The effect of different n on the analysis-trap axial frequency is shown in Fig. 7.6. In the precision trap, each of the protons displayed in Fig. 7.6 is sufficiently damped for measurement, meaning that the cyclotron signal is reduced below the noise floor and the driven axial signal is visible. In the analysis trap, however, the same set of protons exhibits an enormous variation in axial frequency, corresponding to many linewidths of our tuned-circuit amplifier ($\Gamma \approx 150$ Hz). To reduce this variation, which otherwise complicates our initial search for the proton signal in the analysis trap, we ensure that the proton has been damped down to a thermal amplitude in the precision trap before transfer. After a driven excitation for cyclotron measurement purposes, this can require waiting as long as 15 cyclotron lifetimes (Chapter 6). Fully damping the proton in this way serves two purposes. First, by keeping the proton as close as possible to trap center, we minimize the effect of fluctuations $\Delta \rho_c$ on the axial frequency. Second, since our uncertainty in n is now set by the width of a Boltzmann distribution at T_c , our search window for the axial frequency in the analysis trap is greatly reduced. We occasionally observe larger shifts which we attribute to heating during transfer, but if we follow this full-damping protocol in the precision trap, the proton signal in the analysis trap generally repeats to within 100 Hz after each transfer (roughly 1 mV in terms of ring voltage), corresponding to a $\Delta n \approx 4000$ and $T_c \approx 16$ K. (Note that this effective cyclotron temperature includes the effect of any heating during transfer between the traps. We expect T_c in the precision trap to be closer to 4 K.)

As discussed in Chapter 6, we go to great lengths to ensure that there is only one proton in our trap, to avoid unwanted perturbations from a second trapped proton. The deleterious effects of a second proton are magnified in the analysis trap by the strong cyclotron-axial bottle coupling. In the precision trap, the axial signal from two trapped protons looks qualitatively similar to the single-proton axial response, but with twice the frequency width. In the analysis trap, however, the magnetic bottle



Figure 7.6: Shift of the analysis-trap axial signal for protons damped to different cyclotron radii in the precision trap. A proton in a larger cyclotron orbit has a higher axial frequency and thus requires a lower ring voltage to come into resonance with our fixed-frequency drives and amplifier.

produces a substantial axial frequency difference between the two protons, since each has a cyclotron quantum number n selected from a broad thermal distribution. We observe two common signatures of a second proton in the analysis trap. First, we are unable to effectively tune out the trap anharmonicity to produce a narrow drivesweep response for two protons in the presence of the bottle. Second, the axial dip feature has poor signal/noise and a width much greater than the $2\gamma_z$ expected for two protons in a trap with no bottle.

7.3.2 Magnetron Effects

The magnetic bottle couples the proton magnetron and axial motions. Analogous to the cyclotron case above, Eq. 2.30 gives the axial-magnetron relation

$$\Delta \nu_z = 5.2 \times 10^{-7} \ Hz \cdot \Delta \ell \ , \tag{7.6}$$

or roughly 40 mHz per μ m around our typical value of the magnetron radius. For unit change in quantum state or ρ , the magnetron effect on axial frequency is much smaller than the cyclotron effect (Eq. 7.5). However, given the low frequencies of the magnetron and axial-magnetron sideband resonances, unwanted heating is far more common in the magnetron case.



Figure 7.7: Effect of magnetron heating on the proton axial signal in the analysis trap. The response to an identical axial drive sweep shifts to increasingly higher frequency as a sideband heating drive increases the magnetron radius. The observed variation in axial amplitude occurs as the response moves off-center with respect to our tuned-circuit axial amplifier. Detuning of the fixed-frequency sideband drive increases with ν_z , preventing the runaway exponential heating observed in Fig. 7.8b.

As in the precision trap, we rely on axial-magnetron sideband cooling and heating to manipulate the magnetron motion. The signature of magnetron heating in the analysis trap is a visible shift in the axial frequency, shown in Fig. 7.7. Contrast this with Fig. 6.11; in the absence of a magnetic bottle we observe a broadening of the axial signal with heating, but no significant axial frequency shift.

7.4 Probing the Limits of Sideband Cooling

The bottle coupling can be exploited to measure the magnetron state distribution, derived in Section 6.3.1, that results from axial-magnetron sideband cooling in the analysis trap [76]. This serves as the first probe of a sideband cooling distribution for the case where the oscillator coupled to the cooling reservoir has some nontrivial thermal occupation. Earlier work featuring a similar coupling used an axial-cyclotron sideband coupling only in the heating regime, increasing an ion cyclotron temperature from 5 K to 1500 K [75, 77].

To observe the distribution resulting from sideband cooling, we make repeated measurements of the proton axial frequency, applying a strong resonant sideband cooling drive for several seconds between the axial measurements. Each application of sideband cooling results in a different magnetron state drawn from the cooling-limit distribution,

$$p_{\ell} \sim \exp\left[-\ell\hbar\omega_z/k_B T_z\right]$$
, (7.7)

where we have recast Eq. 6.9 in terms of the proton axial temperature and we assume $\ell = k$ at the cooling limit. Each such reselection of the magnetron state ℓ in turn produces a measurable shift in the axial frequency, by Eq. 7.6. Over repeated trials, we build up a histogram of measured axial frequencies which reveals the distribution of magnetron states that results from sideband cooling. To carry out this procedure

in reasonable time, the axial frequency measurements must be made rapidly and with high precision; for this we utilize the single-proton self-excited oscillator, described in Chapter 8.

Histograms generated in this way are plotted in Fig. 7.8a. The gray histogram shows the scatter of repeated axial frequency measurements when no sideband cooling is applied. This histogram is labeled as $T_z = 0$, since it indicates the unavoidable background scatter that we observe even with no explicit changes to the magnetron radius. This background distribution fits to a narrow Gaussian. When sideband cooling is applied (green histogram in Fig. 7.8a), the magnetron motion is cooled to the limit set by the axial thermal reservoir (Eq. 6.10). We measure the temperature (T_z) of this reservoir by matching the observed histogram to a convolution of Eq. 7.7 and the background Gaussian. The axial temperature obtained is 8 ± 2 K, reasonably higher than the 5.2 K realized with one electron in a 1.6 K apparatus [78].

The cooling limit depends on the temperature of the thermal reservoir. We can vary this temperature by applying axial feedback, as discussed in Chapter 8. Feedback cooling to $T_z = 4$ K reduces the distribution of magnetron states (blue histogram in Fig. 7.8a). Feedback heating to $T_z = 20$ K broadens the distribution (red histogram in Fig. 7.8a).

We can also probe the cooling limit by measuring the magnetron radius that results from sideband cooling to the limit. After applying our strong cooling drive to reduce the magnetron orbit as far as possible, we drive on the sideband heating resonance, producing an exponential increase in the magnetron radius with time [1], which by Eq. 7.6 becomes an exponential increase in the axial frequency. As shown



Figure 7.8: Probing the limits of axial-magnetron sideband cooling. (a) Histograms of magnetron states after no sideband cooling (gray), and produced by sideband cooling using feedback cooling (blue), no feedback (green), and with feedback heating (red). Solid curves are convolutions of the gray Gaussian resolution function and Boltzmann distributions at the specified T_z . (b) A weak (-128 dBm) heating drive is applied to produce exponential increase in the axial frequency. A relatively strong (-72 dBm) cooling drive is applied to restore the original frequency. The effective magnetron cooling limit can be extracted from the offset between $t \to -\infty$ and t = 0 in an exponential fit to the heating data.

in Fig. 7.8b, we fit the resulting heating curve to determine the effective magnetron cooling limit, which is the difference between our starting point and trap center.

In order to maintain resonant sideband heating as the proton axial frequency increases, this magnetron heating measurement is performed with the axial signal locked as described in Section 7.1.2. We obtain an effective cooling limit of 11(2) μ m, consistent with our temperature measurement [76] within experimental uncertainty, and roughly twice as large as the theoretical cooling limit for an axial temperature of 4.2 K. Earlier measurements with trapped electrons yielded an effective cooling limit radius 20 times larger than this theoretical limit [1, 79], corresponding to axial temperature in excess of 1000 K. More recent measurements with trapped ions [75, 77]
yielded axial temperatures of 60-70 K, again in an apparatus that was nominally at 4.2 K. We attribute the lower temperatures observed in our apparatus primarily to improvements in detection electronics that affect the axial temperature (Chapter 4).

The low proton temperatures demonstrated here are critical for measurement of g_p . Combining feedback cooling with sideband cooling, the proton magnetron motion is cooled to an effective temperature $T_m = 14$ mK. In terms of a cooling-limit radius, the proton is thus cooled within 6 μ m of trap center, minimizing the effects of electrostatic and magnetic anharmonicities. The power required to drive spin-flip transitions is also reduced (Chapter 9).

Fig. 7.8a also illustrates a key restriction for our eventual measurement of g_p : while attempting to detect spin-flips, the sideband cooling drive must be left off. If sideband cooling is applied, the magnetron state will be randomly reselected from the cooling-limit distribution, producing an axial frequency shift that is larger on average than the 60 mHz shift from a spin flip.

Chapter 8

Self-Excitation and Feedback Cooling of an Isolated Proton

We apply techniques developed with a single trapped electron [78, 80] to realize the first feedback cooling and self-excitation of a single proton [76]. The basic setup for feedback manipulation of the proton is shown in Fig. 8.1.



Figure 8.1: Penning trap electrodes and radiofrequency schematic for feedback cooling and self-excitation of the proton axial motion. G represents gain and ϕ an adjustable phase offset in the feedback loop.

The proton axial oscillation satisfies the equation of motion

$$\ddot{z} + \gamma_z \dot{z} + [\omega_z(A)]^2 = F_d(t)/m.$$
 (8.1)

The proton axial signal is amplified, phase-shifted, and used as a feedback drive $F_d(t) \sim mG\gamma_z \dot{z}$, where G is the feedback gain in Fig. 8.1. Depending on feedback gain and phase, this driving force serves to enhance or suppress the damping force $-m\gamma_z \dot{z}$ provided by effective resistance R. With G < 1 and ϕ chosen for negative feedback, the result is feedback cooling (Section 8.1). With G = 1 and ϕ chosen for positive feedback, the result is self-excitation of the axial motion (Section 8.2.1).

In practice, we avoid direct feedthrough to the amplifier by applying the feedback in a two-drive scheme. For self-excitation, an amplitude limiter is also required, as described in Section 8.2.1. A complete schematic is shown in Fig. 8.2.



Figure 8.2: Radiofrequency schematic for feedback cooling and self-excitation of the proton axial motion.

8.1 Feedback Cooling

Feedback cooling is a suppression of the Johnson noise in the damping resistance R. Feedback cooling has been realized with a trapped electron [78] and also extended to other systems. Recent examples include "cold damping" of one motional degree of freedom of a ¹³⁸Ba⁺ ion in an RF (Paul) trap [81], and cooling the mechanical mode of a microscale silicon cantilever to the millikelyin regime [82, 83].

For feedback cooling of our trapped proton, we use the scheme of Fig. 8.2. A portion of the axial signal detected on the upper endcap and compensation electrodes is mixed down to an intermediate frequency (f_I) , then fed back on the lower endcap along with a second feedback drive at $\nu_z - f_I$. By using two independent frequency synthesizers for the two appearances of $\nu_z - f_I$ in the mixdown chain, we can adjust the phase of one to apply a relative phase shift to the feedback (ϕ in Fig. 8.1). The DSP-controlled voltage-variable attenuator (VVA), used for amplitude limiting of a self-excited signal (Section 8.2.1), is unnecessary for feedback cooling and can be set to a fixed attenuation by switching out the DSP.

Since our feedback is proportional to the proton image current, we can write the axial feedback drive in the form $F_d(t)/m = G\gamma_z \dot{z}$, where G is then the feedback gain. The effect of such a feedback drive is to modify the damping term in the equation of motion for the damped, driven proton axial oscillation, which becomes

$$\ddot{z} + (1 - G)\gamma_z \dot{z} + \omega_z^2 z = 0.$$
(8.2)

For our current purposes, the feedback phase ϕ serves only to set the sign of the feedback drive, selecting either feedback "cooling" (G > 0 in Eq.8.2) or feedback

"heating" (G < 0 in Eq.8.2). The proton axial motion is considered here to be perfectly harmonic. The effects of anharmonicity and feedback phase are treated in greater detail in Section 8.2.1.

To select the proper gain G and phase ϕ for feedback cooling and heating, we utilize the proton axial dip feature, which becomes narrower with feedback cooling (Fig. 8.3c) and broader with feedback heating (Fig. 8.3a). If G is too weak, the dip is unchanged from its standard value (Fig. 8.3b). If G is sufficiently strong to affect the damping, but ϕ differs by more than ~ 20 degrees from the optimal phase for heating or cooling, the dip feature becomes visibly asymmetric.



Figure 8.3: Feedback modification of the axial damping rate. Proton axial dip features are shown with (a) feedback to increase damping; (b) no feedback; (c) feedback to reduce damping.

A noiseless analysis of feedback cooling [35] predicts reduction in axial damping width Γ from Γ_{z0} at G = 0 to $\Gamma_z(G) = (1 - G)\Gamma_{z0}$. The axial temperature is reduced similarly, from T_{z0} at G = 0 to $T_z(G) = (1 - G)T_{z0}$. Fig. 8.4 shows the observed dependence of T and Γ on feedback gain. Damping widths Γ are measured from the half-widths of proton axial dips at the various gain settings (Fig. 8.3). Temperatures Tare measured using the technique described in Section 7.4. The ratio of temperature and damping width is a fluctuation-dissipation invariant. To calibrate the x-axis in these plots, the known setting for G = 0 (no feedback) is combined with the x-intercept value of G = 1 obtained from a simultaneous fit to the damping and temperature data.



Figure 8.4: Measured axial damping widths (a), temperatures (b), and their ratios (c) as a function of the feedback gain G.

The reduction in temperature from feedback cooling is limited by technical noise [78]. In our current apparatus we are able to cool by roughly a factor of two, somewhat less than the factor of six obtained with an electron. We attribute the discrepancy to a higher amplifier noise temperature in our 4.2 K apparatus, compared to the 1.6 K apparatus used for the electron. The recent addition of a second-stage axial amplifier (Section 4.2.3) promises to reduce this noise temperature and allow further cooling.

The expected benefit of feedback cooling for the proton experiment is a reduction in the spin-flip linewidth with temperature. By cooling the axial motion, which sets this linewidth due to the bottle broadening effect, we hope to achieve comparable Rabi frequency for the spin-flip in a shorter drive time (Chapter 9). Since our frequency resolution is limited by background drift effects (Section 9.1), any such reduction in measurement time is critical.

8.2 Self-Excitation

8.2.1 One-Proton Self-Excited Oscillator

A one-proton self-excited oscillator (SEO) is realized using the same basic setup as for axial feedback cooling, but with the feedback gain increased to G = 1. The damping term in Eq. 8.2 goes to zero, and the proton drives its own axial oscillation by positive feedback, generating a strong and narrow axial frequency signal. Fig. 8.5 shows a one-proton SEO signal clearly visible after only four seconds of averaging. As described below and in Section 9.1, the SEO response has several characteristics particularly well-suited to precision measurement of the axial frequency, as needed for detection of a proton spin-flip. In fact, the first report of a single-electron SEO [80] proposed just this application, if self-excitation could be extended to the proton regime.

A key experimental challenge with self-excitation is that if G deviates even slightly from unity, an exponential increase or decrease in the proton amplitude A will result, causing either damping of the signal below the noise floor, or runaway excitation that is limited only by trap geometry. Stabilizing the self-excited response thus requires limiting the oscillation amplitude to some fixed value A_0 [84, 80]. For our proton SEO,



Figure 8.5: Signal from a single-proton self-excited oscillator after four seconds averaging time (two FFT scans with dwell time two seconds each).

we use an electronic limiting scheme similar to that developed for one electron [80]. A digital signal processor (DSP) chip is programmed to Fourier transform the proton signal and output an amplitude-dependent voltage, which controls a voltage-variable attenuator in the feedback line (Fig. 8.2). The performance of our DSP limiter is described in Section 8.2.2.

The self-excited response is governed by a feedback-modified equation of motion similar to Eq. 8.2, but now we must treat the feedback phase explicitly. For a proton axial oscillation $z(t) = A\cos(\omega t)$, the effect of our feedback loop, including the adjustable phase shift (Fig. 8.1), is a force $F_d(t) = -\omega A Gm \gamma_z \sin(\omega t + \phi)$. Plugging z(t) and $F_d(t)$ into the anharmonic axial equation of motion (Eq. 6.4), and separately equating the in-phase $(\sin(\omega t))$ and quadrature $(\cos(\omega t))$ components, we obtain

$$G\cos(\phi) = 1 \tag{8.3}$$

$$G\omega\gamma_z\sin(\phi) = \omega^2 - [\omega_z(A)]^2 . \qquad (8.4)$$

Eq. 8.3 restates the basic condition for self-excitation: the positive feedback drive must exactly cancel the damping term. Eq. 8.4 describes the effect of the quadrature component of the feedback, which shifts the axial frequency from its value in the absence of feedback. This effect is revealed more clearly by combining the equations to obtain

$$\omega(A,\phi) \approx \omega_z(A) + \frac{\gamma_z}{2} \tan(\phi) , \qquad (8.5)$$

where we have applied $\gamma_z \tan(\phi)/\omega_z(A) \ll 1$, valid so long as ϕ does not approach $\pi/4$ (the boundary between positive and negative feedback).

For a fixed gain G, the SEO response appears over the range of feedback phases corresponding to positive feedback. This effect is observed in Fig. 8.6a. The variation in signal strength with phase is a consequence of our amplitude-limiting gain control. As ϕ is adjusted, the steady-state oscillation amplitude A_0 must also change, increasing or decreasing G(A) to maintain Eq. 8.3. For a given set of feedback parameters, the SEO signal is strongest at $\phi = 0$, where the feedback drive is exactly in-phase with the proton oscillation. This phase $\phi = 0$ is also shown to minimize the phasedependent SEO frequency shift, plotted in Fig. 8.6b with a fit to Eq. 8.5 (the change in $\omega_z(A)$ over this phase range is small and is included in the error bars).



Figure 8.6: (a) SEO signal strength vs. feedback phase. Positive feedback results in self-excitation. Negative feedback produces a feedback-broadened axial dip, causing the detected signal to drop slightly below the background (gray) measured without a proton. (b) Measured axial frequency vs. feedback phase (points) fit to the expected Eq. 8.5.

8.2.2 DSP Performance

The digital signal processor for the proton SEO is modified only slightly from the original version developed for the electron SEO [35]. While the proton axial frequency is substantially lower than ω_z for the single electron, the DSP analyzes a mixed-down signal at a frequency much lower than either, typically 5 kHz. Also, after considerable efforts to optimize the proton detection amplifiers, our proton axial damping rate is made comparable to γ_z for the single electron. However, since our frequency stability requirements are far stricter with the proton SEO, we have made several modifications to the electron DSP program, for the purpose of optimizing amplitude control.

Functionality of the basic DSP code is described in detail in Appendix C of reference [35]. The DSP samples the proton axial signal at a rate $f_s = 1/\Delta t$ (typically 25 kHz), then takes a discrete Fourier transform, storing the sine and cosine transform values in a number of frequency bins. The number and spacing of these frequency bins defines the active DSP window around some center frequency (typically 5 kHz). Previous transform data is multiplied after each new sampling period by a constant $0 < \alpha < 1$, producing an exponential weighting in time. The time constant $\tau = \Delta t/(1-\alpha)$ sets the effective averaging time of the DSP and determines the effective bin width of the Fourier transform. The DSP control signal is obtained simply by keeping track of the maximum total power ($\sin^2 + \cos^2$) in any frequency bin, and then applying some output proportional to this maximum power.

Our updates for the proton experiment focus mainly on providing real-time control of various DSP parameters, allowing us to probe the effects of these settings on axial frequency stability. Convenient on-the-fly adjustment of parameters in the DSP code is enabled by a new Ethernet interface; the previous DSP could be modified only by disconnecting from the experiment, switching to a computer test setup, and writing a new program to the DSP Flash memory. We have now made several parameters addressable via Ethernet, to control the strength and speed of the DSP response. In particular, we can address the polynomial feedback coefficients that determine the size of the DSP output signal, and the parameter α that sets the effective time constant of the DSP response. We also control a bit-shift parameter that sets the overall scale of the response and prevents the DSP accumulator register from overflowing as we go to longer averaging times.

Fig. 8.7 shows the effective DSP time constants as we vary α . The observed time constants, obtained by fitting the response to an amplitude-modulated test signal, match well with values τ calculated from α and the 5 kHz sampling rate. We are thus able to adjust the effective DSP response time over a wide range of values. Fig. 8.8



Figure 8.7: Effective DSP averaging time for 5 kHz sampling rate and various settings of the exponential weighting constant α .

demonstrates how this adjustability is used to optimize the proton SEO stability. Experimentally, we observe the best SEO performance when the DSP averaging time is tuned to be slightly longer than the proton axial damping time, $\tau_z = 1/\gamma_z \approx 0.06$ seconds.

Beyond simply manipulating the parameters of the current DSP program, we can consider improving the algorithm for signal control and analysis, up to the restrictions set by DSP memory and processing time (i.e. limited available memory addresses, and a total allowed time Δt before the next signal acquisition occurs). In particular, improving amplitude control of the proton SEO is an ongoing goal. One useful feature of the electron SEO was the ability to tune the amplitude and trap anharmonicity such that the electron oscillated in a locally harmonic potential, with $d\omega_z/d(A^2) = 0$. Frequency stability was shown to be minimized at such a "harmonic point" [80]. The remaining frequency standard deviation at a harmonic point,



Figure 8.8: Scatter in repeated proton SEO measurements, with different values of the effective DSP time constant.

$$\sigma_{\omega_z} = \frac{15C_6 A_d^2 k_B T_z}{4C_2 d^4 m \omega_z} , \qquad (8.6)$$

is then due to thermal fluctuations in the axial amplitude [35], where A_d is the SEO amplitude and C_4 and C_6 are anharmonicity coefficients as defined in Eq. 2.10. In principle, the effect of Eq. 8.6 should only broaden the axial linewidth, not create a frequency scatter, since the thermal fluctuations are rapid compared to the time needed to measure ω_z . However, since we do observe an increase in SEO stability as we tune the anharmonicity (Section 9.1), it remains possible that some combination of anharmonicity and imperfect amplitude control could be contributing scatter on the time scale of an axial frequency measurement.

With the proton SEO, we currently observe an optimal anharmonicity tuning for a given feedback gain (Fig. 9.3b), but without clear evidence that this is a harmonic point in the sense that $d\omega_z/d(A^2) = 0$. There may be no harmonic point at accessible amplitude for our proton trap parameters, or the characteristic frequency shifts with feedback gain may currently be masked by drive strength systematics. Fig. 8.9a demonstrates the behavior observed thus far. We increase the SEO amplitude by increasing the strength of the local oscillator in our two-drive scheme. SEO frequency is more independent of amplitude (flatter slope in Fig. 8.9a) at a comp/ring tuning ratio of 0.78071; however, SEO frequency stability is better (smaller error bars in Fig. 8.9a) at a ratio of 0.78011.

Meanwhile, Fig. 8.9 suggests that amplitude limiting for the proton SEO is not yet optimized. On average (Fig. 8.9b), the DSP does function as desired: as feedback gain increases, the SEO amplitude increases, and the DSP control voltage decreases in order to call for more attenuation in the feedback line. However, considerable short-term fluctuations are evident in the raw data for the DSP output (Fig. 8.9c), and in a set of repeated SEO amplitude measurements (Fig. 8.10). The DSP output fluctuations are in fact much larger than what would produce the observed 60 mHz scatter in the proton axial frequency, indicating that some additional averaging is present and not yet understood. However, clearly we would prefer a better amplitude lock. Two potential improvements currently under investigation are (1) modifying the DSP algorithm for better amplitude control, as described below, and (2) reducing noise in the feedback drive, perhaps by switching to an alternate drive scheme. Our two-drive scheme is experimentally convenient, but requires relatively large drive strength; the single compensated drive scheme ultimately used for the electron SEO [35] might allow gentler feedback for the same SEO signal.

A recent modification to the DSP algorithm, coded but not extensively tested,



Figure 8.9: Proton SEO frequency and DSP control signal with increasing feedback gain. For two values of trap anharmonicity, proton SEO signals shift differently in frequency (a) as feedback gain is increased in regular steps. The averaged DSP control signal (b) demonstrates a similar increase in SEO amplitude for both cases. The raw DSP control data (c), plotted against a corresponding time axis, shows considerable short-term fluctuation that increases with SEO amplitude.

is designed to reduce sensitivity of the control response to background noise spikes. This new program keeps track of the frequency bin in which the maximum axial signal is observed, and then modifies the correction output only if that frequency is within acceptable range of a setpoint value. Both the range and setpoint are specified on-thefly by the user, providing a means of instructing the DSP to respond only to known



Figure 8.10: Correlation between peak signal strength and frequency, as seen by the DSP. Scans over the active window are recorded with averaging time 0.33 seconds that matches the effective DSP time constant. The proton SEO signal is inconsistent in strength, even dropping at times below the background level, such that the DSP responds instead to a noise peak at random frequency.

proton signals, rather than to background noise spikes in the active window, which can appear stronger than the proton signal for short averaging times (Fig. 8.10).

Chapter 9

Attaining Spin-Flip Resolution

Proton spin-flips will be detected via the small (currently 60 mHz) shift in axial frequency created by coupling to a strong magnetic bottle. The one-proton SEO now provides resolution of the axial frequency commensurate with this 60 mHz spin-flip shift, opening a path to spin-flip detection. We describe the capabilities and current limitations of frequency resolution with the SEO, along with prospects and challenges for proton spin-flip detection.

9.1 Frequency Resolution with the SEO

The one-proton self-excited oscillator provides a powerful tool for measuring the proton axial frequency, ultimately enabling frequency resolution at the level of the 60 mHz shift due to a single-proton spin flip. Fig. 9.2 compares axial frequency measurements made with the SEO and the alternate method of axial dips (Section 7.1.1). Since the self-excited axial response (Fig. 9.2a) corresponds to a large-amplitude os-

cillation, while the axial dip feature (Fig. 9.2b) is driven only by Johnson noise in the detection resistance, signal/noise is considerably better with the SEO. The SEO linewidth is also narrower by more than an order of magnitude due to feedback inhibition of the axial damping γ_z .



Figure 9.1: Linewidth of the proton axial SEO response, observed with the specified FFT dwell time.

The SEO signal is measured by taking a Fast Fourier Transform (FFT) of the time trace from our proton axial amplifier. For short FFT dwell times, the SEO linewidth is given by $\Gamma \approx 1/t$, as narrow as allowed by the frequency-time uncertainty relation governing the effective resolution of the FFT. At longer dwell times, the SEO linewidth is limited by trap anharmonicity and fluctuations, such that the observed signal does not continue to narrow indefinitely with effective FFT bin size. Fig. 9.1 shows the linewidth for 160 seconds of SEO data, analyzed with different FFT dwell times; e.g. as 320 sets of 0.5-second traces, or 10 sets of 16-second traces.



Figure 9.2: Comparison of axial frequency measurements using self-excited oscillator and axial dips. (a) SEO peak and (b) noise dip, each averaged 160s. (c) Frequency resolution achieved with a single average of an SEO peak (black x) and noise dip (red x), with the standard deviation (black circles) and Allan deviation (blue circles) of averaged SEO measurements. (d) Drift of 256s averages over sixteen nighttime hours.

This combination of improved signal/noise and decreased linewidth leads to greatly improved "line-splitting" of the SEO signal. The line-splitting resolution of a single dip or SEO measurement is characterized by the uncertainty in center frequency of a Lorentzian fit to the axial frequency spectrum. While an axial dip must be averaged for a few minutes in order to obtain line-splitting sufficient to detect a 60 mHz proton spin-flip (red x in Fig. 9.2c), substantially better line-splitting is possible with the SEO after only a few seconds of averaging (black x in Fig. 9.2c). The FFT dwell time for SEO measurements is chosen to match the averaging time for averaging times up to 16 seconds. For longer averaging times, due to limited capacity of the computer DAQ card, we average the appropriate number of 16-second scans, then fit the averaged scan. Note that SEO linewidth is already well above the frequency-time limit for t = 16 seconds (Fig. 9.1).

The SEO is an ideal tool for rapid, high-precision measurements of differences between axial frequencies, exactly what is needed for eventual spin-flip detection. But line-splitting is not the only contributing factor to frequency resolution. Frequency stability also plays a role; to detect a spin-flip, the scatter of repeated measurements must also be better than 60 mHz. The stability of the proton SEO, plotted in Fig. 9.2c in terms of standard deviation (black circles) and Allan deviation (blue circles) of repeated measurements, is also demonstrated to reach the 60 mHz level critical for spin-flip detection, for an optimal averaging time of 16 seconds in Fig. 9.2c.

The one-proton SEO thus achieves a critical milestone for the proton/antiproton g-factor experiment. In the presence of a very strong magnetic bottle, the axial frequency is resolved to a level just below the 60 mHz frequency shift that would signal a single-proton spin-flip. Current efforts are focused on reducing the remaining frequency scatter of the SEO, which limits the overall resolution since the stability of repeated SEO measurements is not yet as good as the resolution possible from the linewidth of a single measurement. In particular, stability at long averaging times is limited by a random background drift in the axial frequency, observed equally with SEO and dips (Fig. 9.2d). Fluctuations in the trapping potential, mechanical vibrations, temperature variations, and fluctuating patch potentials are being investigated as possible sources of the remaining scatter.



Figure 9.3: Tuning for optimal SEO stability by adjusting (a) feedback phase, and (b) anharmonicity of the trapping potential.

The scatter of repeated axial frequency measurements is minimized by careful tuning of the SEO. We find that the tuning of feedback phase is not particularly critical, as the SEO stability appears optimal over a broad phase range centered on $\phi = 0$ (Fig. 9.3a). SEO stability is somewhat more sensitive to trap anharmonicity (Fig. 9.3b), which is tuned out by adjusting the ratio of potentials on the compensation and ring electrodes.

Techniques for cooling the proton axial (Section 8.1) and radial (Section 6.3.1, Section 7.4) motions are also essential for reducing the frequency scatter. Electrostatic and magnetic anharmonicities, substantial in our small trap and strong bottle, are minimized for a proton near trap center. A probe of these unwanted effects is obtained by measuring the axial frequency scatter as a function of magnetron radius. A weak sideband heating drive is applied to deliberately increase the magnetron radius, as in the exponential heating experiments of Section 7.4. After the heating drive is turned off, the ring voltage is adjusted to re-center the proton response on the tuned-circuit axial amplifier. Applying Eq. 7.6 and accounting for the measured cooling-limit radius (Section 7.4), the size of this voltage adjustment indicates the magnetron radius to which the proton has been heated. Repeated axial frequency measurements are then taken to determine the scatter at this larger radius.



Figure 9.4: Radius-dependent increase in frequency scatter of repeated proton axial dip measurements.

The results of this procedure are plotted in Fig. 9.4, in terms of an Allan deviation of frequency measurements taken using the axial dip method (Section 7.1.1). The observed frequency scatter increases directly with magnetron radius, illustrating the importance of cooling to enable high-precision axial frequency measurements in the presence of our strong magnetic bottle. Effective sideband cooling reduces the magnetron radius, while axial feedback cooling sets the low T_z that determines the sideband cooling limit.

9.2 Multiple Spin Flip Simulation

Our demonstrated axial frequency resolution makes it possible to begin looking for proton spin flips. Given the various technical complications associated with an actual spin-flip drive (Section 9.3-Section 9.4), simulated spin flips are utilized as an initial means of testing our detection capabilities. Small adjustments to the trapping potential shift the axial frequency by an amount ($\sim 60 \text{ mHz}$) equivalent to a spin flip. Since the analysis trap electrodes all have long RC time constants that prevent rapid voltage adjustments, we typically produce simulated spin flips by applying the bias adjustment to transfer electrodes T2 and T3.

Simulation in this manner demonstrates a potentially valuable technique for initial detection of multiple proton spin flips. Compared to a data set where the proton spin state is unchanged, the scatter of repeated axial frequency measurements is increased when a (simulated) spin flip is attempted between each axial measurement, due to the added 60 mHz shifts from successful spin flips. Measuring this increase in axial frequency scatter could serve to test and diagnose the spin-flip drive, and also to locate the spin-flip frequency. Moreover, this effect should become readily visible even at a frequency resolution that would barely suffice for detection of a single-proton spin flip.

The multiple spin-flip simulation is shown in Fig. 9.5. First, a prediction of the effect is generated numerically. We analyze a data set of repeated proton axial frequency measurements, each from a Lorentzian fit to the SEO signal averaged for 40 seconds. Differences between successive measurements are used to calculate an Allan deviation, removing the effect of linear background drift. Without any spin flips, the



Figure 9.5: Simulation of increase in axial frequency scatter due to spin flips. The scatter of repeated SEO measurements is shown to increase as simulated spin flips are applied with a given probability (red), in agreement with a numerical prediction (blue).

Allan deviation of the SEO data is 58 mHz. Frequency shifts that mimic spin flips are then added with probability $P_{\uparrow\downarrow}$ to the SEO raw data, and the Allan deviation is recalculated. The modified data sets demonstrate a scatter that increases with $P_{\uparrow\downarrow}$ (blue points in Fig. 9.5). At the limit of $P_{\uparrow\downarrow} = 1$, the spin-flip shift adds in quadrature to the background scatter.

To fully simulate the multiple spin-flip effect, we then repeat the set of axial frequency measurements, this time adjusting the trapping potential to simulate spin flips as described above. Again the proton SEO signal is measured repeatedly, and between each measurement a random-number generator determines whether or not to apply a voltage offset to electrodes T2 and T3. The voltage adjustment, if applied, produces a shift in the axial frequency by ~ 60 mHz, hence the probability of applying

the adjustment represents the probability $P_{\uparrow\downarrow}$ of a spin flip. While in reality this probability can be at most 0.5 (Section 9.3), for our simulation we are free to specify any value. Running the simulation with $P_{\uparrow\downarrow} = 0$, $P_{\uparrow\downarrow} = 0.5$, and $P_{\uparrow\downarrow} = 1$, we observe increases in axial frequency scatter (red points in Fig. 9.5) that agree with expected values (blue points in Fig. 9.5). A calibration of the size of our simulated spin-flip is obtained by averaging every other frequency difference measured in the $P_{\uparrow\downarrow} = 1$ data set, where a simulated spin-flip occurs between each SEO measurement. In the simulation presented here, this shift was 65 mHz, within 10% of the 60 mHz intended.

As described in Section 9.3, this calibration step, averaging repeated frequency differences to reduce noise and extract the size of the spin flip, is unfortunately not possible in our actual experiment, since it requires a spin-flip probability $P_{\uparrow\downarrow} = 1$. However, the general technique demonstrated, observing multiple spin flips by looking for increased scatter in a large data set, remains valid even for realistic spin-flip probabilities. The increase in scatter from $P_{\uparrow\downarrow} = 0$ to $P_{\uparrow\downarrow} = 0.5$ in Fig. 9.5 is small but detectable, and the contrast of course would improve with further reduction of the background scatter.

9.3 Driving Spin Flips

A remaining challenge to proton spin-flip detection is the application of a sufficiently strong spin-flip drive. In the recent electron g-factor measurements [2, 72], spin-flip transitions are produced by making use of the radial component of the magnetic bottle, a technique dating back to early electron/positron experiments [85]. The trapped electron is driven axially at its anomaly frequency, such that amplitudemodulation of the $B_2 z \rho \hat{\rho}$ term in the bottle field (Eq. 2.21) produces an effective spin-flip drive. The Rabi frequency for this transition is given by [86]

$$\Omega_a = B_2 z_a \rho_c \frac{\mu}{\hbar} . \tag{9.1}$$

For this technique, some axial excitation is required at the anomaly frequency $\omega_a = \omega_s - \omega_c$. The amplitude z_a of an axial oscillation driven at ω_a satisfies [87]

$$z_a \propto \frac{\omega_z^2}{((\omega_z^2 - \omega_a^2)^2 + \gamma_z^2 \omega_a^2)^{\frac{1}{2}}} \frac{d^2 V_d}{z_0} V_0 , \qquad (9.2)$$

from the general result for a damped harmonic oscillator driven off-resonance, where V_d is a drive voltage and V_0 is the trapping potential.

For a proton, however, this axial-anomaly technique is not necessarily the best option. Frequencies in the electron g-2 experiment are arranged with ω_z close to ω_a , partially to reduce the drive voltage V_d required to produce a sufficient amplitude z_a . However, a similar frequency coincidence is not possible for our proton experiment. The anomaly frequency is $\omega_a \approx 2\pi \times 140$ MHz for a proton, while the axial frequency is kept low ($\omega_z = 2\pi \times 553$ kHz in the analysis trap) in order to maximize the axial frequency shift due to a spin-flip (Eq. 2.23). An initial estimate, comparing the 2008 electron experiment with parameters from our current proton analysis trap, suggests that the Rabi frequency for the proton transition would be close to 10^4 times slower, given the same anomaly drive ratio V_d/V_0 .

Direct excitation of the spin-flip transition is a more attractive technique. Segments of the Penning trap electrodes may be wired together to create effective current loops, producing a magnetic field transverse to the trap axis [86]. This technique was used in the 1987 electron g-2 measurement to produce an anomaly drive [29]. For a direct spin-flip drive, we require a configuration approximating Helmholtz coils (Fig. 9.6).



Figure 9.6: Idealized picture of current loops for driving spin-flip transitions.

Driving a current through the paths shown in Fig. 9.6, at a frequency ω_s , we produce a small radial magnetic field \vec{B}_1 , such that the total field is now given by $\vec{B} = \vec{B}_0 + \vec{B}_1 = |B_0|\hat{z} + |B_1|\hat{x}\cos(\omega_s t)$. Our radial magnetic field can then be rewritten as a sum of co-rotating and counter-rotating components,

$$\vec{B}_{1} = \frac{|B_{1}|}{2} \left(\hat{x} \cos(\omega_{s}t) - \hat{y} \sin(\omega_{s}t) \right) + \frac{|B_{1}|}{2} \left(\hat{x} \cos(\omega_{s}t) + \hat{y} \sin(\omega_{s}t) \right) .$$
(9.3)

The co-rotating piece of this AC Helmholtz field drives transitions from spin-up to spin-down, with Rabi frequency

$$\Omega_R = \frac{|B_1|\mu_p}{\hbar} \,. \tag{9.4}$$

The transition probability for a spin-flip then follows from a standard calculation for the two-state system,

$$P_{\uparrow\downarrow} = \left(\frac{\Omega_R}{\Omega_R'}\right)^2 \sin^2\left(\frac{\Omega_R' t}{2}\right) , \qquad (9.5)$$

where the spin-flip drive is detuned by δ from $\omega_s = 2\mu_p |B_0|/\hbar$, and $\Omega'_R = \sqrt{\delta^2 + \Omega_R^2}$.

Ideally we could use this setup to drive a π -pulse on resonance, with $\Omega'_R = \Omega_R$, such that the proton spin would flip in a time $t = \pi/\Omega_R$. However, in our Penning trap, coupling of the proton to the axial detection resistor produces a constant thermal fluctuation at temperature T_z , producing a significant broadening of the spin transition line as described in Section 2.2.2. As discussed below, the effective Rabi frequency (Eq. 9.4) for the proton spin transition in our experiment will be of order 100 Hz at best; meanwhile the spin transition has a linewidth of almost 10 kHz due to our strong magnetic bottle. Consequently, the spin-flip behavior suggested by Eq. 9.5 lasts only an infinitesimal time before fluctuations destroy the coherent spin rotation. A full analysis, accounting for the nonzero linewidth of the spin transition, suggests that Eq. 9.5 is modified in our actual Penning trap to [1]

$$P_{\uparrow\downarrow} = \frac{1}{2} \left(1 - \exp\left(-\pi \,\Omega_R^{\prime 2} \,\Delta t \,\chi_s(\omega)\right) \right) \,, \tag{9.6}$$

where the resonant spin-flip drive is applied for a time Δt , and $\chi_s(\omega)$ describes the spin transition lineshape. Using the normalization $\int_{-\infty}^{+\infty} \chi_s(\omega) d\omega = 1$, we can make the good approximation

$$P_{\uparrow\downarrow} = \frac{1}{2} \left(1 - \exp\left(\frac{-\pi \,\Omega_R^2 \,\Delta t}{\Delta \omega_s}\right) \right) \,, \tag{9.7}$$

valid for a drive near resonance, where $\Delta \omega_s$ is the bottle-broadened linewidth (Eq. 2.32) of the spin transition.

The experimental consequence of Eq. 9.7 is that we have only probabilistic control over the proton spin state. When driving spin flips, the best we can hope to do is to saturate the transition probability at 1/2 by applying a strong resonant spin-flip drive for sufficiently long Δt .

The largest possible Rabi frequency Ω_R is desired in order to minimize this drive time Δt . A long Δt limits the rate at which we can make repeated spin-flip measurements, while also reducing our frequency resolution due to the background drift described in Section 9.1. In the idealized picture of Fig. 9.6, we are driving current loops on the compensation electrodes. Our trap geometry is such that this configuration forms a good approximation of a Helmholtz pair, producing a radial magnetic field

$$\vec{B}_1 = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 I}{\rho_0} \hat{x} . \tag{9.8}$$

For our trap dimensions, Eq. 9.8 predicts $|B_1| \approx 3$ gauss/ampere. However, trap geometry and RF wiring considerations prevent full realization of this Helmholtz pair in the experiment. The spin-flip drive is applied on two segments of a compensation electrode (Fig. 3.4), forming only a portion of the current path shown in Fig. 9.6. A full calculation of the field generated by currents through the compensation electrode segments predicts $|B_1| \approx 0.75$ gauss/ampere [88]. Inserting this value into Eq. 9.4, we obtain a Rabi frequency of 3 kHz/ampere for the spin-flip transition. For a current of 10 mA in the drive line, it should therefore be possible to saturate the spin-flip probability at 1/2 (Eq. 9.7) in roughly 10 seconds.

To carry out the double Penning trap measurement scheme (Section 2.3), proton

spin flips must also be driven in the precision trap. However, since no explicit magnetic bottle broadens the spin transition line, driving spin flips in the precision trap should be substantially easier. A spin-flip drive is wired in the same configuration as for the analysis trap, on two halves of a compensation electrode. The precision trap is not infinitely far from the iron ring of the analysis trap, so there is still some residual magnetic bottle, but reduced by a factor of 10^4 (Chapter 2). Spin-flip probability in the precision trap thus saturates at 1/2 in less than 0.1 second, even for modest current of order 1 mA in the drive "coils."

9.4 Proposed Sequence for Spin-Flip Detection

A proposed sequence for proton spin-flip detection would proceed as follows:

- Apply sideband cooling to reduce the proton magnetron radius to the cooling limit.
- 2. Use the SEO signal to measure the proton axial frequency.
- 3. Switch from self-excitation to axial feedback cooling, and apply a spin-flip drive for a time Δt . Feedback cooling narrows the spin resonance linewidth and reduces the Δt required to obtain a significant spin-flip probability $P_{\uparrow\downarrow}$ (Eq. 9.7).
- 4. Turn off the spin-flip drive, re-establish the SEO, and measure the axial frequency. A shift of 60 mHz from the previous measurement would reveal a spin flip.

Though actual proton spin flips have not yet been observed in our apparatus, we have begun to realize the necessary steps outlined above. For a proton that has been sideband cooled to the cooling limit, unwanted magnetron heating causes only a slow increase in ω_z , typically by 0.3 Hz/hr (Fig. 9.2d). An attempt to detect the spin flip can thus proceed for several minutes without the need for additional sideband cooling, which would shift the axial frequency by more than 60 mHz on average, as described in Chapter 7.

Axial frequency resolution at the 60 mHz level has been demonstrated with the SEO (Section 9.1). The scatter of repeated SEO measurements is shown to increase only slightly with the application of a full-strength (+13 dBm at the hat) spin-flip drive, an encouraging sign given the possibility of large systematic effects from applying such a strong RF drive to a trap electrode. When axial feedback cooling and a full-strength spin-flip drive are applied for $\Delta t = 10$ seconds between SEO measurements, the axial frequency scatter increases from 60 mHz to 70-80 mHz. Further work on the spin-flip sequence is needed to understand and reduce this systematic increase, which likely comes at least partially from the long wait times presently required. Besides the spin-flip drive time Δt , we find it necessary to wait several seconds after re-establishing the SEO before the frequency stabilizes sufficiently for measurement at the 60 mHz level.

However, this 70-80 mHz scatter (Allan deviation below 60 mHz) is already suitable for the sort of demonstration proposed and simulated in Fig. 9.5. This technique, looking for increased scatter due to multiple spin flips, is a likely candidate for initial detection, given our uncertainty in the actual spin-flip frequency. Without a proton

cyclotron amplifier, we lack a high-precision measurement of the analysis trap magnetic field, which is shifted considerably from its value in the precision trap due to the magnetic bottle. Numerical calculation of B_0 (Eq. 2.29) for our bottle suggests a reduction by 0.457 Tesla, or 8% from the value measured in PRING, but there is considerable uncertainty due to machining tolerances and the value of saturation magnetization for our iron ring. Our best estimate of $|\vec{B}|$ in ARING is extracted from the magnetron frequency, which can be measured at the Hz level by tuning the avoided-crossing feature (Section 7.2) for maximum symmetry. This measurement suggests that our bottle is slightly stronger than predicted, with $B_0 = 0.470(3)$ Tesla. The resulting uncertainty in $|\vec{B}|$ translates to uncertainty of order 100 kHz in the spinflip frequency, larger by an order of magnitude than the expected bottle-broadened linewidth, $\Delta \omega_s \approx 2\pi \times 10$ kHz, for $T_z = 4$ K. Finding the spin-flip frequency will require covering the uncertainty range in steps smaller than $\Delta \omega_s$; an improved measurement of the magnetic field in ARING would aid in shortening this search interval. Prospects for such a measurement include a precise determination of the magnetron frequency, or an attempt (likely destructive since we lack cyclotron damping in AR-ING) to measure the cyclotron frequency by sweeping a weak drive and watching for the large shift in axial frequency that would indicate the cyclotron heating resonance.

Chapter 10

Conclusion and Future Directions

10.1 Summary and Status of the Experiment

Detection of a single-proton spin-flip transition would lead to a novel precision measurement and comparison of proton and antiproton magnetic moments. The first one-proton self-excited oscillator now opens a path to these measurements, demonstrating resolution at the level of the shift from a proton spin flip in a Penning trap with an extremely large magnetic gradient.

To overcome the relative weakness of the nuclear magneton, the proton "analysis" Penning trap is built with a magnetic bottle 50 times stronger than was used for recent electron g-factor measurements. This large magnetic gradient creates a potentially resolvable shift between spin-up and spin-down states, but also introduces significant magnetic field inhomogeneity, making a double-trap scheme essential for precision resolution of the cyclotron and spin-flip frequencies needed to measure g_p . As proof of principle for a double-trap measurement, a single proton is loaded into a second Penning trap with no magnetic bottle, which is separated by roughly two inches from the analysis trap. The proton cyclotron, magnetron, and axial oscillation frequencies are measured at high resolution in this "precision" trap, before the proton is transferred to the analysis trap for attempts at spin-flip detection.

The central challenge of observing a proton spin-flip is detecting the tiny shift in axial frequency that results from coupling between the magnetic bottle and the proton magnetic moment, while at the same time overcoming the unwanted effects of the extremely strong magnetic gradient needed to produce this shift. We have made significant progress on both fronts. Proton axial frequency resolution is now demonstrated to reach the level required for spin-flip detection, utilizing feedback techniques realized previously only with a single electron. Our one-proton self-excited oscillator provides a powerful tool for rapid, high-precision measurements of the axial frequency, ideal for monitoring spin-flip shifts. One-proton feedback cooling reduces the axial temperature, narrowing the spin transition line and promising to increase the spin-flip transition rate. Unwanted bottle effects are studied and minimized by careful axial and radial cooling, reducing couplings that otherwise threaten to produce axial frequency shifts larger than a spin flip. In the cyclotron case, we have shown we are able to operate without a cyclotron amplifier in the analysis trap, decoupling the proton cyclotron state from its thermal reservoir and locking in whatever state is obtained at the moment of transfer from the precision trap. In the magnetron case, we have shown that effective sideband cooling is possible despite the strong magnetic gradient, and we have in fact exploited the bottle along with axial feedback cooling to demonstrate cooling to a low-temperature theoretical limit, reducing the magnetron orbit to a radius of 6 μ m.

With the self-excited oscillator, our current axial frequency resolution stands at 60 mHz in the analysis trap, sufficient in principle to detect the shifts from spin flips, which in this trap are also 60 mHz in size. Our resolution is currently limited by a yet-unexplained background scatter. Ongoing efforts are focused on characterizing this remaining frequency scatter, with the goal of improving resolution by another factor of two or three, to the point where spin-flip detection would be less challenging.

In parallel, we are pursuing a different approach. Along with improving our frequency resolution, it may be possible to increase the size of the spin-flip signal. One straightforward way to increase the spin-flip shift (Eq. 2.23) is to increase the size of the magnetic bottle. To do this, we must put the ferromagnetic material closer to trap center, which requires building a smaller Penning trap. The design and initial construction of a trap with $\rho_0 = 1.5$ mm (half the size of our current proton trap) is shown in Fig. 10.1. This trap has been designed to replace the analysis trap in our existing electrode stack, with conical transfer electrodes to reduce the diameter from our $\rho_0 = 3$ mm precision trap.

The magnetic bottle in this smaller trap is almost four times larger than in our current analysis trap. The resulting spin-flip shift would be 220 mHz, assuming we maintain the same axial frequency. If our frequency resolution remained at 60 mHz, the shifts due to spin-flips would thus be easily detectable. However, retaining this high resolution is by no means guaranteed. If our current background scatter is in fact due to some effect of the strong magnetic bottle, then the scatter may well increase with bottle size, such that we see no net improvement with the smaller



Figure 10.1: Proposed smaller trap for future proton/antiproton spin-flip detection. (a) Current 6 mm diameter trap. (b) Proposed 3 mm diameter trap. (c) Test assembly of 3 mm trap electrodes.

trap. Trap anharmonicity is also likely to be a greater challenge in the smaller trap, since machining tolerances and inter-electrode gap sizes do not improve as we scale down. Concerns about tunability were in fact why we chose the current trap size of $\rho_0 = 3$ mm; however, the optimal size for efficient g-factor measurement remains to be determined.

Our proposed smaller trap, at 3 mm diameter, already pushes the limits of conventional machining and assembly, but it may eventually become possible to extend below the millimeter scale using microfabrication techniques. Due to considerable interest in the use of trapped-ion arrays for quantum information processing, methods for constructing and characterizing small traps have been advancing rapidly [89, 90, 91, 92, 93]. It is still unclear at this point whether such fabrication methods can produce a very good approximation of our cylindrical Penning traps. However, at least in principle, the geometry of an analysis trap for measurement of g_p is somewhat flexible; we require only a magnetic bottle strong enough to generate a clear
spin-flip shift $\Delta \omega_z$, sufficient tunability to resolve the axial frequency to better than $\Delta \omega_z$, and an open path that allows transfer to a more conventional trap for precision measurement.

10.2 Towards a Proton Measurement

Attaining a resolution the size of the frequency shift that signals a spin flip is an important milestone in the road to measurement of g_p . The obvious next step is to demonstrate detection of a single-proton spin flip, which we have closely approached but not yet observed. We are proceeding on two fronts: (1) attempting to reduce the background drift that limits frequency resolution in our current apparatus, as described in Chapter 9; and (2) experimenting with a smaller trap as described above, in which the signal from a spin-flip could be made significantly stronger.

The next phase of the experiment will involve conversion from the current proofof-concept to an actual high-precision g-factor measurement. We address here several aspects of the experiment that could be further optimized for an effective measurement.

First, while we believe that our current trap wiring should be sufficient to drive spin-flips in the analysis trap, we have not explored the tradeoffs of drive power versus drive time. For example, inclusion of a power amplifier could allow us to boost the drive strength and saturate $P_{\uparrow\downarrow}$ in a shorter time. Whether this would be a net gain (reduced time in which the axial frequency could drift) or a net loss (possible drive-strength systematics and increased noise from the power amp) is an open question. There is also the possibility of using external spin-flip drive coils, rather than winding effective current loops into our trap electrodes. We have been skeptical of this approach due to the greater physical distance between the spinflip coils and trap center, and the significant shielding factor of the trap electrodes. However, a competing effort at Mainz believes that sufficient drive strength can be obtained with external coils [41]. If successful, this alternate approach could allow a somewhat simpler wiring scheme for the trap electrodes.

Next, measuring g_p with the target precision of 1 ppb will require more careful treatment of the magnetic field. Techniques described in reference [71] can be used to measure the magnetic bottle in the precision trap, where B_2 may vary between 1 and $10~{\rm T}/m^2$ depending on the low-temperature magnetization of macor spacers (Chapter 2). The position of precision and analysis traps should likely also be exchanged. For our current proof-of-concept demonstrations in the strong magnetic bottle, we position the analysis trap at field center and shim the field to be as flat as possible between the two traps. However, unless field homogeneity in ARING proves critical for our resolution there (untested, but unlikely given the far stronger gradients from the magnetic bottle), the opposite arrangement seems preferable for measurement of g_p . For the best resolution of cyclotron and spin-flip frequencies, the precision trap should be placed at field center, and the shims set to optimize field homogeneity in PRING. As a figure of merit, magnetic field gradients must be reduced such that the \vec{B} field is good to a ppb within the small region occupied by the proton during a spin-flip attempt; this region should be roughly defined by the effective magnetron radius at the sideband cooling limit.

Time-variation of the magnetic field must also be considered. Measuring the

spin-flip transition line via the double-trap sequence (Section 2.3) could be a lengthy process, during which the magnetic field and ω_s are likely to drift around. Such drifts could potentially be removed by using the cyclotron frequency to monitor the field. However, considerable time (currently several hours) is required to measure the cyclotron frequency and damp back to a thermal level (Section 6.1), so the realistic number of intermediate cyclotron measurements is limited.

Finally, we will likely want to address some technical issues in our current setup that are not well-suited to an actual measurement. One such issue is the relative size of tuned-circuit amplifiers. Our current efforts focus on axial detection, and we have partitioned tripod space accordingly, with two axial amplifiers of considerable size, and a relatively small cyclotron amp squeezed in the remaining available space. For measurement of g_p , however, the precision-trap axial amplifier serves primarily just for damping, while the cyclotron amplifier becomes critically important. The trap-modified cyclotron frequency ω'_c must now be determined to a ppb, and also the cyclotron damping time becomes a rate-limiting step in the overall measurement. Trading some physical size from the small axial to the cyclotron amplifier, which should decrease γ_z but increase γ_c , is likely worthwhile.

Similarly, our current setup is responsible for a looming inconvenience in the time required to transfer between traps. To improve voltage stability, we have added long RC time constants to virtually all the trap electrodes, with the unwanted side effect that transferring a proton from PRING to ARING now requires several minutes. Our measurement of g_p , however, will require enormous numbers of transfers in order to build up histograms of spin-flip probabilities. For an efficient measurement, we may need to sacrifice some stability for time, keeping the long time constants only where essential.

Another tradeoff in transferring involves the number and size of transfer electrodes. A larger number of shorter electrodes would allow better tuning of the potential well during transfer. With transfer electrodes closer to radius-length, it would be possible to maintain essentially the same well depth throughout the transfer process, rather than using the current inchworm technique. Transfer might be possible at lower voltages than the ~ 100 V currently utilized, and without the magnetron heating currently observed in a substantial fraction of transfers. However, each additional electrode also requires an extra feedthrough pin, DC line, and BiasDAC channel. One solution might be to move to larger-diameter electrodes for the transfer in order to cover the same axial length with fewer electrodes. Conical electrodes would then be required to transition back to the diameter of the Penning trap; we plan to experiment with a similar configuration in the forthcoming $\rho_0 = 1.5$ mm trap (Fig. 10.1b).

A next-generation apparatus incorporating several of these features is shown in Fig. 10.2.

10.3 Towards an Antiproton Measurement

As described in Chapter 1, a key feature of this experiment is our ability to quickly follow any measurement of g_p with a measurement of $g_{\bar{p}}$ at equal precision. To this end, we have begun to incorporate plans for antiproton measurement into our nextgeneration apparatus.

Antiprotons are loaded from the Antiproton Decelerator at CERN in Switzerland,



Figure 10.2: Preliminary design of next-generation apparatus for proton/antiproton g-factor measurement.

which delivers a pulse of 5 MeV antiprotons. The antiprotons are slowed to the keV energy scale by passing through a beryllium degrader, then cooled by collisions with trapped electrons [94]. The electrons can be pulsed out of the trap, leaving a cloud of antiprotons which can then be manipulated using our standard methods. Previous single-antiproton experiments in our group have demonstrated measurement of the \bar{p} cyclotron frequency to 0.1 ppb [16], despite the generally noisier environment at CERN compared to our lab at Harvard.

Preliminary design for a $g_{\bar{p}}$ apparatus is shown in Fig. 10.2. The electrode stack provides open access for antiprotons. Protons can still be loaded off the degrader, which would sit just below the bottom of the stack. This apparatus also includes several improvements from the current design. Extraneous small flanges on the pinbase are removed; these were never utilized in the current experiment, and served only as a (frequent) source of vacuum leaks through the additional indium seals. Removing these flanges also allows for slightly larger amplifiers in the tripod region, and the cyclotron amp can is enlarged at the expense of the small axial can, as suggested in Section 10.2. The positions of analysis and precision traps are reversed to locate the precision trap at field center. The second iron ring (not pictured in Fig. 10.2), which nulls out linear gradient B_1 in the precision trap, is placed below the degrader, so as not to interact with antiprotons after they have been slowed. Despite the small trap diameter, estimates indicate that a sufficient flux of antiprotons will enter the electrode stack in this configuration.

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