

The Road to Antihydrogen

A thesis presented

by

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to

The Department of Physics

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

in the subject of

Physics

Harvard University
Cambridge, Massachusetts
October 2001

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The Road to Antihydrogen

Abstract

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Four advances were made towards the production of cold antihydrogen. First, a new mechanism of accumulating positrons was discovered and explained which increased our accumulation rate by over two orders of magnitude. Second, an intricate Penning trap was built to enable our group to study the antihydrogen formation process. Third, we demonstrated, for the first time, antiprotons cooling inside a positron plasma. And, finally, a detailed theoretical description of the dynamics of a charged particle in a Penning-Ioffe trap was developed.

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Acknowledgements

I would like to thank Gerald Gabrielse, my thesis advisor, for his support and effort in making this the best thesis possible - almost perfection.

I'd like to acknowledge the crew of ATRAP: J. Estrada, J.N. Tan, N.S. Bowden, P. Oxley, T. Roach, C.H. Storry, M. Wessels, J. Tan, D. Grozonka, W. Oelert, G. Scheppers, T. Sefsick, W. Breunlich, M. Carngelli, H. Fuhrmann, R. King, R. Ursin, H. Zmeskal, H. Kalinowsky, C. Wesdorp, and J. Walz. Thanks for the help guys.

I would like to salute Todd Squires who helped to create a new understanding.

My most sincere gratitude for Janet Ragussa who on more than one occasion helped me to find things I did not even know I had lost.

I want to acknowledge two people who when to great lengths to push my career forward, George Clark of MIT and Edward Fenimore of LANL.

I would like to thank my parents, Irene and Michael. I would like to give special mention to my father for correcting the grammar of this thesis during the eleventh hour. Thank you to Maarten who provided enormous support; it is a shame he didn't survive to read this.

I would like thank Dorothea who has greatly increased the dimensions of my life. And finally, I would like to thank my best buddy in the whole world, Phillip, who kept me company during the greater part of this endeavor.

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Chapter 1

Introduction

Antihydrogen promises to be a fertile vessel for testing the predictions of the CPT theorem and the gravitational properties of antimatter. This thesis reports four significant strides towards the production and storage of cold antihydrogen:

1. The discovery of a new physical mechanism for accumulating positrons in an ultrahigh vacuum, cryogenic environment. This method currently stands as the only demonstrated way to load positrons in an environment suitable for cold antihydrogen production.
2. A complete theoretical description of the dynamics of a charged particle in a Penning-Ioffe trap. Our theory provides the first clear path to trapping of antihydrogen atoms.
3. The design and construction of the world's most intricate Penning trap. This trap was built to investigate cold antihydrogen production. It is now midway through its third year of operation at CERN.
4. The first demonstration of antiprotons losing kinetic energy to a cold positron plasma.

1.1 Motivations

It seems very reasonable to believe that the laws of physics do not make a distinction between left and right, ie., physical laws should be invariant to parity transformations. A

parity violating process would not have a mirror image in nature. However, in 1957 C.S. Wu measured a violation of parity in the decay of Cobalt 60 [1], thus disproving this naïve assumption.

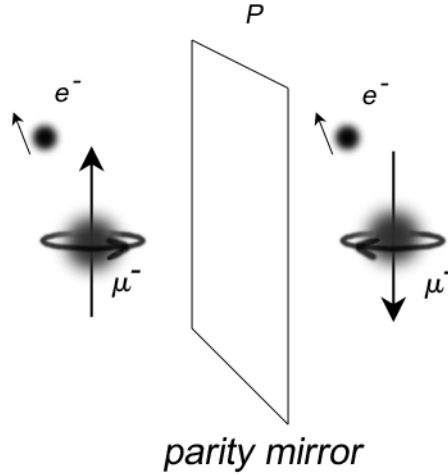


Figure 1.1: The decay of the muon violates parity conservation because the mirror image of the process does not exist in nature.

Many parity violating phenomena have now been found. One example is the decay of the muon¹. The principle decay path of the muon is

$$\mu \rightarrow e + \nu_{\mu} + \bar{\nu}_e. \quad (1)$$

It is parity violating as can be seen by the electron coming out along the axis of the magnetic moment (Fig. 1.1). In the mirror image of the decay, the magnetic moment of the particle "flips over" but the electron still leaves along the original direction of motion. The mirror image process does not occur in nature. If it did, we would observe electrons coming out of muon decay in both directions relative to the magnetic moment.

¹ In fact this experiment is so straight forward that it is completed several times a semester in MIT's undergraduate physics laboratory.

The physics community rectified the situation by including C, the charge conjugation operator with parity. The charge conjugation operator transforms a particle to its antimatter equivalent [2]. CP invariance would dictate that no process could be found whose antimatter mirror image did not also exist. For a while it appeared that nature was invariant to CP transformations. But in 1964 Cronin and Fitch found CP invariance to be violated in the kaons system.

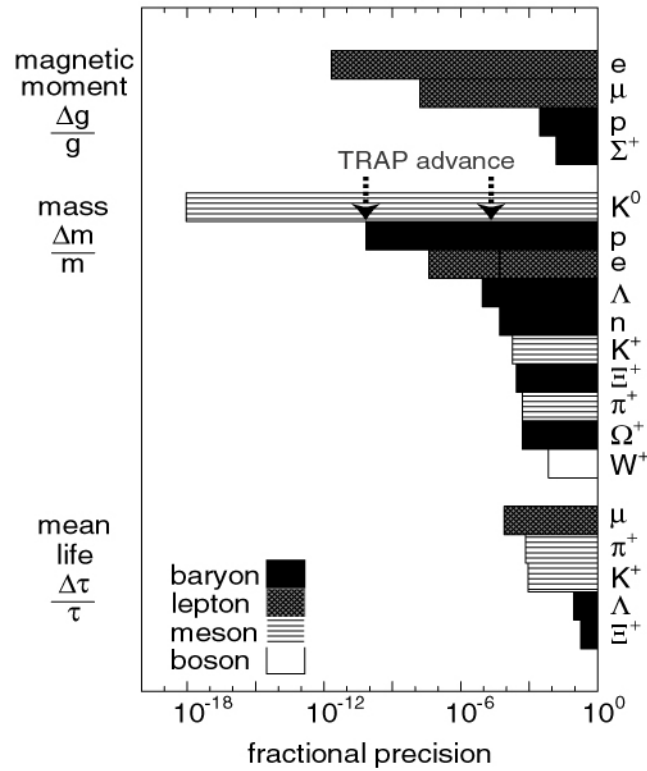


Figure 1.2: Comparison of the accuracy of baryon, lepton and meson CPT test [3]

Again the symmetry paradigm shifted; this time a third symmetry, T, time reversal symmetry, was included. Time reversal symmetry states that the laws of physics should be invariant when the direction of time is reversed. The classical example is two billiard balls having an elastic collision on a pool table. Imagine that we film this collision with a movie camera. If we play the movie backwards, nothing seems unusual because the laws of physics

governing the elastic collision are time invariant. The collision process can run either way in both theory and experiment.

The CPT theorem states laws of physics are invariant under CPT transformations. The symmetry operators may be applied in any order and the physics of the system will remain the same. The CPT theorem stands on solid theoretical ground, but whether it really applies to nature must be tested by experiments. As of yet nobody has detected CPT violation. Fig. 1.2 summarizes the CPT experiments so far.

One way to look for a violation of CPT invariance is to carefully compare two systems that the CPT theorem says should have identical properties. The CPT transform of hydrogen is antihydrogen; hence, hydrogen and antihydrogen should have exactly the same electromagnetic spectra. Hydrogen and antihydrogen make an interesting test case because their spectra can be measured with high precision and accuracy. The two photon, 1S-2S transition of hydrogen has an extremely narrow fractional line width, only 5×10^{-16} . In principle, splitting this line by a factor of 200, would lead to a spectroscopic comparison of hydrogen and antihydrogen at an accuracy of 10^{-18} . This would make possible a CPT test [3] that rivals the most precise test already done, Fig. 1.2.

There are serious experimental obstacles to obtaining the ultimate 10^{-18} accuracy. But if we could repeat the current narrowest observed 1S-2S line width [4], 8.5 parts in 10^{13} , with antihydrogen we would provide a large increase in accuracy over the current CPT tests involving baryons and leptons particles, Fig. 1.2.

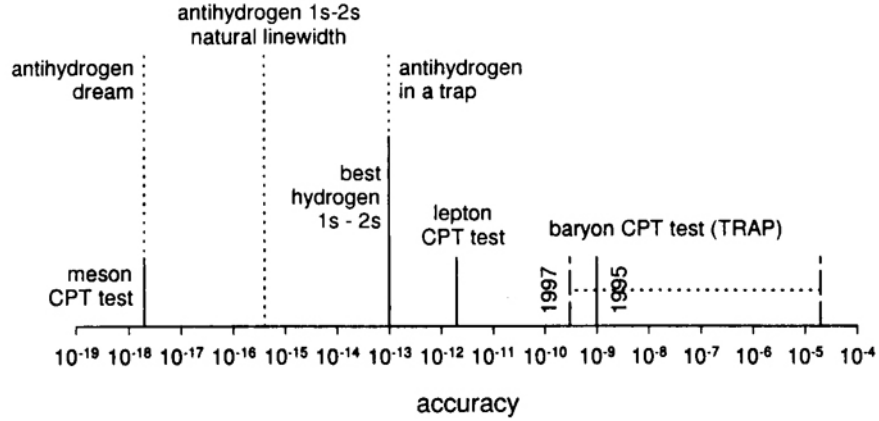


Figure 1.3: The accuracies for the precise 1s-2s spectroscopy of antihydrogen are compared to the most stringent test of CPT invariance carried out with the three types of particles: mesons, leptons and baryons.

Of course we can't measure the spectra of antihydrogen if we do not have any cold antihydrogen. That was the mission of this doctoral work to make and store cold antihydrogen atoms.

1.2 The Apparatus

Early in my graduate career we discovered a completely new method of accumulating positrons within a cryogenic environment [5]. The method is simple and robust. A thin foil (single crystal) is placed across the opening of a cylindrical Penning trap. A positron source is on one side. When voltages are applied to the electrodes on the other side, the trap fills with positrons. Our motivation for trying this experiment was the thought that placing an electron plasma in the path of the positron beam would remove energy from the positrons and cause them to be trapped in an adjacent well. Imagine our surprise when the positrons filled the well with no electrons present. The mechanism behind this result is the subject of Chapter 3. Our new method of loading positrons stands as the most efficient demonstrated way to accumulate positrons into a cryogenic, ultrahigh vacuum environment.

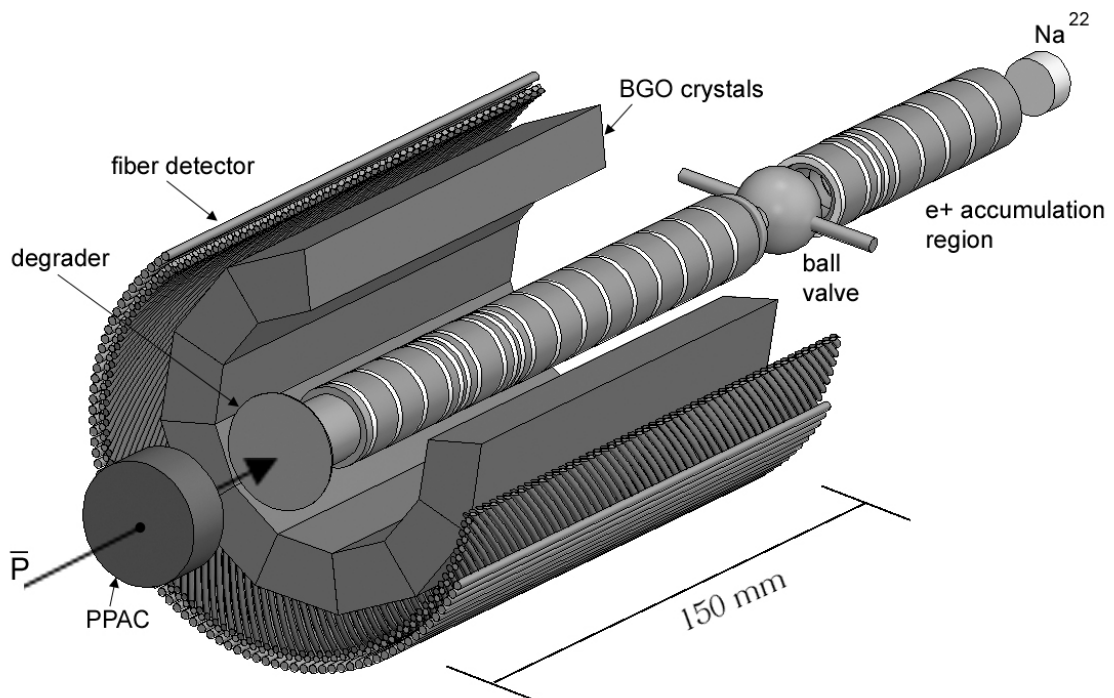


Figure 1.4: The main components of the ATRAP I experiment.

In December 1995 (during the last 10 days of LEAR, the Low Energy Antiproton Ring), we brought a Penning trap, equipped with the new positron loading mechanism to CERN to try to make antihydrogen [6]. We learned that exposure to antiprotons would destroy our novel mechanism for accumulating positrons. Consequently, we were doomed unless we built a beam stopper for the antiprotons inside our trap. This resulted in the development of a cryogenic electrode-valve, which operates in an ultrahigh vacuum at 4.2 K in a 6 Tesla magnetic field (Ch. 2).

During the two years after the LEAR run, we designed and built an intricate Penning trap in which we could attempt to make antihydrogen. This new trap is the subject of chapter 2. The new trap required 81 AutoCAD drawings to specify its design and over 400 more AutoCAD files were required to specify the trap installation. It is separated into two

parts by an antiproton stopping ball valve (Fig. 1.4). Positrons originating in a ^{22}Na source are accumulated in the trap section above the valve, while antiprotons from CERN are simultaneously accumulated below it. Surrounding the trap is an antihydrogen annihilation detector. The inner layer is a barrel of BGO crystals for detecting the gamma rays from the annihilation of the positron. External to the BGO are three layers of scintillating fibers for observing the antiproton annihilations (Ch. 4). These detectors work in conjunction with 18 scintillator paddles on the outside of the solenoid (not shown in the figure). Also included in Fig. 1.4 is the Parallel Plate Avalanche Counter (or PPAC) for centering and counting the antiprotons delivered to the trap.

Our intricate trap, now in its third year of operation, is at the heart of our experiment at CERN. Figure 1.5 gives an overview of the CERN installation. Our trap is inside a large magnetic solenoid, which stands almost 2 meters tall and provides a 6 Tesla field. Below the solenoid is a line of bending magnets that steer the antiprotons into the trap.

Above the magnet is a 2800 lb lead chest, mounted on wheels, which holds a 110 mCi positron source when it is not in use. The source must be lowered down to the particle trap under remote control to prevent human exposure. The 60 cm walls of concrete surrounding the magnet protect the experimenters from the positron source. Above the lead chest is a dewar of liquid nitrogen to cooled the source to, 77 K, which prevents the source from dramatically disturbing the, 4K, cryogenic environment of the trap. Just outside the concrete is a laser room, which will house the many lasers needed for performing antihydrogen spectroscopy.

Our experiment was built by four separate member groups of the international ATRAP collaboration. Part of this thesis work was the successful coordination of all the dimensions of the entire installation pictured in Fig 1.5.

ATRAP

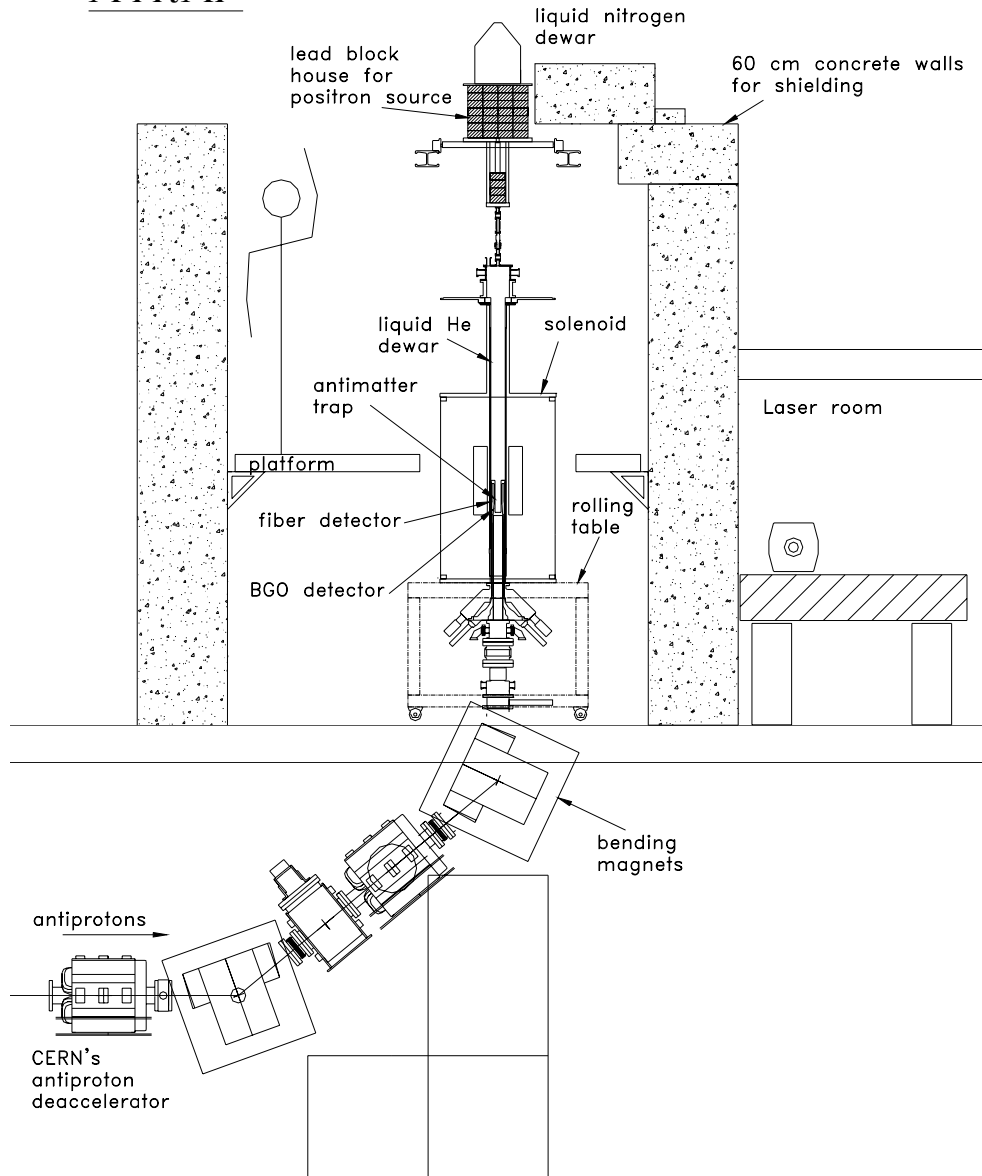


Figure 1.5: An overview of the ATRAP I instillation.

1.3 Attempting Antihydrogen

The apparatus was designed to enable us to pursue four different approaches to making antihydrogen. To make antihydrogen the positron must have enough energy to approach the antiproton. This energy then has to be removed so the particle forms a bound

pair. To conserve energy, a third body (massive particle or photon) must always be involved. The third body distinguishes one method from another. Table 1.1 list four different recombination techniques and the rate we might expect with each.

Method	formula	rate or number
three body recombination	$\bar{p} + e^+ + e^+ \rightarrow \bar{H} + e^+$	600 /sec/antiproton [7]
radiative recombination	$\bar{p} + e^+ \rightarrow \bar{H} + \gamma$	3×10^{-4} /sec/antiproton [7]
stimulated radiative recombination	$\bar{p} + e^- + \gamma \rightarrow \bar{H} + \gamma$	1.0 /sec/antiproton [7]
pulsed field recombination	$\bar{p} + e^+ + \vec{E} \rightarrow \bar{H}$	10-100 /pulse [8]

Table 1.1: A list of the four different methods of producing antihydrogen in the ATRAP Penning trap. The rates given assume 10^7 positrons at 4.2 Kelvin. The rates for SRR assume a CO₂ laser at 10 W/mm²

During the 5-month run in the year 2000 we investigated the three body recombination and pulsed field recombination methods of producing antihydrogen. Although we did not observe any antimatter atoms, we were able to demonstrate the first positron cooling of antiprotons (Ch. 4). We also demonstrated (to my surprise) that we could use nanosecond timing to toss particles plasmas from one well to another in the trap almost as if we were throwing a baseball. By pulsing a plasma of positrons across the trap we can clean it of contaminate ions. The acceleration of the heavier ions is thousands of times slower than the positrons and so the ions are left behind [8].

1.4 Storing Antihydrogen

To trap antihydrogen once it is produced, ATRAP plans to use a neutral particle trap like those developed to hold hydrogen. However, the stability of a Penning trap, used to confine the ingredients of antihydrogen, is closely related to the axial symmetry of its electric

and magnetic fields. The axial symmetry leads to the conservation of angular momentum and hence to a confinement theorem [9]. Neither one charged particle nor a dense single-component plasma can spread perpendicularly to the magnetic field and leave a Penning trap. The addition of a radial quadrupole field of an Ioffe trap breaks the axial symmetry and invalidates the confinement, which requires axial symmetry. Using a three pronged approach, numerical simulations, perturbation analysis and the theory of adiabatic invariants we were able to develop a full theory of the dynamical behavior of a charged particle in a combined Penning Ioffe trap [10]. This theory enables us to predict how the gradient Ioffe field would affect the familiar motions of a Penning trap. We were delighted to predict that a charged particle will remain in a stable orbit, not leaving the trap, for many minutes. This study is the subject of Ch. 5.

Chapter 2

Design and Construction of the ATRAP Trap

For the accelerator run that began in 1999 we designed a Penning trap in which we could make antihydrogen. This effort resulted in the most intricate Penning trap ever built. Many additions were made to the previous designs of the group [11]. We included a cryogenic valve to separate the positron and antiproton accumulation regions. We designed the trap to accommodate up to 6,000 volts, more than 3 times what had been done before. The trap has increased in length by a factor of 1.4, increasing the antiproton capture efficiency. We reduced the diameter of the trap enclosure by a factor of 2.3 to 1.282", to make room for a prototype antihydrogen detector (Ch. 4). We more than doubled, to 53, the number of electrical feedthroughs into the trap without using any additional area. And we added a fiber optic laser port. Figure 2.1 illustrates similarities and differences between the new and older trap designs.

2.1 A New Vacuum Enclosure

Around the Penning trap is a copper can, which is a vacuum enclosure. By pumping out the gas inside this can and then cooling it to 4.2 K we are able to get a vacuum better than 1×10^{-17} torr [12]. This ultrahigh vacuum prevents the antimatter from annihilating with the residual matter gas.

An antiproton proton annihilation releases 2, 4 or even more charged pions. By recording where these particles hit the detector we should be able to triangulate where in the trap the antiprotons annihilated. The position of the annihilation could be used to determine the plasma distribution in the trap. This could be done by leaking nitrogen gas into the trap and recording the position of the antiproton annihilations.

To have position sensitivity we need to make the vacuum enclosure as thin as possible, so that its material will make a minimum perturbation to the trajectory of annihilation particles. The thickness of the can was selected using the failure formula for a short tube with its ends held circular [13],

$$q = .807 \frac{Et^2}{lr} \left(\left(\frac{1}{1-\nu^2} \right)^3 \frac{t^2}{r^2} \right)^{\frac{1}{2}}. \quad (2)$$

Here E is Young's modulus, ν is the poisson ratio, r is the radius of the tube, t is the thickness, and q is the pressure on the wall that will collapse the tube.

Copper	E=138.6Gpa, ν =.344 @0°K
pressure	thickness
1 atm	0.021"
5 atm	0.036"
10 atm	0.046"
Stainless Steel 304	E=29.0 Gpa, ν = 0.29 @0°K
pressure	thickness
5 atm	0.032"
10 atm	0.040"

Table 2.1: Comparison of the thickness of a thin walled tube

Table 2.1 compares the thickness of copper and stainless steel required to withstand different pressures. It may have been possible to make the wall a bit thinner than the 0.036"

we chose, but a thinner wall probably would not have survived the working environment of the laboratory. A test can, with a vacuum inside, was significantly deformed when

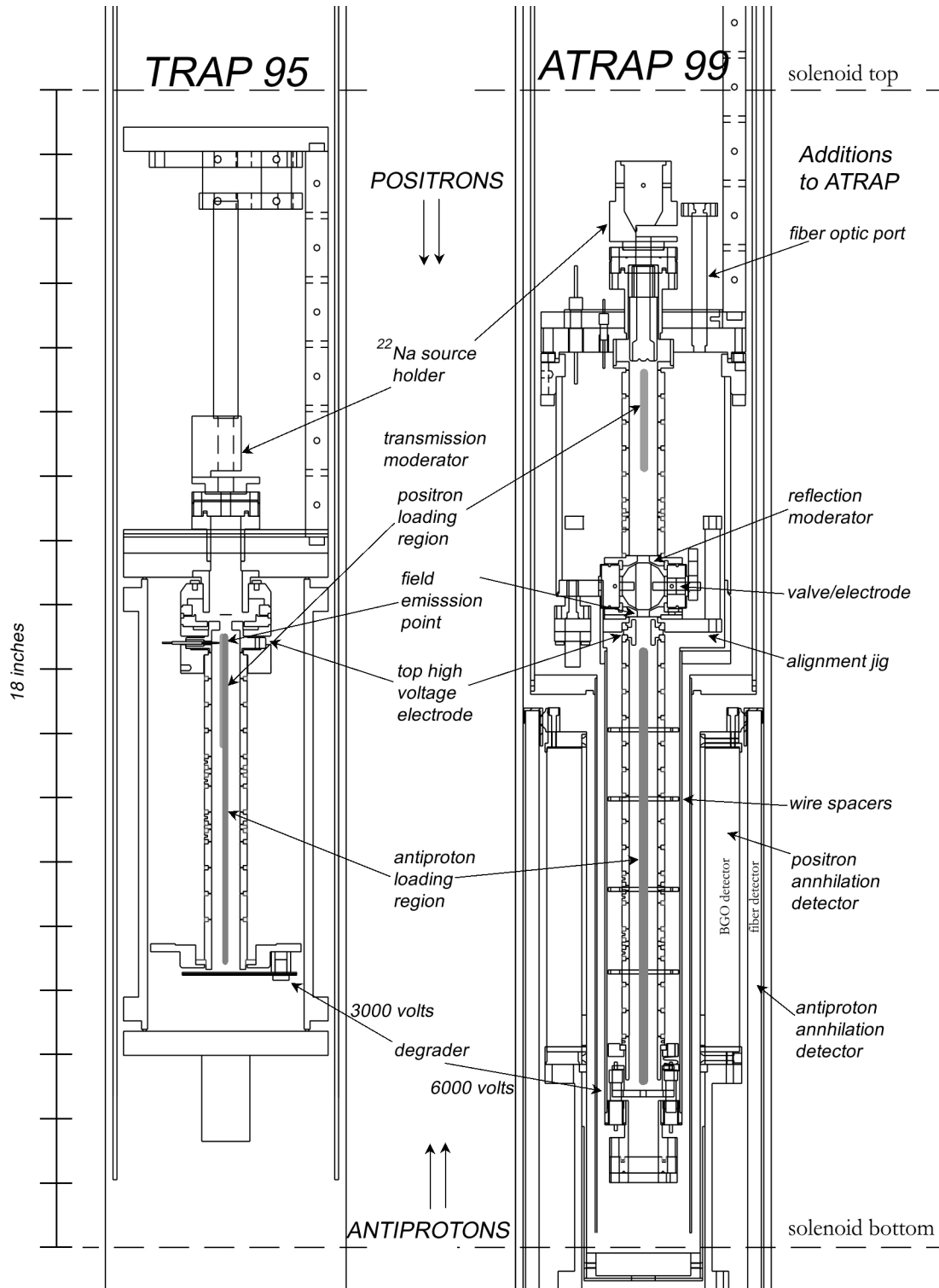


Figure 2.1: A comparison of the apparatus for LEAR and ATRAP

“whapped” with a wooden pencil.

To make room for the antihydrogen annihilation detector we reduced the diameter of the vacuum enclosure by a factor of 2.3. This significant challenge necessitated several innovations. Because we have 53 feedthroughs into the vacuum space, we needed the largest possible diameter on the top of the can. Thus, the vacuum enclosure changes size. To route the wires around the change in diameter I constructed a circuit board which moves the wires from the small diameter to the larger one in an organized way. A second circuit board also inside the trap enclosure arranged the wires into a configuration that matched the pattern of the ceramic feedthroughs. To keep the signal lines, which are thin straps of copper, from shorting with each other, we put in teflon wire guides.

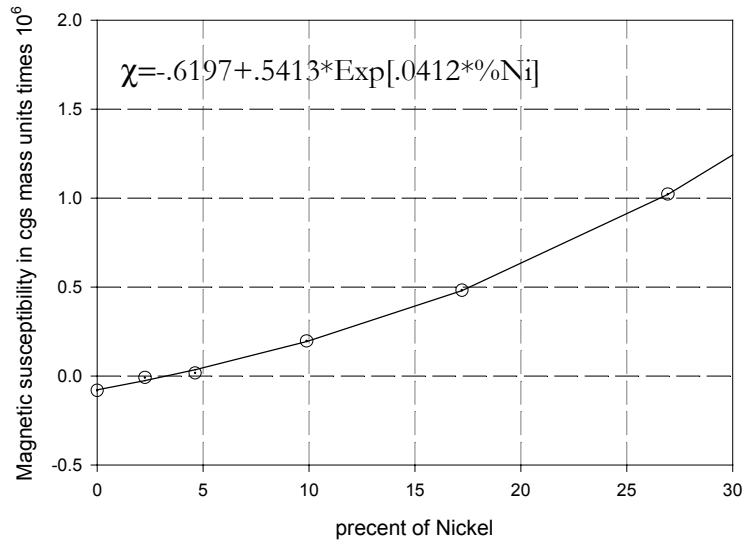


Figure 2.2: A plot of the magnetic susceptibility of 70/30 copper nickel alloys at 4.2 Kelvin [15,16].

Because the experiment is operated at 4.2 Kelvin, there is a lot of leeway in what materials can be introduced into the vacuum environment. The gas that would normally flow out of a porous material and ruin a room temperature vacuum becomes frozen in place. But it is important to insure that all the materials can survive thermal cycling. In our trap the

heads of the nylon screws, which were holding the circuit boards in place, were sheared off after one thermal cycle.

To bring electrical signals into the vacuum enclosure we used custom built ceramic feedthroughs. Normally these have stainless steel weld skirts. We had the weld skirts custom made of 70/30 copper-nickel, which is *not* another name for constantan [14]. The 70/30 copper nickel is an alloy with similar properties to brass. It is non-magnetic because the dia-magnetism of the copper is nearly cancelled exactly by the ferromagnetism of the nickel [15,16]. Figure 2.2 shows a plot of the magnetic susceptibility of 70/30 copper nickel at 4.2°K. For comparison, pure copper's susceptibility at room temperature [17] is 5.46×10^{-6} in cgs units. The feedthroughs were e-beam welded into the pinbase, a process that was assisted by the addition of a weld prep to the surface of the copper. A weld prep makes a lip of metal to which the weld skirt of the feedthrough is welded. This is done so that the thermal mass of the copper is nearly the same as the thermal mass of the weld skirt. The advantage of e-beam welding is that it heats locally and doesn't anneal the part, which you are welding to; hence, the pinbase is more rigid than it would be if we had placed it in an oven. On the other hand the pieces from the e-beam welder often dirty and required substantial cleaning.

2.2 New Thoughts on High Voltage Isolation

In this design we followed a new line of thinking about high voltage isolation. We moved away from any special materials, such as sapphire balls or alumina, used in the past, to stand off the high voltages. Instead we found that the key to withstanding high voltages is the distance left between surfaces. Figure 2.3, shows how high voltage region of the trap

was constructed. The electrode is held with macor, like all the others, but I used the rule of thumb, 0.001" of vacuum can stand-off 250 Volts [18].

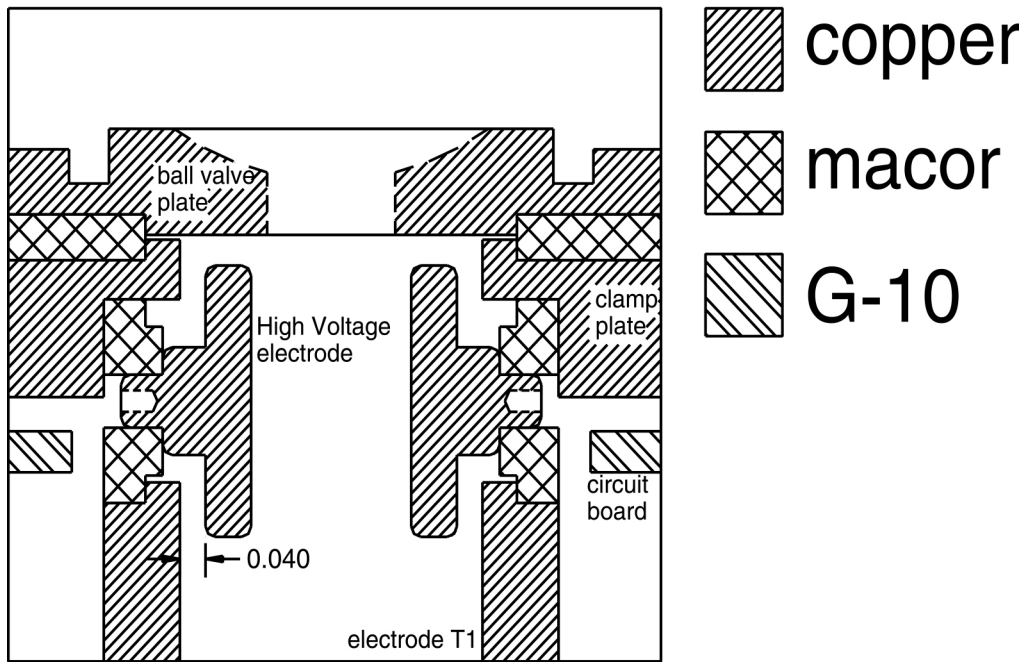


Figure 2.3: The mechanical environment high voltage electrode

OFE copper actually breaks down at a much higher voltage. Kobayashi investigated the high voltage properties of OFE copper [19] by making ultra-smooth, mushroomed shaped electrodes and then measuring at what voltage the electrodes would break down. The break down voltage was found to be 500 Volts/0.001" for untreated OFE copper. The break down voltage continues to increase with each breakdown. After 500 breakdowns the breakdown field rose to 4600 Volts/0.001", a substantial improvement. Kobayashi has measured that this increase was mainly due to the removal of impurities from the surface of the electrode, *"at the beginning of the breakdown measurements, the observed breakdown voltage is the value of the contaminant layer other than the electrode material itself"*. Using electrochemical buffing he polished the copper surfaces to a mirror-like surface and found that the break down voltage was not increased, *"the removal of protrusions does not always*

contribute to an improved breakdown if the electrode surface is carefully machined.” The paper also discusses a number of techniques to improved the breakdown voltage such as annealing, diamond turning and sputter cleaning, but the most important technique is just letting the surface breakdown a few hundred times to remove the impurities. It seems like this ought to work for us too, but we have yet to try it in our traps.

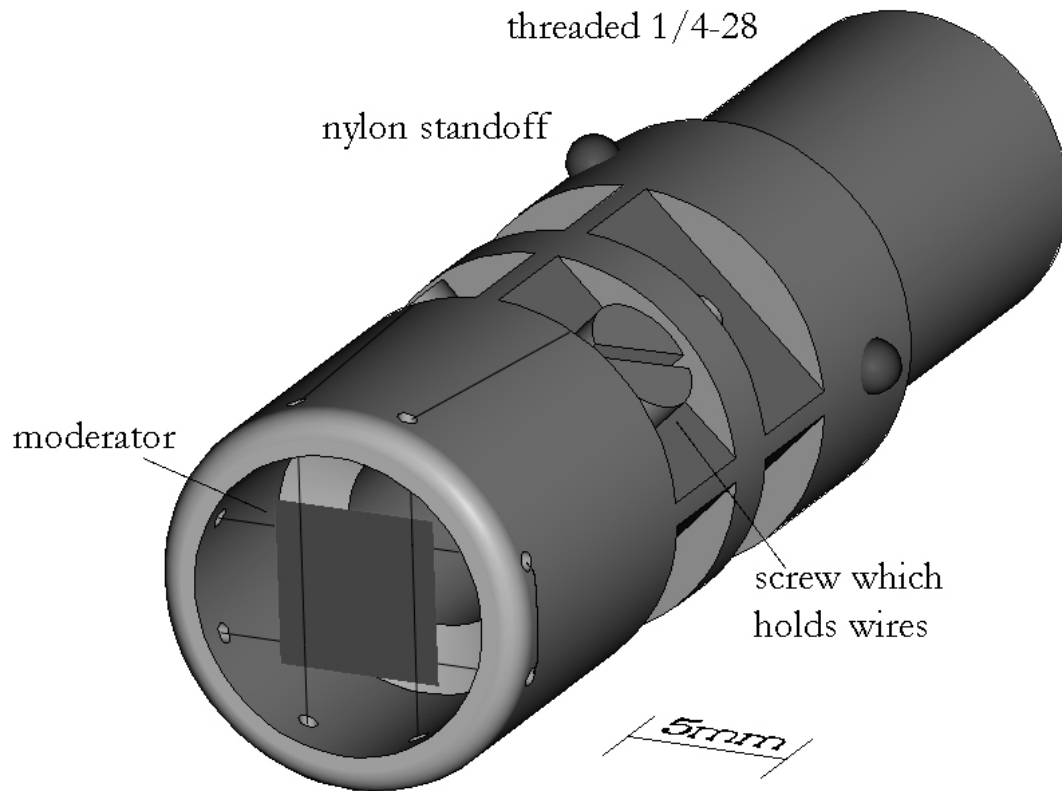


Figure 2.4: The transmission moderator holder

We have had good luck inserting teflon in the vacuum between our copper surfaces to insulate against breakdown. By placing heat-shrink tubing and teflon around every exposed surface near the degrader we raised the breakdown voltage from 3000 to 6000 volts. The added insulator helps by stopping the electrons as they leave the surface. These electrons

build up on the insulator and reduce the electric field which was causing the potential breakdown.

2.3 Holding the Transmission Moderator

Figure 2.4 shows the system developed to hold the transmission moderator in place. It is basically a $\frac{1}{4}$ -28 screw, which has been hollowed out. It screws into a macor nut which itself is glued (the blue glue [20]) into the 1.25"OD copper flange that seals the central port on the pinbase. The moderator is held between two sets of .0035" thoriated tungsten wires, which are offset by 0.01". The lower wires are passed over the upper wires to create the tension, which holds the moderator in place. This technique works well but it scores the moderator and it looks as though it would be impossible to recover the moderator after it has been mounted. The tungsten wires are held in place with custom built M2, large head, screws which tap into the holder. The 0-80 titanium screws used in the first incarnation did not have the strength to hold the wires. For more on transmission moderators see Ch. 3.

2.4 The Ball Valve

The most radical addition to the antihydrogen trap was a rotating electrode which serves to isolate the positron accumulation region from the trap where we capture anti-protons. In 1996 we discovered that antiprotons impinging on the thin tungsten positron moderator disrupt the positron loading [6]. We realized that we had to have a valve to block the antiprotons from ever reaching the positron moderator at one time while leaving open the ability to pass particles along the axis of the trap at other times. This task was complicated by the operating environment of the Penning trap. The cryogenic nature of the experiment makes it difficult to actuate a valve with a motion feedthrough into the vacuum.

In addition the solenoid that provides our 6 Tesla trapping field only has a 4" bore. So our valve has to be internally actuated, cryogenically compatible, work in a very strong 6 Tesla magnetic field, and fitted into a tight space.

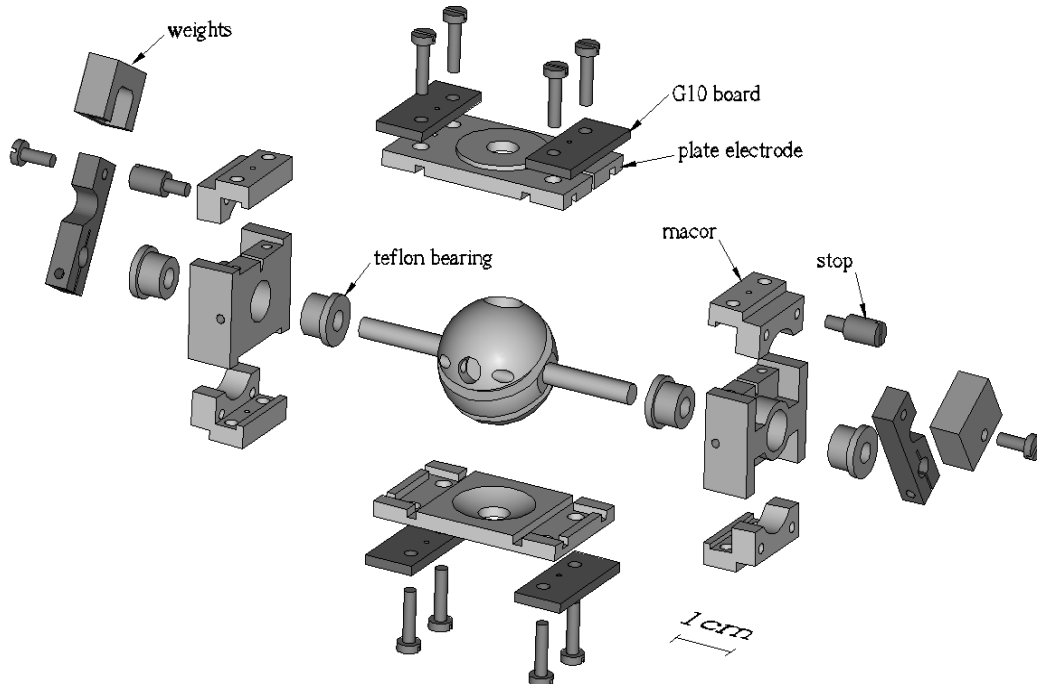


Figure 2.5: Exploded view of the ball valve

Fig. 2.5 shows an exploded view of the ball valve. On either end of the balls molybdenum axils we have attached weighted arms. The arms stop the balls' motion at preset points. The alignment of the open electrode was accomplished by inserting a soft, snug fitting, Delran rod into the tube of the ball, which was then joined to a second rod, which in turn registered to an adjacent electrode. The arm weights are made of molybdenum and help the ball remain open (or closed) after the power has been turned off. These weights were necessary to offset the tension in the wires connected to the coil on the ball.

The open position is a cylindrical hole 0.750" long and 0.250" in diameter, though the end plates restrict the diameter to 0.020". The closed position presents a recessed cavity to each trap. In one cavity we have mounted a tungsten crystal which serves as a reflection moderator. On the opposing side we have mounted a very small tungsten field emission point used to load the trap with electrons. The ball is rotated from open to closed (or visa-versa) by passing a current through the coils epoxied to the ball with blue glue [20]. Care was taken to orientate the coils so the ball would never experience a zero torque, (Fig. 2.6). It would get stuck if the torque were to equal zero. The axles run through teflon bearings. These are held in place by aluminum side walls. At 4.2 Kelvin the teflon bearings shrink much more than the metal components, which hold them. At room temperature the axles turn inside the bearings. While at 4.2 Kelvin the bearings shrink down and clamp onto the axles and then turn inside their housings. The wires for the electro-magnet are brought out along the axis of rotation to minimize the amount of free wire.

Because it is difficult to hold a sphere in a vice, all the structural features of the ball were first machined into a cylinder. The last step of the machining process was to round the ends of the cylinder to make spherical ends. The waist of the ball was held in a lathe during this rounding step and hence remains cylindrical. The ball is made of OFE copper plated with gold to prevent oxidation of the electrode surface. Then the molybdenum axles were pressed into place and approximately 80 turns of .009" enamel coated copper wire were wrapped in the channels carved in the ball. These turns form the magnet used to actuate the ball.

The reflection moderator was mounted into the ball without making electrical contact. The moderator has two small holes through which we could pass .0035" tungsten wires. These wires are then passed through alumina tubes that are imbedded in the ball. A

small pad of macor was placed behind the moderator to prevent it from making electrical contact. A teflon sheath was placed over the end of the tungsten wire, which was then etched into a fine point to make a field emission point (FEP). The teflon was then glued into the ball opposite the reflection moderator [21]. The FEP used to inject electrons into the antiproton trap; it fires with a current of 1.5 nanoamps at 875 volts.

The ball is enclosed in a housing, which integrates it into the rest of the Penning trap. The housing is made of both aluminum walls and copper plates for the top and bottom. These parts are held apart by macor spacers (which were a bit too fragile). The whole structure is held together by custom-made M3, copper beryllium screws. The G10 board, shown in Fig. 2.5, provides a place to anchor the wires going to the ball.

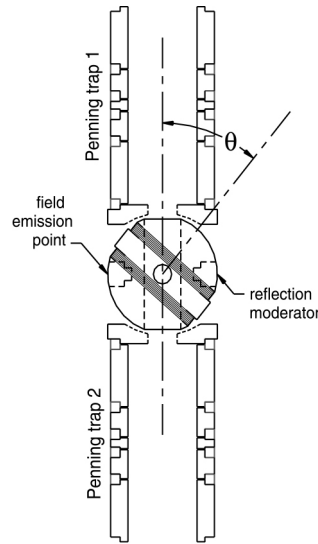


Figure 2.6: The orientation of coils with respect to the field

To turn the ball from one position to the other we apply a current (typically about 0.25 Amps) to coils. As the ball turns, eddy currents develop within its structure, which counteract the turning. If we set the torque from the applied current equal to the torque from the induced eddy currents and then, by making the simplifying assumption that the

eddy currents are constrained with in a loop geometry (instead of a sphere with a cylindrical hole and lots of awkward cuts) we can derive the equation,

$$\tau \propto \frac{\kappa B}{IR(T)}, \quad (3)$$

which says the time for the ball to flip, τ , is proportional to the magnetic field and inversely proportional to the applied current and the resistance of copper (which changes by 3 orders a magnitude between 300 K and 4.2 K). This equation only applies in the regime when the ball has overcome static friction and has begun to move and when the angular acceleration of the ball is not dominated by its moment of inertia (a term which I have neglected). Here κ is a geometrical constant associated with the actual distribution of eddy currents. We have verified Eq. 3; see Fig. 2.7a.

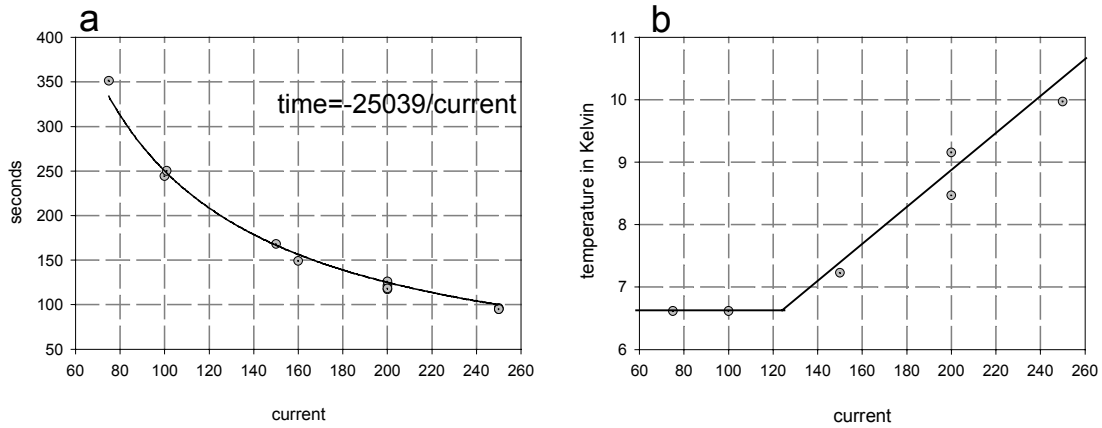


Figure 2.7: (a) Time for the ball to complete a turn vs. the current applied to its windings. (b) Temperature of the ball vs. the current applied to the windings, with lines added to assist the eye.

We were concerned that the currents running through the ball would heat it up causing it to outgas. This is a major concern because the storage time of antimatter decreases as the vacuum deteriorates. We measured the temperature increase of the ball by

attaching a carbon-glass resistor to the arm, which is coupled to the shaft of the ball.

Currents less than 0.100 amps did not measurably heat the ball even when applied for 5 minutes (Fig. 2.7b).

2.5 Passing Positrons Through the Ball Valve

The long aspect ratio of the hole through the ball valve, a length three times the diameter, greatly reduces the electric field felt by particles within the electrode. This makes it impossible to transfer particles through the ball valve using the slow stepping method developed by David Hall [22]. We found that transferring the particles quickly worked.

Figure 2.8 shows the key steps of the positron transfer. We start with a 12 volt well on the XR electrode where the positrons initially reside. We quickly jump to curve 2, by stepping down the voltage of the next electrode every 10 msec, until we reach T2. Then we go to curve 3, by applying 0 volts sequentially again starting with the XR electrode. This results in a 140 volt well on the T2 electrode. This well is then raised in 20 volt steps, every 5 seconds, to get to curve 4.

This move sequence is implemented by the LabView vi, `matrixsender.vi`, using the file `fasc4T10ms`. After the well has been raised to 12 volts, the positrons can be transferred downward in the trap using the standard moving routines. Using this technique we were able to consistently transfer 80% of positron clouds, which had on the order of million particles².

The faster you make the transfer, the better it works. We were limited by the filters on the lines of the high voltage amplifiers. And we were limited to 10 msec between the

² The data for the study of positron transfers through the ball valve is recorded in the lab book which has several titles, “Development Log”, “AD2HU”, “Log-0HU”, beginning on March 1999, pages 162 thru 213.

electrodes by the DAC hardware. This is a good example of a place where we could use a DAC that could change multiple channels simultaneously. We used the largest voltages accessible with our hardware to get a small electric field to penetrate into the long cylinder of the ball valve.

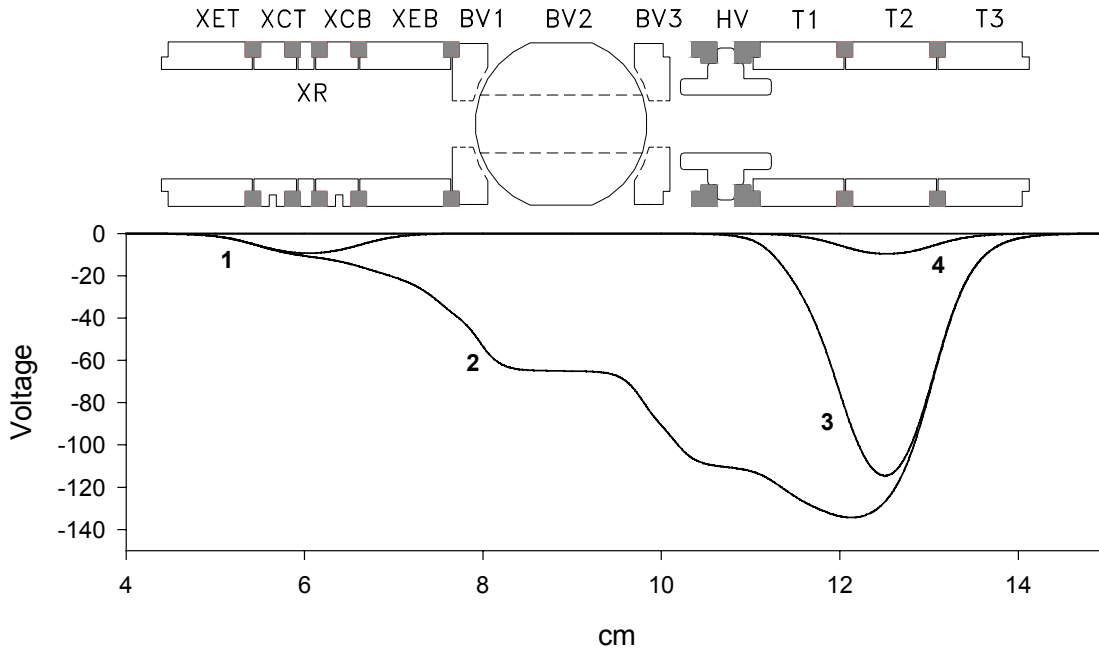


Figure 2.8: The top figure shows a scaled drawing of ball valve and the electrodes on either side. The lower plots show the 4 potential set ups we use to pass positrons through the ball.

2.6 The Source Elevator

To accumulate many positrons we use a 150 mCi, ^{22}Na source. Because of the obvious danger we constructed an elevator to remotely remove the source from its 2800 lb lead enclosure and insert it into our 4.2 Kelvin experiment while no one is near. Figure 2.9 shows a schematic diagram of our first elevator system with the key element labeled. The radioactive source is attached to a brass rod, which shields the radioactivity in the upward direction. The string is run through a tension sensor that is built from a load cell purchased from OMEGA. A load cell is a piece of metal designed to deform appreciably, yet linearly, over a large range of applied stresses. Strain gauges are placed on the cell to convert the

deformation into a voltage. The string is wound on a spool, which is attached to a stepping motor.

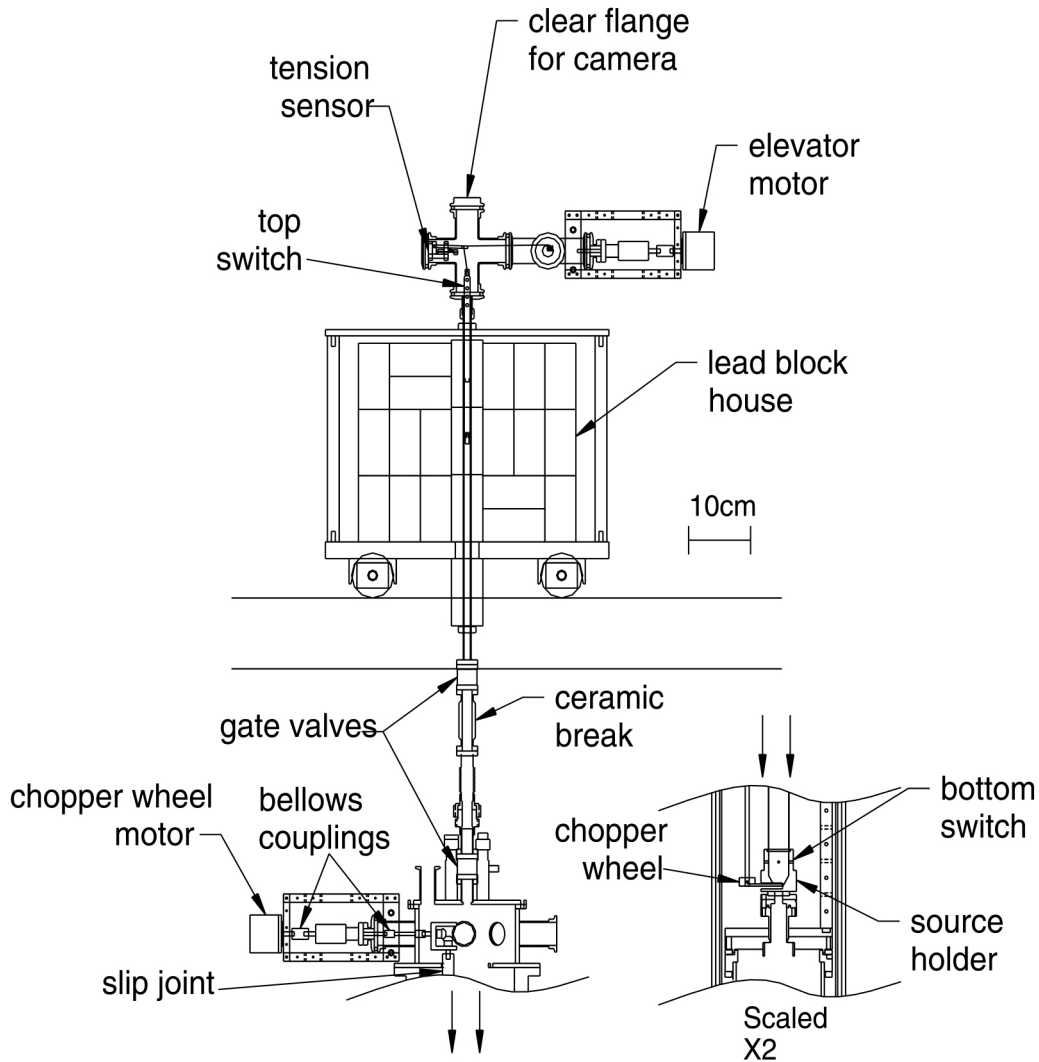


Figure 2.9: A schematic diagram of the elevator system which lowers the ^{22}Na source into the experiment.

We use the stepping motor controller MAX-410 from a small company, Advanced Micro Systems, located in New Hampshire. I have been very happy with AMS; their manuals are a bit difficult to understand but their prices are fair and their service has been extraordinary. The MAX-410 is a 4 AMP single axis unit controllable over a RS-422 bus. It has its own internal computer and can respond to simple ASCII commands.

A standard stepping motor has 200 stable positions. Applying full power to its windings causes it to lock into one position. So it maintains its largest torque when it is not moving (the opposite of a standard continuous DC motor). To get the stepping motor to increment to the next stable position, one has to switch the current to a different winding. There are typically 4 separate windings in a motor that are arranged in an alternating fashion. The motor includes iron magnets, which increase the holding torque. When the power is removed from the motor, the permanent magnets will cause the shaft to snap to one of 200 stable positions.

The MAX-410 motor controllers have the ability to “micro-step”, meaning they can divide the standard 1.8° steps into 256 increments. This is done by proportioning the current among the 4 windings of the motor. The motor cannot maintain a position achieved with micro-stepping when the power has been removed from its windings.

The MAX-410 controllers also have the ability to detect when the shaft of the motor has become stuck. A stuck shaft is called a stall. The controller does this by noticing that it has requested the motor to rotate but the encoder, attached to the shaft, has not advanced. Some care must be taken when setting the stall parameters.

To guard against accidents while remotely lowering the source, we monitor the tension and position of the source continuously, about 4 times a second, using a LabView vi. LabView is not a real time control system but it is just adequate for this application provided that the computer is not sharing its resources with other concurrently running applications and that the code has been optimized to execute quickly. To optimize a LabView vi for speed, you can use the profiler. This utility records how much time each part of the code takes. I’ve found that calling a subvi requires a substantial overhead per call. Hence to get the fastest possible code one should incorporate *all* the code into one vi. In addition, the

interrupt priority of the vi can be set to prevent the operating system of the computer from interrupting the vi excessively.

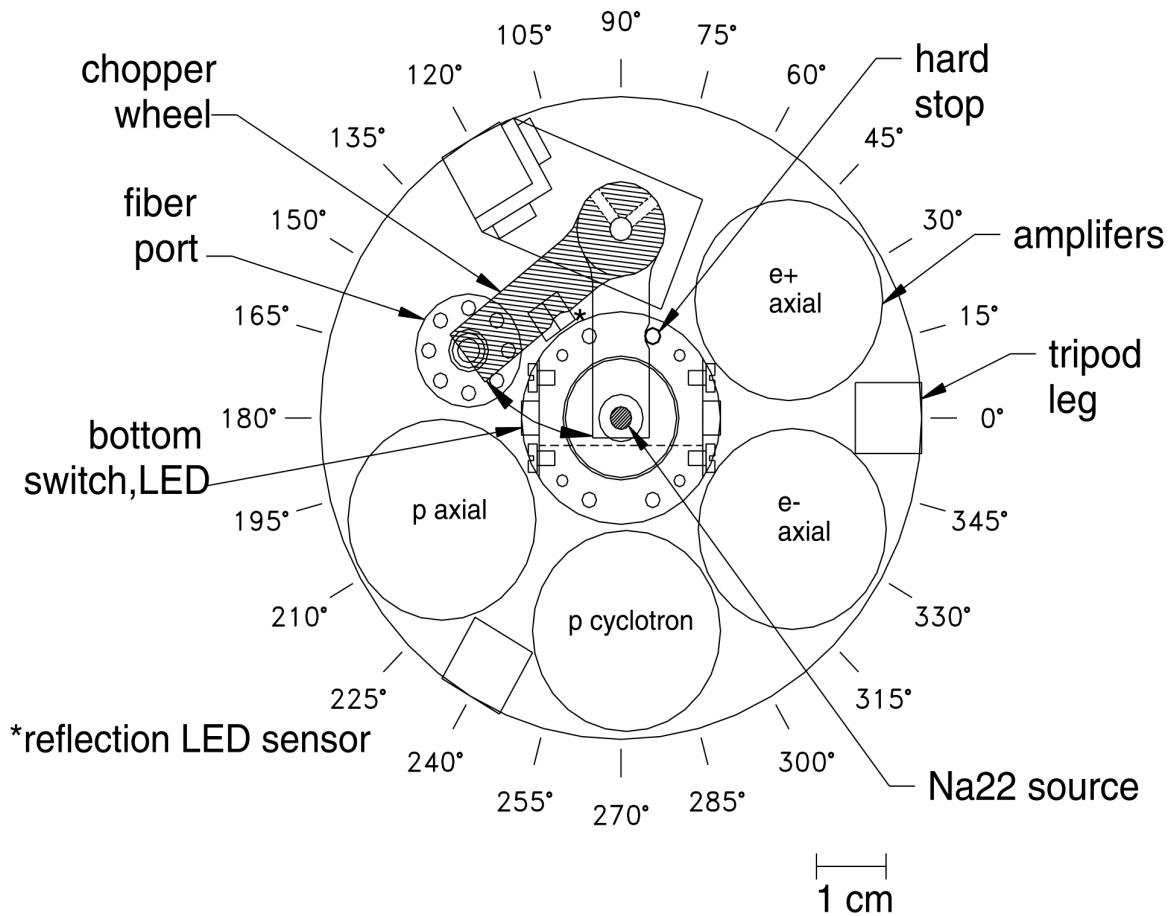


Figure 2.10: The chopper "wheel" in its operating environment.

To turn off the positron loading we have installed a beam chopper in the cryogenic part of the experiment. This enables us to count the number of positrons trapped without contaminating the measurement with new positrons. The beam chopper can also modulate the tiny picoamp positron current striking the opposite end of the trap. This lets us use phase sensitive detection. Figure 2.10 shows the chopper wheel implemented on the ATRAP experiment. It is a copper disk, 0.065" thick, which when rotated in front of the radioactive source absorbs the positrons from entering the trapping region. As with most systems, it is important to have a variety of ways to ensure the chopper wheel is where you

asked it to be. In our experiment we do this in 3 ways. First we have the mechanical feedback of the chopper wheel. When rotated into position, it hits a mechanical stop and stalls the stepping motor. In addition to the mechanical stops, we have an LED sensor that works in reflection mode to detect the position of the chopper wheel. Finally, we can keep track of the numerical value of the encoder which indicates where the angular position of the wheel. In practice, using the encoder alone will not suffice because its value becomes inaccurate after successive moves due to backlash in the gear system and elasticity in the shaft. In practice, we first reset the values for the encoder limits by running the copper wheel up against the mechanical stops on both extremes of its motion and reinitialize the encoder.

2.7 Electrical Connections

Figure 2.11 and 2.12 show the pinbase circuit that was used during the fall 2000 run at CERN. This circuit is similar to the pinbase circuits that have been used in the past; the thesis of D. Phillips gives a good description [23]. We did change the design of the GaFET amplifiers from a dual gate design to a single gate model; for further information one should refer to the thesis of Brian D’Urso. There are three coaxial lines on electrodes, T3, T8 and EET. The best way to terminate these lossy coaxial lines is with a resistor that matches the resistance of the stainless steel line; which in this case is $300\ \Omega$.

A substance modification from the LEAR system is a new method of applying the high voltage rapidly to the degrader of the antiproton trap. At LEAR we used a krytron for the rapid high voltage switching which could switch 3000 Volts in 20 nanoseconds [24]. The klystron worked reliably for more than a decade, but it required substantial knowledge to repair. We replaced it with a packaged MOSFET switch, the Behlke HTS 301, that has the

potential to switch much higher voltages. Unfortunately this switch is packaged with a driver that manufactures an intermediary voltage, ~ 200 volts, from the 5 volts supplied, using a DC-DC converter. This causes a lot of noise to appear on the output of the switch. An improvement to the circuit would be to ask the manufacture to remove the DCDC converter and let us supply the 200 volts from an external supply.

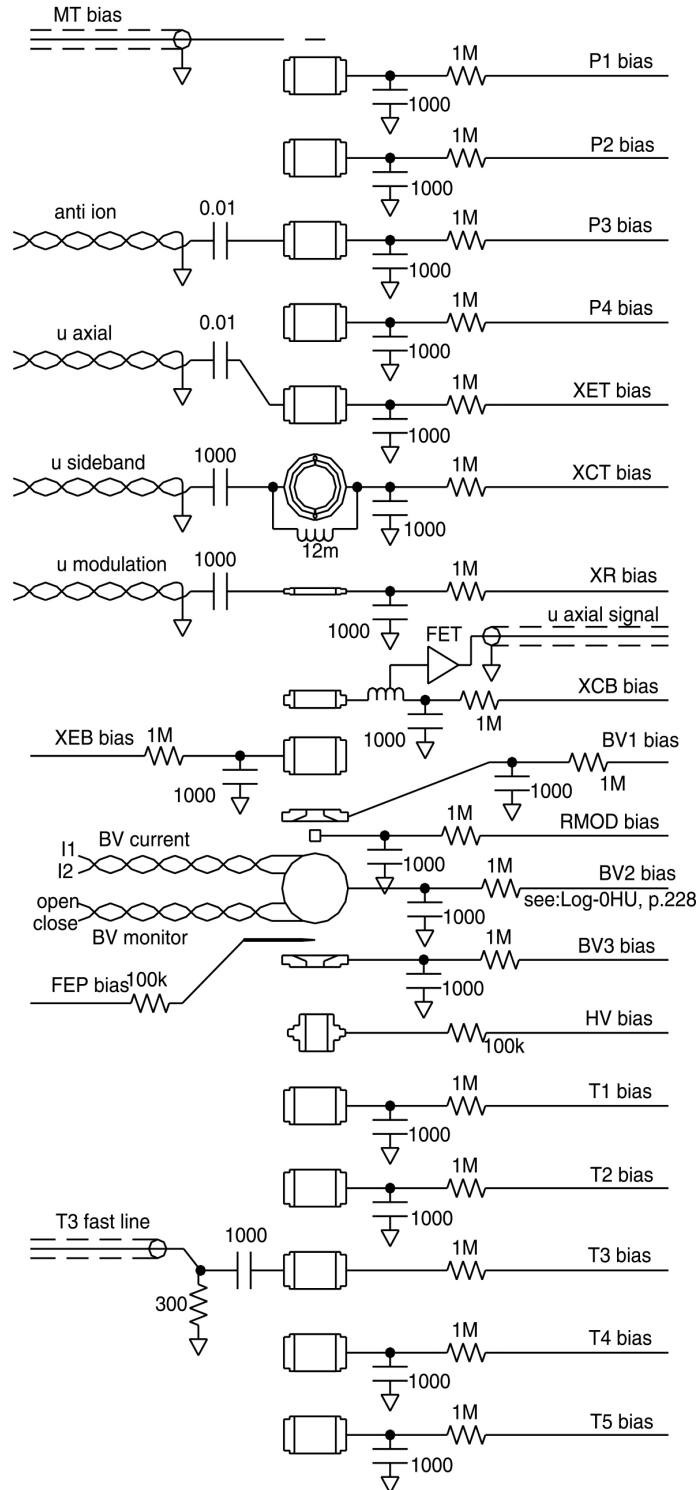


Figure 2.11: Electrical connections for the upper half of the trap

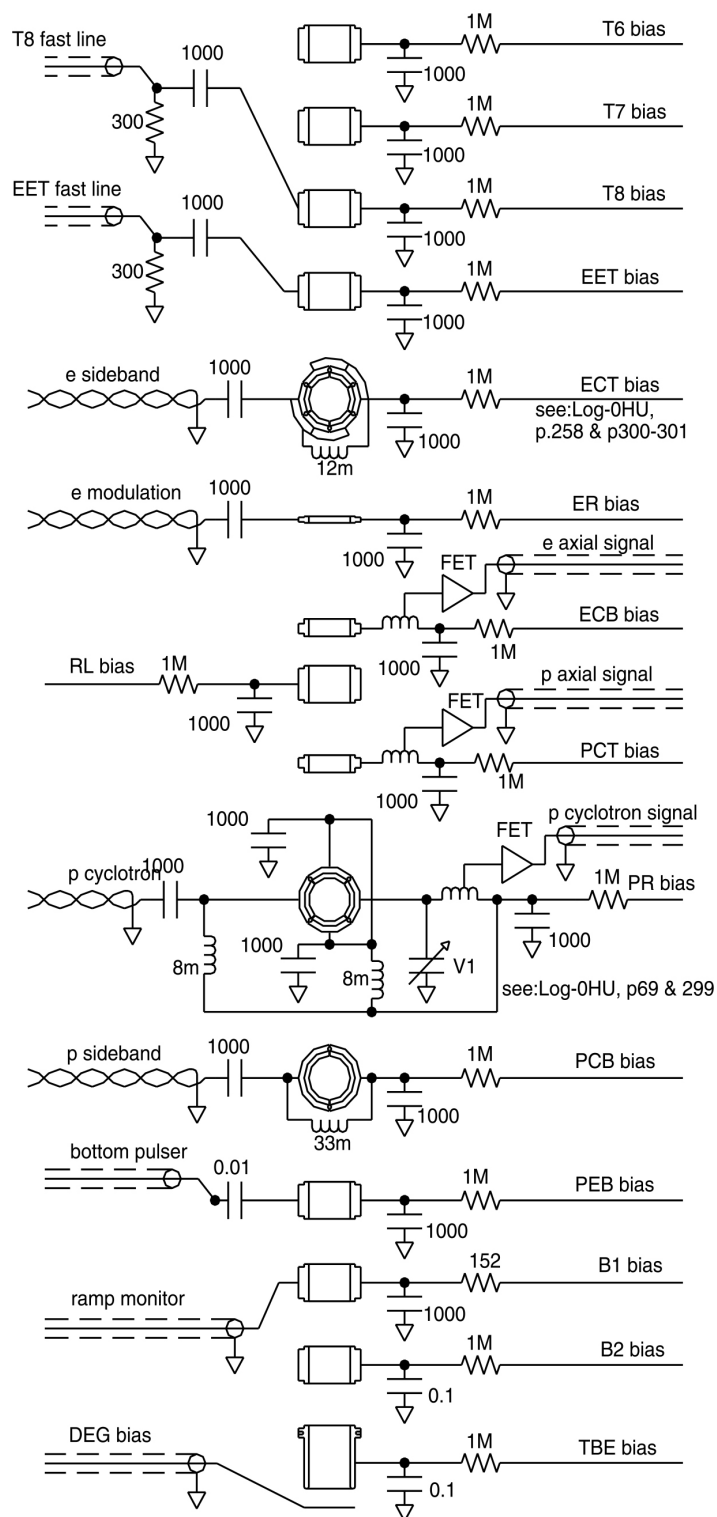


Figure 2.12: Electrical connections for the lower half of the trap

2.8 Steering and Normalization: The PPAC

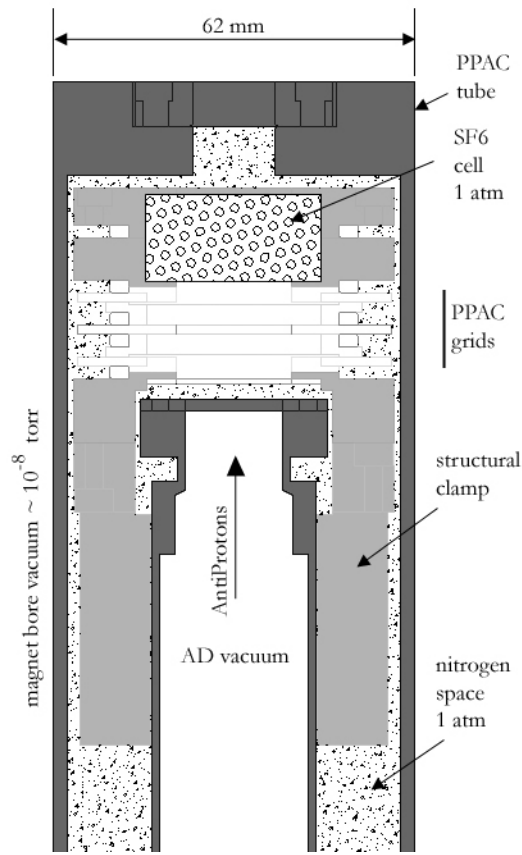


Figure 2.13: A schematic illustration of the hardware internal to the aluminum PPAC tube

To steer the antiprotons into the trap, we have a Parallel Plate Avalanche Counter (or PPAC for short). The initial version was built by our Vienna collaborators. This detector is squeezed between the exit window of the accelerator and the entrance window of the trap can (Fig. 2.13). For a good reference on how these detectors work, see Stelzer et al. [25,26]. Our detector, has 5 horizontal and 5 vertical strips, which together form a grid. Each strip is 2 mm wide and the distance between them is 0.5 mm. The strips are held at positive 200 Volts when running with antiprotons. It is common to use isobutene as the ionization gas inside a PPAC detector but we use a 95% Argon, 5% methane (at 200 mbar) mixture, because it is less flammable. To optimize the antiproton loading we tune currents

in the magnets of the accelerator, to center the particle flux on the center of the PPAC grid.

We also use the PPAC signal to count the number of anti-protons loaded per shot.

Figure 2.13 shows how the PPAC and surrounding gas spaces are organized. At the bottom of the figure, in the center of the bore, is a tube, which is the extension of the AD accelerator poking into the ATRAP solenoid. Although the beam pipe is from the AD we are responsible for maintaining the vacuum in the last two meters of the pipe. CERN requires the vacuum, in this tube, to be better than 2×10^{-5} torr before they will open the gate valve that connects ATRAP to the accelerator.

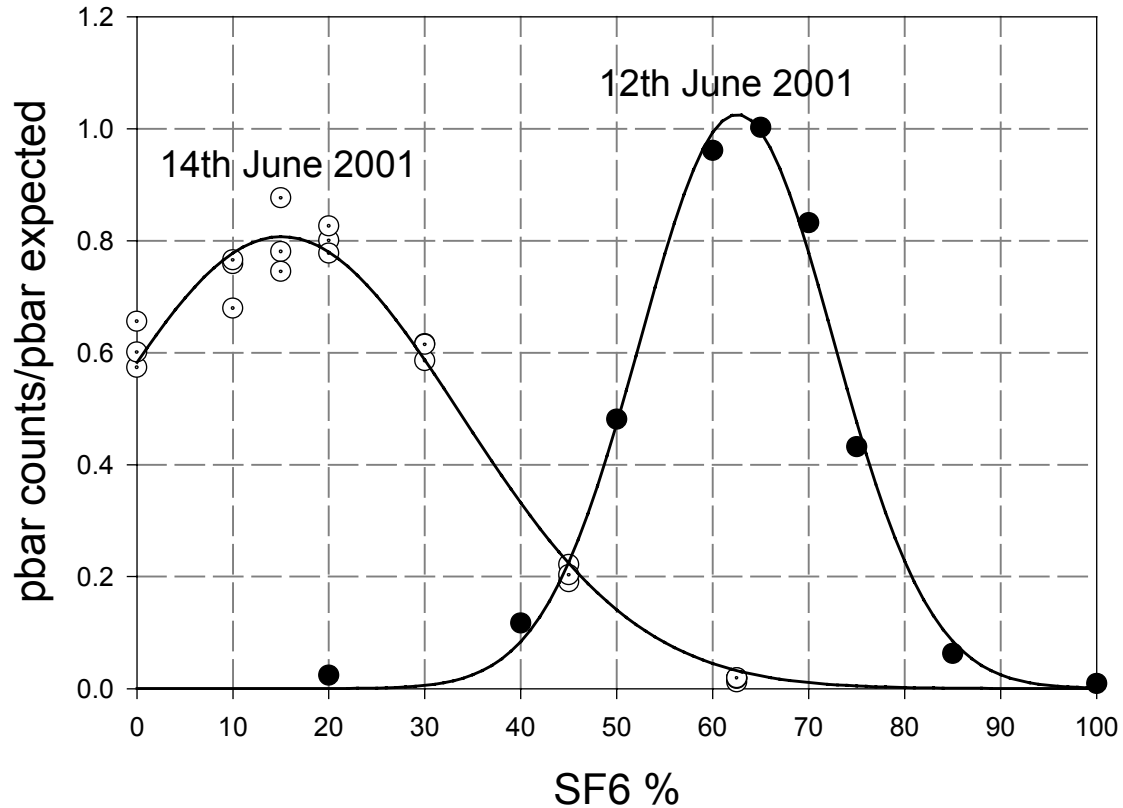


Figure 2.14: The Sulfur Hexafluoride tuning curves taken on two different days. The shift of the curves indicate the ice was building up on a window inbetween the PPAC and TRAP.

Exterior to the AD vacuum pipe is a space that is filled with nitrogen gas at atmospheric pressure and near room temperature. This region exists to keep the PPAC

warm and as a possible tuning space. It is important that this space remain vacuum tight so that no water sneaks in and condenses against the window of the PPAC. Finally, exterior to the nitrogen space is the vacuum of the magnet bore. This space is pumped from the top of the magnet bore, and we usually get a pressure around 1.0×10^{-6} (if the bore is cold). With the inclusion of the BGO detector it will also be necessary to simultaneously rough pump through both the bottom and top of the bore.

Material/Object	Δ energy keV	antiproton energy keV
	starting energy	5328.9
10 μ m Titanium window on AD beam pipe	212.7	5116.2
4.3mm N ₂ gap	36.8	5079.4
PPAC	463	4616.4
SF ₆ Cell, 100% He (100% SF ₆)	107.1, (691.2)	4509.3, (3925.2)
9.3 mm N ₂ gap	87.7, (97.6)	4421.6, (3827.6)
10 μ m Titanium window on PPAC tube	242.7, (268.3)	4178.9, (3559.3)
10 μ m Titanium window on trap can	333.1, (374.2)	3845.8, (3185.1)

Table 2.2: This table shows a summary of the energy loss by the antiprotons in the PPAC assembly

Directly after the PPAC detector is a gas chamber 15 mm long, which is filled with a sulfur hexa-fluoride/helium mixture. The pressure of the mixture is kept at 1 atm but the percentage of SF₆ to He is changed, which changes the density of material the antiprotons travel through. By adjusting the gas mixture we can tune the energy of the incoming antiprotons. In Fig. 2.14 we have plotted the normalized number antiprotons loaded into our trap vs. the percentage of SF₆ in the chamber. This tuning plot tells a little story. Between June 12 and 14, ice was building up on either the exit window or the PPAC tube or the entrance window of the trap can. This thin layer of ice removed energy from the antiprotons and hence we had to remove more SF₆ from the tuning cell to compensate. By

the June 17 enough ice was present that it was no long possible to load antiprotons. The moral of the story is that it is important to insulate the room temperature PPAC tube from the 4 Kelvin trap can. As Fig. 2.14 illustrates it is very important to know how each layer of material placed in the antiproton path effects their energy. When building the PPAC assembly we calculate the effect of each layer using a program called SRIM, which stands for Stopping and Range of Ions in Matter. This program is freely available at <http://www.srim.org>. A summary of these calculations, done later by Jochen Waltz, is given in Table 2.2. The table illustrates the difference between 100% He vs. 100% SF6 in the tuning cell. The table also shows how energy loss, dE/dx , is a function of energy.

Chapter 3

A New Mechanism for Positron Accumulation

Early in 1995 the best way we had for loading positrons was to use resistive damping in a hyperbolic trap [27, 28]. The loading technique was as follows: the positrons originate in the radioactive ^{22}Na source and travel through the hyperbolic trap to the reflection moderator on the other side. The moderated positrons leave the reflection moderator and reenter the hyperbolic trap, losing enough energy, via electrical damping in the tuned circuit elements attached to the electrodes, so that they can not exit the trap along their entry path. With a 10 mCi ^{22}Na source [28] $120\text{ e}^+/\text{hr/mCi}$ were accumulated.

Desperate for improvement in the loading rate and simplification in the loading mechanism, we decided to try to load positrons the same way we load antiprotons [29], via sympathetic cooling with a plasma of electrons. The novelty was that the electrons and positrons have opposite charge and hence we would need to use the new nested well techniques that had been recently developed [30]. We built an apparatus similar to the one shown in Fig. 3.2a and found that it loaded positrons. But to our surprise they loaded even when there were *no electrons* in the trap! The loading rate was $40000\text{ e}^+/\text{hr/mCi}$, a rate over two orders of magnitude higher than with the resistive damping technique. This was a fantastic discovery. It took us several years to prove exactly how the positrons were loading. In the fall of 1999 we submitted a paper to PRL, “Field Ionization of Strongly

[5]. This chapter explains the new method.

3.1 Preparing the Moderators

Positrons originating from the ^{22}Na source are traveling with anywhere from 0 to 511 keV worth of kinetic energy. In order to trap them in wells that are a few volts deep, we must remove nearly all their energy. This is done with a crystal moderator. The positron enters the moderator, in our case tungsten, and thermalizes with the electrons in the bulk of the material. If a positron thermalizes near the surface of the metal with a positive work function, it will be expelled from the surface as a moderated positron traveling with a few eV worth of energy [31] (the work function of the surface). The efficiency for positron moderation is on the order of 10^{-4} for transmission moderators and 10^{-3} for reflection moderators.

To investigate positron loading we used a standard cylindrical trap [32] with a thin transmission moderator, a 2 μm tungsten crystal W(100) at the trap entrance and a thick reflection moderator, a 2 mm tungsten crystal W(110), at the bottom, see Fig 3.2a. Both the thin transmission moderator crystal and the highly polished reflection moderator crystal were treated using standard techniques described by R. G. Musket in his review article on preparing atomically clean surfaces:

“Carbon, which originates in the bulk of [Tungsten] is the most difficult contaminant to remove. The two most widely used techniques for the initial cleaning of a tungsten surface are (1) prolonged heating at a high temperature in UHV, and (2) reaction with oxygen to remove the carbon in the form of CO, followed by flashing at high temperature in UHV to desorb the oxygen as tungsten oxides.” [33]

Specifically this translated into heating our moderators with an electron beam to 1200 °C in 10^{-6} torr of oxygen for 30 minutes and then holding them at 2000 °C for 3 minutes in a vacuum better than 10^{-7} Torr. After five repetitions the moderators were slowly cooled over several minutes to room temperature, exposed to 1 Torr of oxygen, then placed into our apparatus. Both moderators were exposed to air for at least 3 days before we could pump down the trap can.

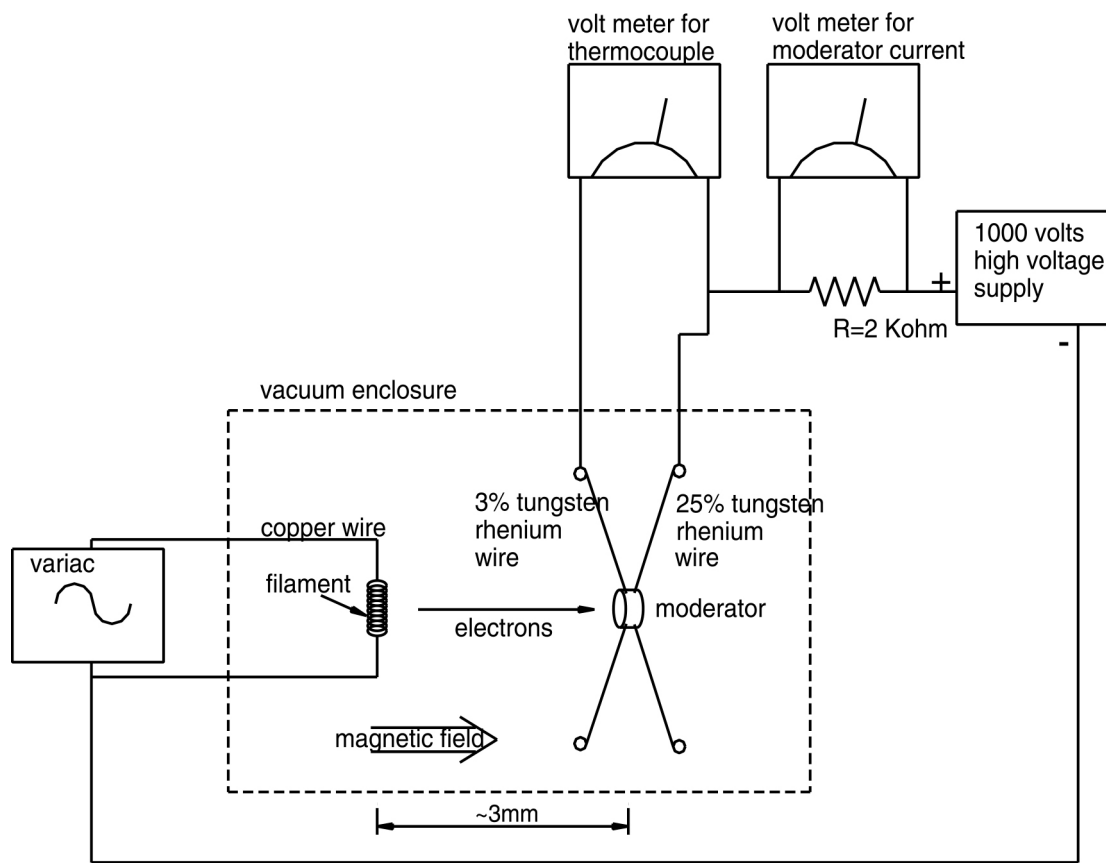


Figure 3.1: A schematic diagram of the reflection moderator heating setup.

We heated the reflection moderator using an accelerated electron beam (Fig. 3. We used a filament from an ordinary 100 Watt light bulb. These filaments have resistance on the order of 100 ohms but only when heated. The moderator is mounted on two wires, which run through small holes drilled through the moderator with an EDM machine. The

composition of one wire is 25% tungsten 75% Rhenium, while the other is 3% tungsten 97% Rhenium. Together these wires make a type D thermocouple, which is able to measure temperatures up to 2300 °C. Here we are using the thermocouple in an unusual way. The two wires do not physically touch each other. Rather they make electrical contact through the reflection moderator. The measured voltage drop across the thermocouple will still be accurate as long as the two joints with the tungsten moderator are both at the same temperature. There is also a resistor which is used to measure the current flowing to the moderator.

It is critical that a magnetic field is provided to help guide the electrons to the surface of the moderator. Also, the filament should be placed close (a few millimeters away from the moderator) or the current will strike the wires holding the moderator, not the moderator itself.

We tried heating the transmission moderator with a filament as well but were worried that doing so put holes into the thin moderator surface. So we used a 780 nm diode laser instead. This was brought into the vacuum via a fiber optic cable. In hindsight it appears that the laser also causes holes to appear in the transmission moderators, presumably because it melts away the thinnest parts of the surface. The thermocouple trick does not work as well with a transmission moderator, because the moderator is too thin to conduct much heat from the center where the laser hits to the edges where the thermocouple wires are. So an optical pyrometer was used to measure the temperature.

Initially we hoped to heat the moderators up to 2000 °C inside the trap can, when it was at 4.2 K, to remove the surface layer of tungsten oxides. I found that one could light a light bulb inside a 6 Tesla magnetic field as long as the drive frequency for the current was above 300 Hz. Below 300 Hz the filament would break due to the forces caused by the

magnetic field. Above 300 Hz the filament could no longer react mechanically to the magnetic force and hence the force was averaged to zero. We also found that we could make a light bulb light when submerged in liquid nitrogen. We only attempted to light a filament inside of the trap can once, and it broke when we applied the high voltage to the moderator. In hindsight we were lucky; if we had annealed the moderator in situ, we would have never observed the new Rydberg positronium loading technique since this effect goes away after the moderator has been heated. Even so, the possibility of lighting a filament in the 6 Tesla magnetic field should be noted even if, for the moment, there is no application.

3.2 New Mechanism for Positron Accumulation

Our experiments at Harvard used positrons from a radioactive source 2.5 mCi ^{22}Na with a 2 mm diameter. The positrons originate in the source and travel along magnetic field lines (5.3 Tesla), first past the chopper wheel, discussed in Ch. 2, which either blocks them or allows them to enter the trap can through a 10 μm Titanium window. They accumulate in the location shown in Fig. 3.2a. The potentials and electric fields used to accumulate positrons (the solid curves in Figs. 3b and 3c) are produced by separately biasing the electrodes. Electrons are accumulated at the same location when the potential in the trapping region is reversed in sign (dashed curves in Figs. 3b and 3c).

The new physical mechanism for capturing positrons arises when a moderated positron leaves the transmission moderator followed by a secondary electron. To be captured the positron must have an electron partner. The strong magnetic field keeps the positron and electron on nearby field lines. If their Coulomb attraction energy exceeds their kinetic energy in the center-of-mass frame, they are bound in a very highly magnetized state of Rydberg positronium. This positronium is ionized by the electric field within the trapping

well if this field is strong enough. If the kinetic energy of the positron is sufficiently low, it will be captured.

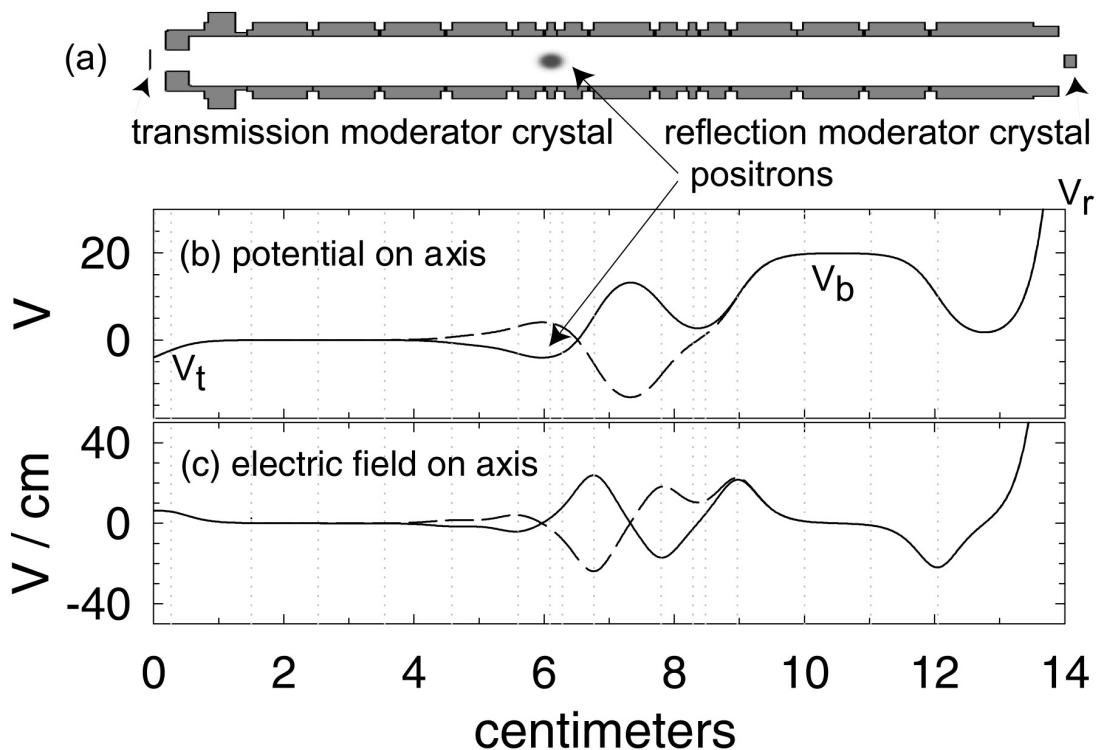


Figure 3.2: The electrodes of an open access Penning trap (a) are biased to produce an electric potential (b) and field (c) along the central axis that confines e^+ (solid curves) or e^- (dashed curves). A 5.3 Tesla magnetic field parallel to this symmetry axis guides fast positrons entering from the left through the thin crystal and towards the thick crystal.

Biasing the transmission moderator to a potential V_t with respect to neighboring electrodes adds energy eV_t to one species and removes eV_t from the other. Optimizing V_t (Fig. 3.2b) thus reduces the axial spacing between the positron and electron and improves their axial velocity matching as they approach the potential well of the trap.

Positrons accumulate one at a time; the loading mechanism does not depend upon the interaction of successive positrons from the source. The most direct evidence is that the number of accumulated positrons is proportional to the incident flux of positrons from the

radioactive source. This flux is deduced from the current measured on the reflection moderator and was varied by pulling the radioactive source away from the trap.

The strong magnetic field is crucial to the new physical mechanism. It keeps the “guiding center” of any slow moderated positron or electron that emerges from the transmission moderator on a magnetic field line as it makes one pass through the trap. The tiny magnetic moment associated with a small radius cyclotron orbit about the guiding center has a negligible effect on the trajectories. This magnetic moment $\vec{M}$ is an adiabatic invariant (for lengthy discussion of adiabatic invariants see Ch. 5). The electric fields of the trap (or from a partner particle of order 1 μm away) will accelerate or decelerate a charged particle along its magnetic field line. But these electric fields are not strong enough to cause the $\vec{E} \times \vec{B}$ drift motion to move the particle appreciably off its one dimensional axial field line path during one pass through the trap.

As a quick numerical example, imagine a positron which has left the moderator with an axial energy of 1.5 eV. It would be traveling along the magnetic field with a velocity around 726000 meters/sec. The $\vec{E} \times \vec{B}$ velocity of the positron, from an electric field of the electron, would be of the order of 1900 m/sec (in a 5.3T field).

A distinct signature of this new physical mechanism is that the rates for accumulating positrons and electrons are the same. Positrons are captured in the potential well represented by the solid curve in Fig. 3.2b. Inverting only the well potential (dashed curve in Fig. 3.2b) instead confines electrons. The striking equality of the superimposed accumulation rates in Fig. 3.3a, 3.5a and 3.5b for positrons (filled circles) and electrons (open circles) provides the confirming evidence. The rates depend identically upon the trap

potentials which are not inverted-the transmission moderator potential V_t (Fig. 3.3a) the reflection moderator potential V_r (Fig. 3.3a) and the barrier potential V_b (Fig. 3.3b).

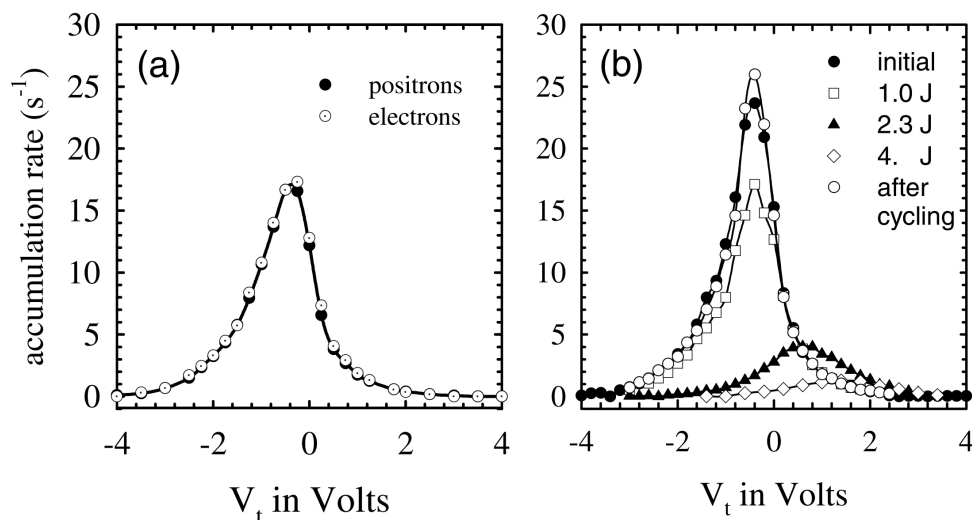


Figure 3.3:(a) Accumulation rates, strikingly equal for positrons and electrons, depending upon the potential of the transmission moderator, and hence upon the electric field at the moderator surface. (b) Changes in the positron accumulation rate when adsorbates on the transmission moderator surfaces are desorbed using laser pulses with the total energy indicated. The cycling is to 300K and then back to 4.2K.

As a further test that positronium enters the trap, we raised the potential between the transmission moderator and the trapping well by up to 6 Volts, so that one of the charged species by itself could not enter the trap well. The potential changes gradually enough as a function of position that the electric field does not increase significantly. If the loading mechanism does not involve neutral positronium, this would essentially eliminate the accumulation. It does not.

The positronium that is ionized must be in a high Rydberg state, with positron and electron well separated, because the weak electric field of the Penning trap (Fig. 3.2c) is able to ionize it. Fig. 3.4 shows the accumulation rate as a function of the magnitude of the maximum axial electric field within the Penning trap. The electric field E_z necessary to

counter the attraction of the positron and electron, spaced by r , is $E_z = 14(\mu\text{m}/r)^2 \text{ V/cm}$ in the simplest linear model, neglecting the kinetic energies. In this model, most of the positronium ionized thus seems to have the positron and electron spaced by 1-5 μm . As the electric field in the trap well is increased farther than shown in the figure, the accumulation rate begins to drop slightly, presumably because the electric field starts to influence the tuning of the relative velocity previously optimized by changing V_t and more field ionization takes place before the trapping well.

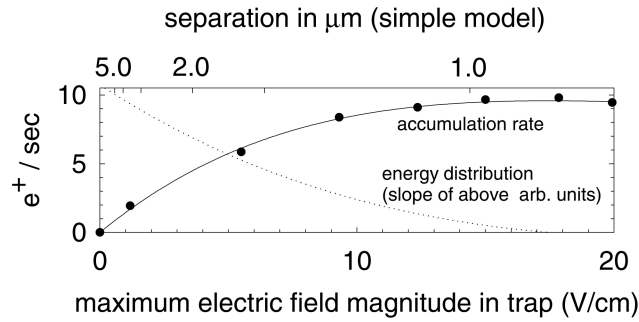


Figure 3.4: Measured dependence of accumulation rate upon the maximum electric field magnitude within the confines of the Penning trap (points) and deduced shape of the ionization energy of the Rydberg positronium (dotted curve).

The magnetized Rydberg positronium that we load into the trap is formed via two distinguishable channels that are represented in Figs. 4c and 4d. The first channel (Fig. 4c) is most direct. Energetic positron from the ^{22}Na source slows down in the transmission moderator, from which it emerges accompanied by a secondary electron and is ionized as described above.

The second channel opens when moderated positrons from the reflection moderator are directed back to the transmission moderator. Fig. 3.5a shows a sharp increase in the positrons (or electrons) accumulation rates, which more than double when the potential V_r

on the reflection moderator is raised to allow the moderated positrons to leave it. Fig. 3.5b gives direct evidence of these low energy positrons for $V_r = 100$ Volts. Varying the height

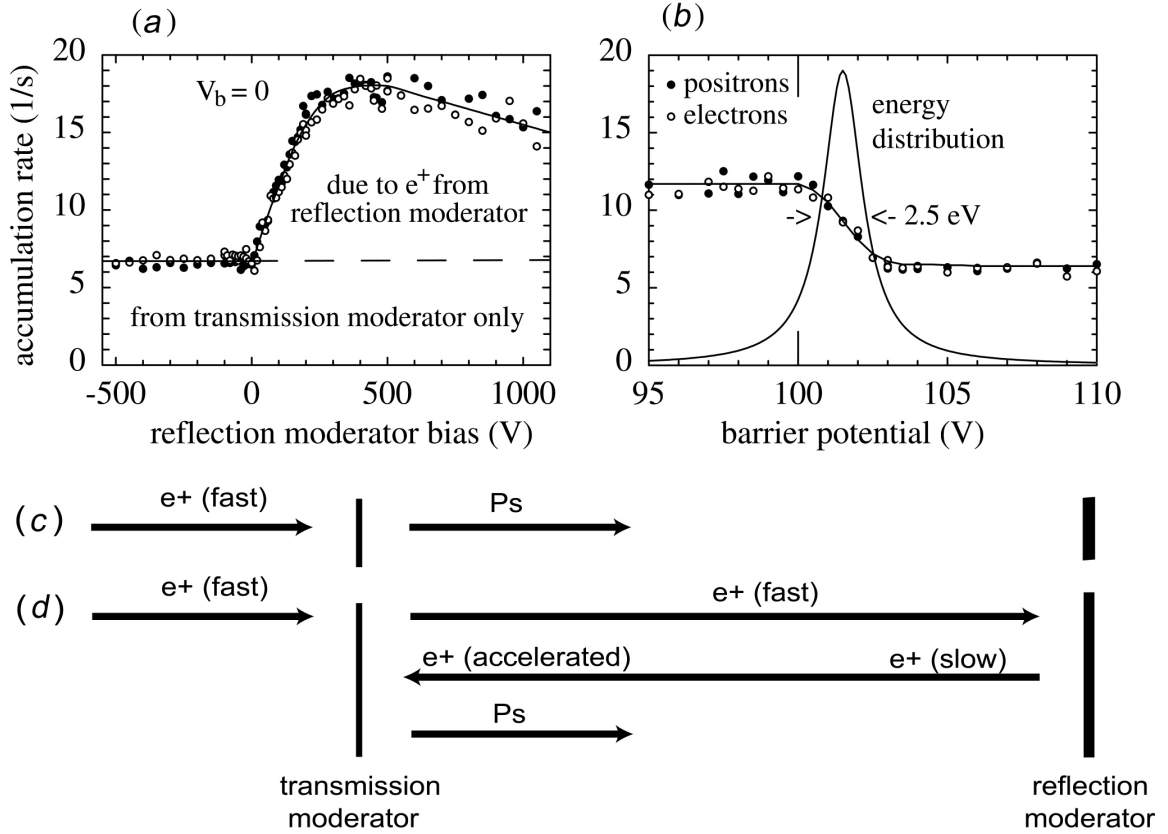


Figure 3.5:(a) Increasing the potential barrier V_r of the reflection moderator opens a second channel for e⁺ to return to the transmission moderator. (b) Increasing the potential barrier V_b above $V_r = 100$ Volts first shows that these e⁺ have an average energy of 1.5 eV and a 2.5 eV width, then blocks the returning e⁺ to stop the additional accumulation.

of a potential barrier V_b , Fig. 3.2, placed in the path shows them to be positrons moderated in the thick reflection moderator, with an average kinetic energy of 1.5 eV and an energy width of 2.5 eV. Most incident positrons from the source pass right through the thin transmission moderator and strike the reflection moderator on the other side of the trap. A fraction $\eta_r \sim 10^{-3}$ of these slow and diffuse near the entrance surface of this crystal, then emerge with low energies [34]. Upon returning to and entering the transmission moderator

some, fraction of the backward traveling positrons are slowed in the transmission moderator and emerge accompanied by a secondary electron just as for the first channel. Above $V_r=400$ volts the accumulation rate gradually decreases, probably because the accelerated positron penetrates too deeply into the transmission moderator to a location from which it is less likely to emerge.

The loading rate for positrons into our trap depends upon the gas adsorbed on the surface of the transmission moderator, presumably because the gas absorbed on the surface affects the way in which the secondary electron joins to the positron. We gradually removed the gas layer absorbed on the surface with 100 msec pulses of up to 4 Watts of 818 nm radiation from a diode laser while the trap was at 4.2 K. Fig. 3.3b shows the resulting decrease in the positron accumulation rate. The peak in the accumulation rate also shifts to a value of the transmission moderator potential V_t that is higher by 2 V. The adsorbed gas layer and higher accumulation rate are restored when the trap and its vacuum container are simply warmed to 300 K and then cooled back to 4.2 K. The restored accumulation rate is slightly higher than initially observed. We observed similar changes in positron efficiency when we used antiprotons and electron-beam heating to remove absorbed gas [35]. Over months of loading and repeated thermal cycling of the apparatus between 3000 K and 4 K, the peak loading rate remains stable as long as the absorbed gas is not deliberately removed from the surface of the transmission moderator crystal.

At CERN we moved from a 2.5 mCi to a 110 mCi source. The loading rate of the large source is $1.4 \times 10^4 \text{ e}^+ \text{ h}^{-1} \text{ mCi}^{-1}$ which is about a factor of 3 smaller than the rate observed for the smaller source, $4.0 \times 10^4 \text{ e}^+ \text{ h}^{-1} \text{ mCi}^{-1}$. The decrease in rate is probably due

to the increased radial size of the source. With the large source we are able to load 2 million positrons in under an hour, Fig. 3.6.

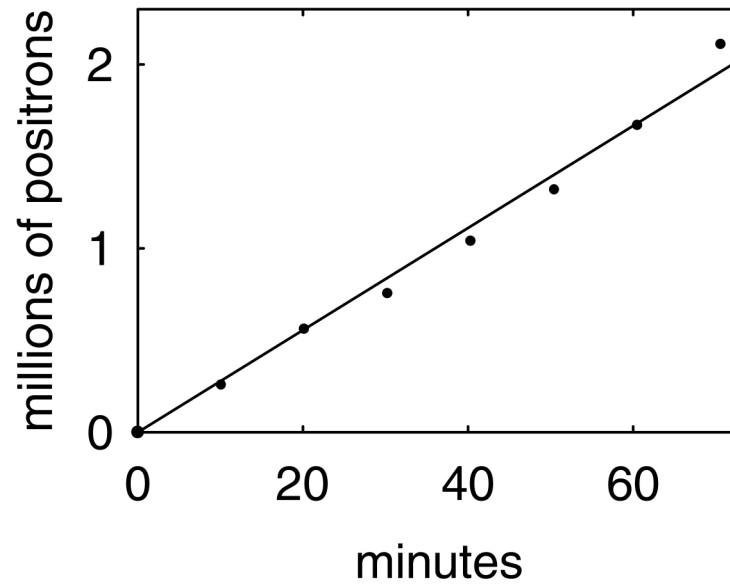


Figure 3.6: The number of positrons accumulated in our high vacuum cylindrical Penning trap vs. time for the 110 mCi source.

Chapter 4

Attempting Antihydrogen

During the five months in 2001, when CERN provided antiprotons, we investigated two separate techniques, three body recombination and pulsed field recombination. Although we did not detect any antihydrogen we did observe the first ever demonstration of antiprotons losing kinetic energy inside a cold positrons plasma. And we learned how to pass particles among electrodes on nanosecond timescales. We can now pass a particle plasma through the trap structure almost as if were a baseball, throwing the plasma from one end and catching it at the other.

4.1 Preparing to Make Antihydrogen

An experiment begins by capturing multiple shots of antiprotons from the accelerator and cooling them to 4.2 Kelvin [30]. The antiprotons accumulate in multiple (6 or 7) wells, see Fig 4.1. Shots come every 144 seconds³ and we usually take between 5 and 10 shots capturing about 8000 antiprotons per shot. The antiprotons are then brought together in a single well. Next the cooling electrons are pulsed out of the antiproton well by dropping one side of the antiproton well for 200 nanoseconds. The electrons are much lighter and are traveling much faster relative to the antiprotons. Thus when the well drops they race out and the antiprotons barely move.

³The time between antiproton shots fluctuates depending on the other demands made of CERN's infrastructure.

Meanwhile at the other end of the trap our 150 mCi source has been lowered into the experiment from its lead block enclosure under remote control and we have been accumulating positrons using the techniques of Ch. 3. When we are done accumulating positrons the radioactive source is remotely pulled back from experiment to avoid having the positron source saturates the particle detectors. Next the ball valve is rotated open and the positrons are passed through the valve (Ch. 2). After all these steps have been completed we are ready to begin making cold antiprotons and positrons interact.

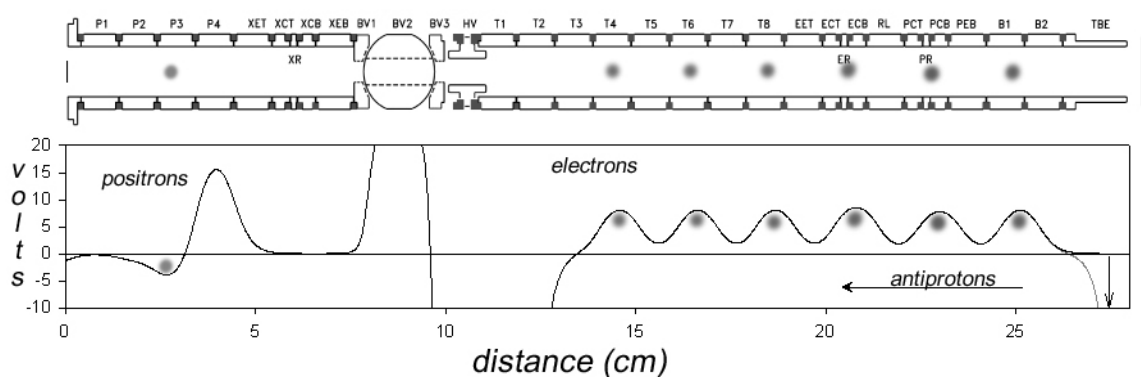


Figure 4.1: Double loading. On the left end of trap positrons are being loaded into a well on the P3 electrode. Simultaneously antiprotons are loading in the right hand side of trap into 6 separate wells located on electrodes T4,T6,ER,PR,B1.

4.2 Cooling Antiprotons with Positrons.

Conceptually the cooling of antiprotons with positrons is a one dimensional process in which we remove the energy from the antiproton's axial motion. A nice analogy for this experiment would be to imagine a bowling ball rolling back and forth in a large bowl, like the ramps used by skate board enthusiasts. Now imagine that in the bottom of the bowl there are a bunch of ping-pong balls and every time the bowling ball rolls through the ping pong balls it stirs them up losing some of its energy. After a number of oscillations the bowling ball would lose all of its energy and settle down into the bottom of the bowl. A similar

energy loss happens with an antiproton and positrons, except that because the positrons and antiprotons have different charges, the two species separate and wind up at different places in the bottom of the bowl.

In fact to trap particles with both negative and positive charges one has to use a special nested well structure [30]. In a Penning trap positrons require a negative going well and the antiprotons require a positive going well. For example, to hold the antiprotons with the positrons we would have a large positive well with a smaller negative going well in the middle; see the solid line Fig. 4.2c.

To start the cooling experiment a potential well containing cold antiprotons and some residual electrons is adiabatically elevated as shown in Fig. 4.2(c). The antiprotons are then launched into the nested trap structure by dropping potential barrier; in Fig 4.2(c) launching the particles corresponds going from the solid curve to dashed curve. The potential barrier drops in less than 20 nanoseconds. Simultaneously, the potential barrier at the opposite end of the nested well is dropped by over half its value so any electrons still confined with the antiprotons will leave the nested well structure. To confine the antiprotons, barriers are restored to full height after 1.5 μ sec, before the slower antiprotons can escape. The antiprotons are now in the nearly symmetrical nested well structure.

Two minutes after the antiprotons are injected into the nested wells, the energy distribution of the trapped antiprotons is analyzed by slowly lowering the potential barrier nearest the launch point and looking at the annihilation signal using the fast mode of the fiber detector in coincidence with the external scintillators. When no positrons are present in the nested trap, Fig. 4.2(a) shows the number of annihilations of antiprotons released from the trap as a function of the remaining barrier height. In this example about 4000

antiprotons had kinetic energies distributed around 7 eV relative to the bottom of the potential well.

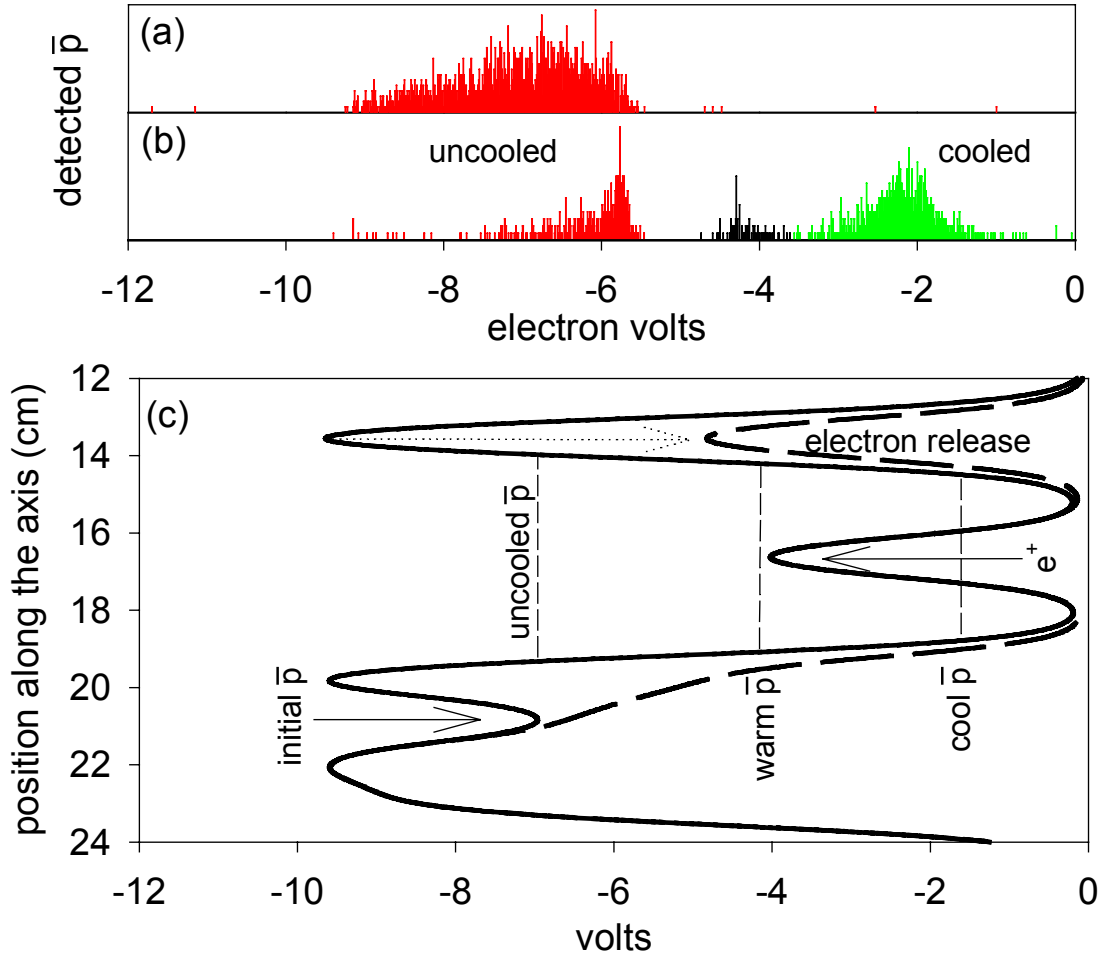


Figure 4.2: (a) Uncooled antiproton spectrum. (b) Cooled antiproton spectrum shows some antiprotons are not cooled, the warm ones have cooled to the level of the positrons and some have mysteriously cooled below the level of the positrons. (c) Potential wells for the positrons and antiprotons.

To demonstrate positron cooling we repeat this process but with approximately 250,000 positrons preloaded into the inverted central well that is nested within the longer outer well. These positrons cool via synchrotron radiation to thermal equilibrium with their 4.2K environment in only 0.1 seconds. They collect in a volume that is a couple of millimeters in radius and length. Antiprotons are launched into the nested trap exactly as

before. As the antiprotons pass through the positron cloud they are cooled by collisional transfer of energy to the positrons.

When we analyze the antiproton energy as before we see in Fig. 4.2(b) that some of the antiprotons remain uncooled, presumably because they are located away from the center axis of the trap where there are no cold positrons. Some are cooled to a level where we believe the positrons to be. And most of the antiprotons have cooled to level in the well below the level occupied by the positrons. This super-cooling is quite mysterious. It could be a result of a redistribution of the antiproton energy, a sort of evaporative cooling without the evaporation. Or maybe it has to do with a more complicated interaction between the positrons and antiprotons. We made this observation in the last six hours of the year 2000 beam time, so further investigation of this spectrum had to wait until 2001.

With or without positrons present, about half of the antiprotons are lost during the launch into the nested well structure. This loss was not observed in earlier experiment done with protons and electrons, Hall [22]. We think the antiproton loss might disappear if we use sideband cooling (a radio frequency technique) to decrease the radial size of the antiproton cloud. This is difficult because sideband cooling requires hours and hence would greatly curtail the number of experiments we could do in a day.

The real goal of the positron cooling of antiprotons is to make antihydrogen. As the antiprotons transverse the positron plasma their energy is dissipated through many collisions with the positrons and towards the end of the process the two species come into contact at very similar energies, and at this moment antihydrogen should form [36]. Even if antihydrogen would have formed it is unlikely we would have seen it using the deep potential wells employed so far. The high Rydberg antihydrogen would be ionized by the electric field

used to trap positrons and antiprotons. The well of the positrons has be lowered to allow the antihydrogen to escape.

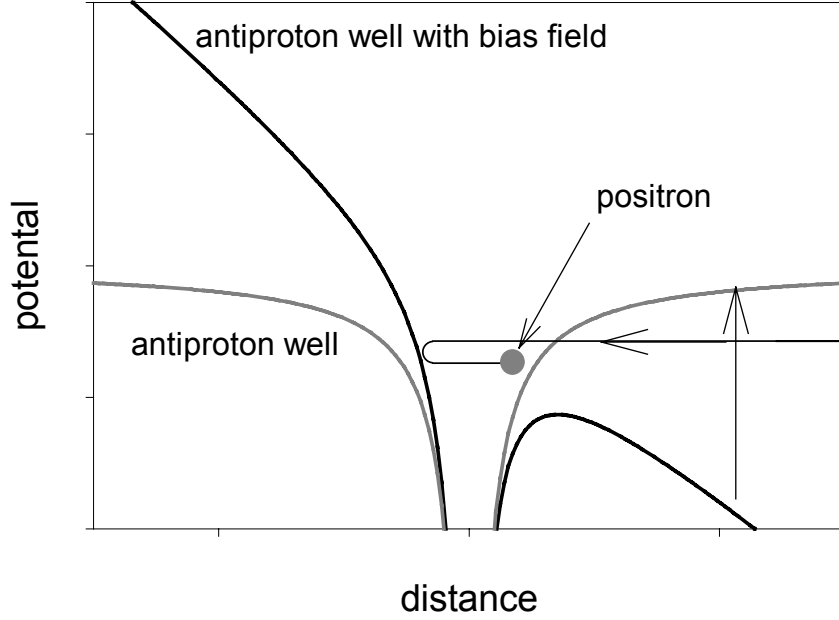


Figure 4.3: The PFR technique works by launching a positron, represented by the gray dot, into the well of a antiproton in a bias field. When the electron is close to the proton the bias field is taken away and the electron is caught in the well of the proton. An atom is born.

4.3 Pulsing and Catching Particles

Pulsed field recombination [37], PFR, is a novel technique demonstrated to allow the binding of an electron to an ion. The first step is to tilt the potential of an antiproton with a linear bias field (Fig 4.3). Because this is done for only 400 nsec the antiproton does not move appreciably during the experiment [37]. The next step is to launch a positron at the proton with an energy such that it will turn around while over the unperturbed proton well. Then we remove the bias field, which shuts the well of the proton, trapping the positron

The number of recombinations, N_{rec} , can be estimated with the formula [38],

$$N_{rec} = \rho_{\bar{p}} \rho_e V_{over} V_{it}, \quad (4)$$

where $\rho_{\bar{p}}$ and ρ_e are the antiproton and positron densities, V_{over} is the macroscopic overlap volume of the antiprotons and positrons plasmas and V_{it} is the volume of space for which a positron with an initial velocity of v_e will recombine with the antiproton. For our experiment we can estimate the number of antihydrogens we should have seen [38]

The interaction volume is defined as the volume in space in which the positron has to possess a certain velocity to get recombined. For the voltages and slew rates of the electric field pulses used in this experiment, this V_{it} is on the order of 10^{-10} cm^3 . If V_{over} is maximal the expected number of recombinations is: $10^5 \times 10^6$ (or 10^7) $\times 10^{-10} = 10^1$ (or 10^2).

In the fall of 2000 one of the participants in the PFR demonstration, C. Wesdorp,

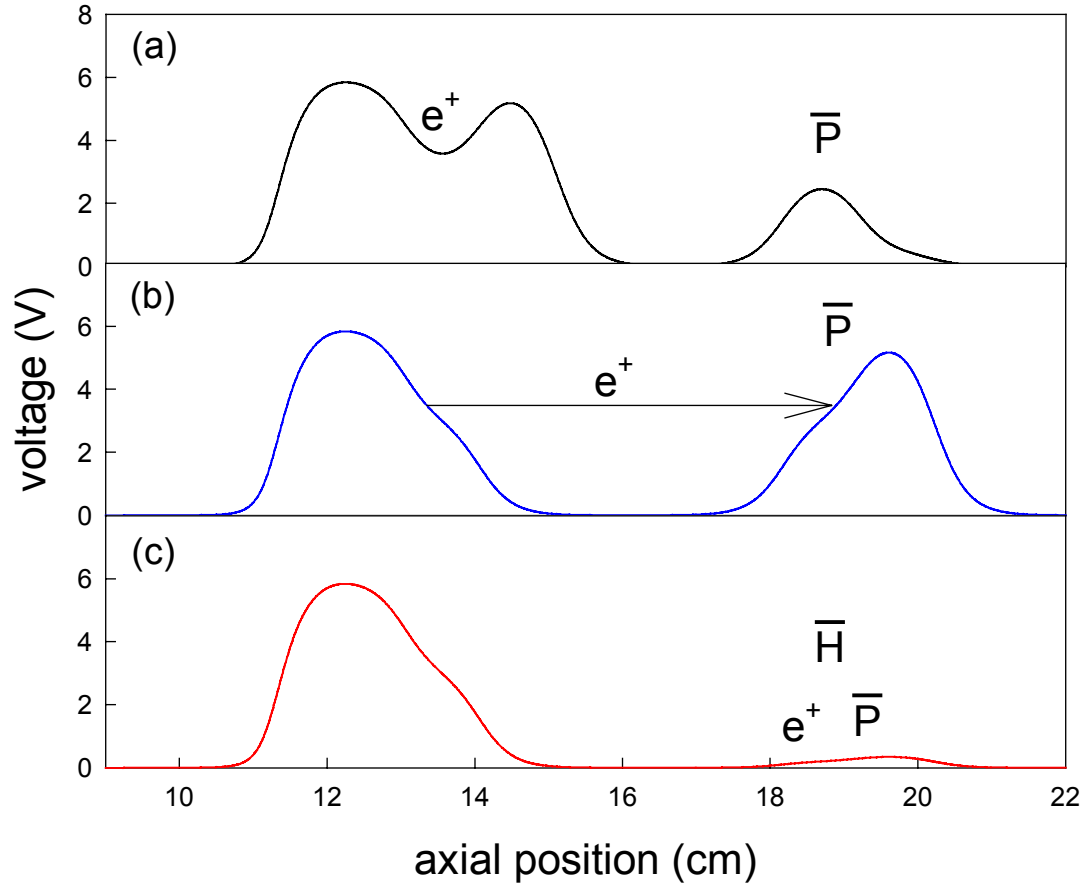


Figure 4.4: The three steps of the pulsed field recombination scheme we tried at ATRAP in the fall of 2000.

came to CERN to help us implement PFR at ATRAP. The first step of the experiment it to set up positrons, typically 5×10^5 , at one end of the trap and antiprotons, typically 10^5 , at the other (Fig 4.4a). Next we pulsed down the potential barrier so the positrons head towards the antiprotons. At the same moment the well next the antiprotons is pulsed up to provide the linear ramp which will tilt over the field of the proton, Fig 4.4(b). At the instant when the positrons have arrived at the antiprotons the antiproton well is pulsed down which should capture some positrons in the well of the antiproton, making antihydrogen.

Unfortunately we have not yet identified any antihydrogen events, probably for several reasons. Pulsed field recombination would make antihydrogen atom in a very high Rydberg state, with a principle quantum number around, $n = 200$. It is possible that any atoms traveling perpendicularly to the magnetic field, were reionized by the motional Stark field. When stripped the antiproton would be recaptured into the ion well and go undetected. It is also possible that antihydrogen atoms traveling parallel to the magnetic field of the apparatus would have been stripped when reaching the end of the trap by the electric field used to contain the unbound antiprotons.

But the most profound problem facing antimatter PFR experiments is the extremely long duty cycle for repeating the experiment. In the PFR experiment done with matter (rubidium atoms) the experiment was repeated 30 times second. If the experimenter, had to adjust the voltages and timing of the experiment for 20 seconds before seeing a signal, then he would have repeated the experiment 600 times before he got a result. At ATRAP it takes 1 hour collect the particles together to do an experiment. And we have time to do about 6 experiments per day. This means that 20 seconds of tuning with ordinary matter translates to 5 months with antimatter.

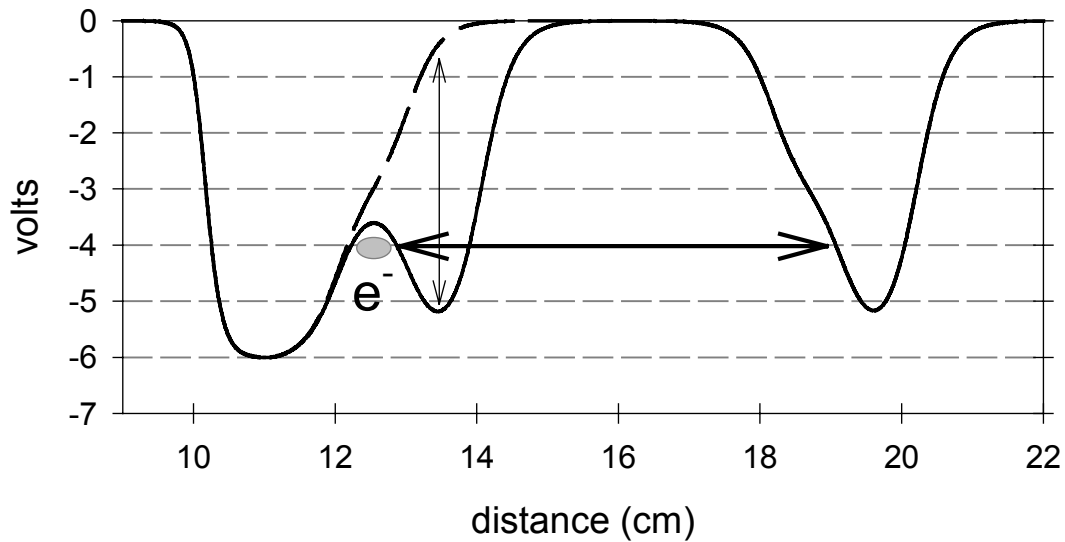


Figure 4.5: The potential well sequence for pulsing positrons.

To get PFR to work with antimatter one would have to know the apparatus settings exactly. This would require doing an accurate computer simulation or practicing with particles of matter inside a device similar to ATRAP. Both of these suggestions are complicated. Magnetohydrodynamic simulations are very challenging. While doing matter experiments faces two practical problems, first, how to get a large source of protons and second how to detect the hydrogen once it is made.

But even if the PFR experiments have not yet been successful they did bring a new capability to the ATRAP group. We are now able to pulse particles through the trap, a feat I thought was impossible until I saw it done. To propagate fast pulses on to the trap electrodes we have to remove the millisecond filters, which keep radio frequency noise out. So adding fast lines to the trap comes at the cost of more noise, which heats up the particle plasmas. Before starting the PFR experiments we did a proof of principle experiment where we threw an electron cloud against a potential barrier and caught it on its return.

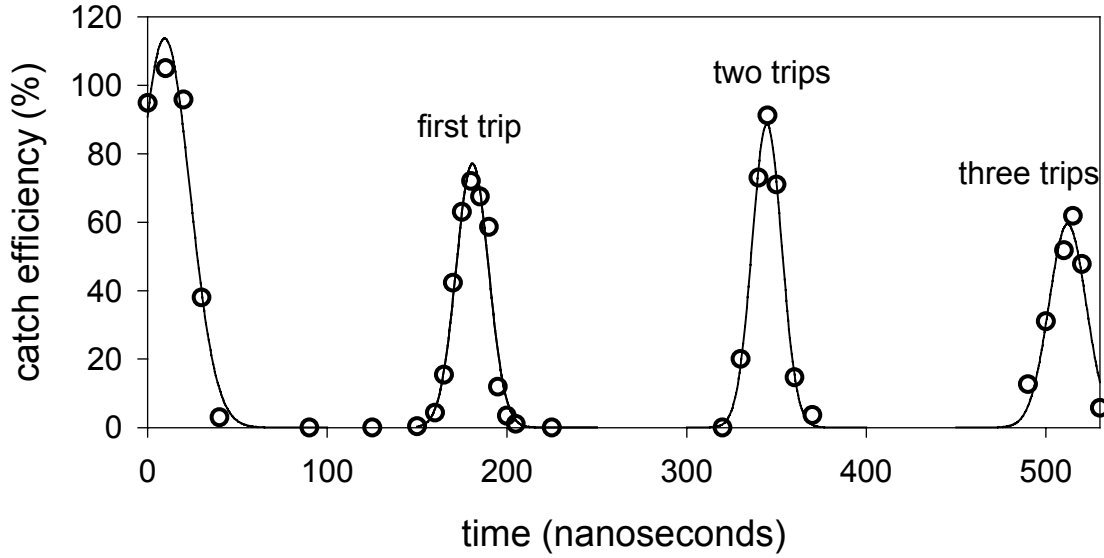


Figure 4.6: The peak sequence.

The potentials for this experiment are shown in Fig 4.5. First the electrons are loaded into a well. Then we pulse down the barrier and the electrons travel approximately 3.5 cm to a potential wall, turn around and upon their return the original potential barrier is raised again, catching the particles. In Fig 4.6 we plot the time we leave the well open vs. the number of electrons caught. As seen in the plot the electrons can make at least 3 round trips before being caught. This technique is now used to cleanse clouds of positrons of ions. Launching the positrons plasma and catching it in a new location assures that any heavy ions, which had accumulated in the plasma during the initial loading of the positrons, are left behind.

4.4 Proposed Initial Detection of Antihydrogen

To looking for antihydrogen production ATRAP has three layers of detectors surrounding the trap. Around the solenoid, which provides the magnetic field for the Penning trap, is a double layer of scintillator paddles arranged in a hexagon about 40 cm

away from the trap center, see Fig. 4.7. The paddles are made of standard scintillating plastic and cover much of the solid angle as seen from the center of the particle trap. Because of the several charged pions produced during an antiproton annihilation, these scintillators have an efficiency of 50% for detecting antiproton annihilations due to absorption of particles in the material of the magnet. The scintillators have a very low efficiency for detecting 511 keV gamma rays of the positron annihilations.

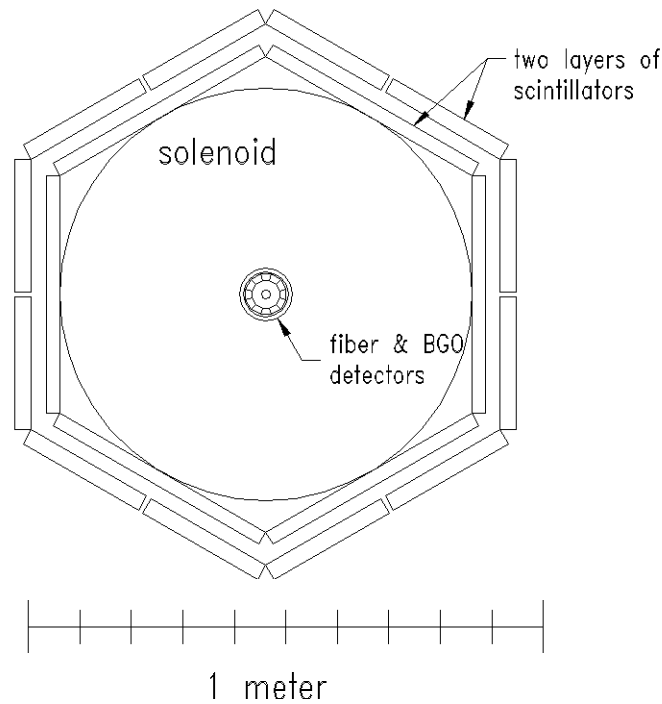


Figure 4.7: The overview of the annihilation detectors at ATRAP.

Closer to the trap are additional detector systems. Finding the volume for this detector inside the 4 inch bore of the solenoid was a Herculean task. The outer ring is a position sensitive barrel of fibers for detecting antiproton annihilations and the inner shell is made of BGO crystals for detecting the positron annihilations. Fig. 4.8 which shows a scaled view of these detectors viewed from above.

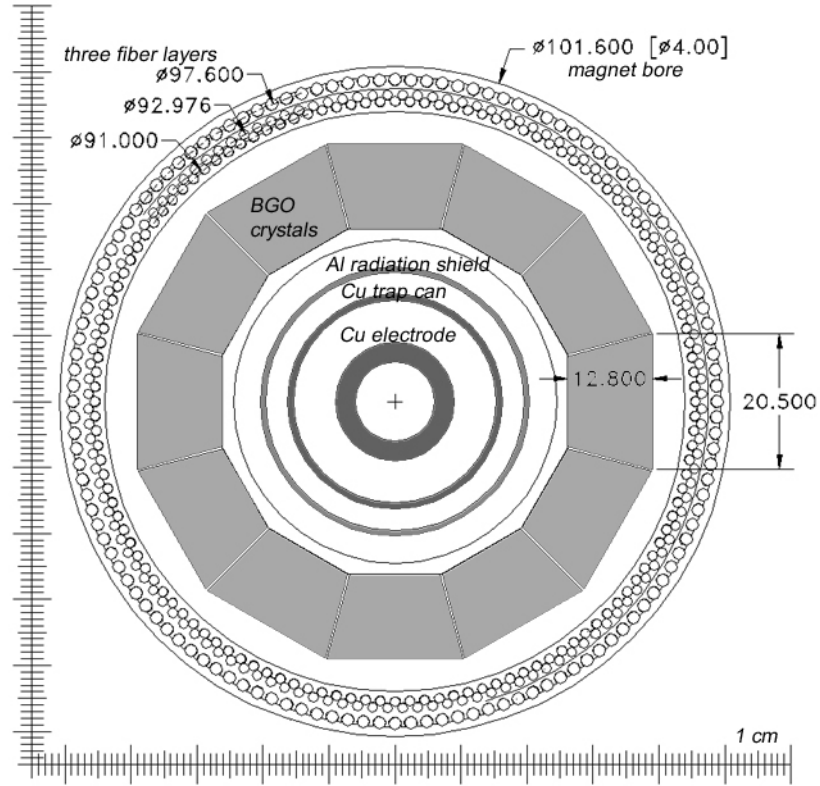


Figure 4.8: A horizontal cross section of the ATRAP experiment.

The positron detector consist of 12 BGO, $\text{Bi}_4\text{Ge}_3\text{O}_{12}$, crystals rods with trapezoidal cross sections are are 120mm long. BGO was selected for its high density $\rho=7.1 \text{ g/cm}^3$ [39] to maximize the probability that a gamma ray from a positron annihilation would be absorbed, given the very small volume we had for this detector. The crystals cover $\sim 92\%$ of the solid angle as seen from their geometric center. This barrel of crystals is centered near the ring electrode of the electron trap. The relative vertical location of the BGO detector changes by a few mm as the trap and detector cool to 4.2 K and 77 K respectively.

To estimate what the BGO crystals will record a GEANT simulation was run [40]. For 10000 511 keV photons, 40% are completely absorbed by the BGO crystals and show up in the 511 keV peak of the spectrum, while an additional 34% of the photons deposit energy between 150 keV and 511 keV. Since each positron annihilation gives out two 0.511

keV photons, counting all the events recorded in the BGO detector above 150 keV then we will see more than 73% of all positron annihilations.

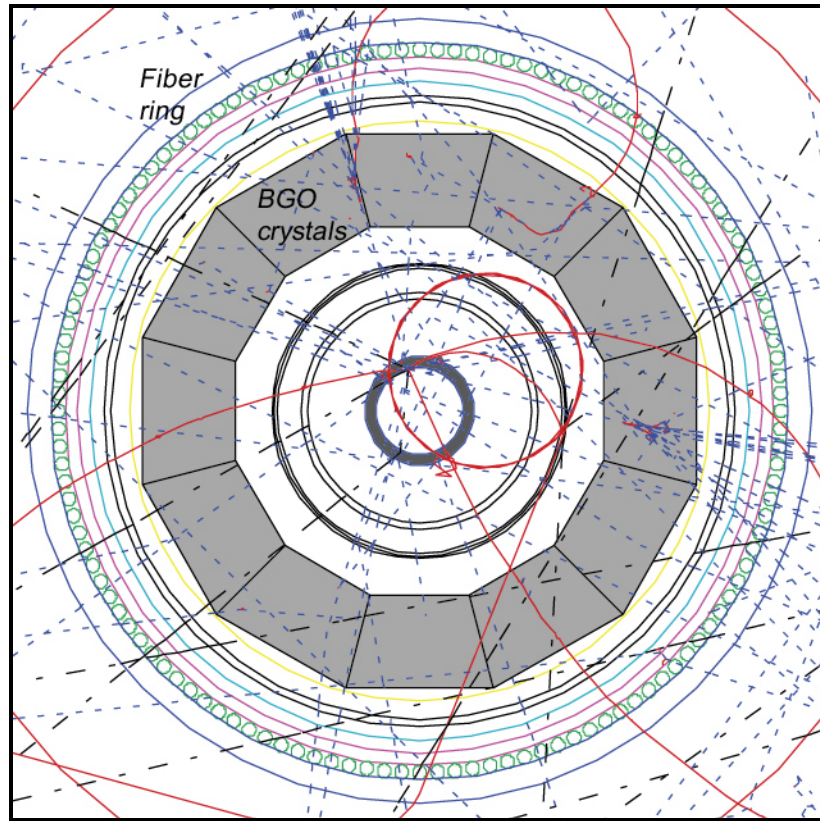


Figure 4.9: This figure is a simulation of one antihydrogen annihilation. Many particles are produced which greatly complicates the antihydrogen identification.

If we say that the fiber detector can count 100% of antiproton annihilations (which is not far from the truth) and the BGO detector can count more than 73% of all positron annihilations then we might expect to be able to positively identify at least 73% of all antihydrogen events. Unfortunately, *antiproton annihilations also produce positrons as secondary particles!* The high energy pions produce electrons and positrons when they transverse the metal of the experimental apparatus. And it is difficult to distinguish between positrons produced from the pions and positrons present in the original antihydrogen. So for a single event we have no way to distinguish between an antiproton or antihydrogen

annihilation. Moreover pions and secondary photons from the antiproton annihilation will also deposit energy in the BGO detector.

The fiber detector consists of 3 layers of scintillating fibers. Each layer has 128 fibers, which extend 150 mm along the axis of the detector. The two inner layers of 1.5 mm diameter fibers wind around in a gradual helix of 150° ; these two layers are offset to close the gaps between fibers (Fig. 4.8). The outer layer consists of 1.9 mm diameter fibers that are aligned vertically. The fiber helix gives the detector some position sensitivity. A particle transversing the detector can only hit a particular pair of fibers in a unique way (Fig 4.10). The fiber layers cover $\sim 86\%$ of the solid angle as seen from center of the trap. The absorption of 511 keV gamma rays is very small for the fibers. But they are very sensitive to charged pions from the antiproton annihilation.

All detector signals are coupled to discriminator modules resulting in fast (< 50 ns) logic NIM pulses which are used for trigger signal generation and counting. Any logic combination of the fired detector modules can be used to define an appropriate trigger for the data acquisition system. For a fast analysis within a time window of about 50 ns several detector signals are prepared. One signal is the "fiber signal" which says that at least two fibers in two different layers detected a hit. Every antiproton annihilation produces many particles (Fig. 4.9) so the fibers can detect 100% of the antiprotons annihilations (so long as each annihilation is separated by more than 50 nanoseconds). The background rate for the "fiber signal" is 60 counts/second. If we also require a simultaneous count in two of the scintillating paddles which surround the magnet then we get a background of 2.5 counts/sec. But the decrease in background comes with a sacrifice in signal. The scintillating paddles have an antiproton annihilation detection efficiency of about 50%. So requiring that they

register a count reduces the detection rate of antiprotons from 100% (with the fiber detector alone) to 50%.

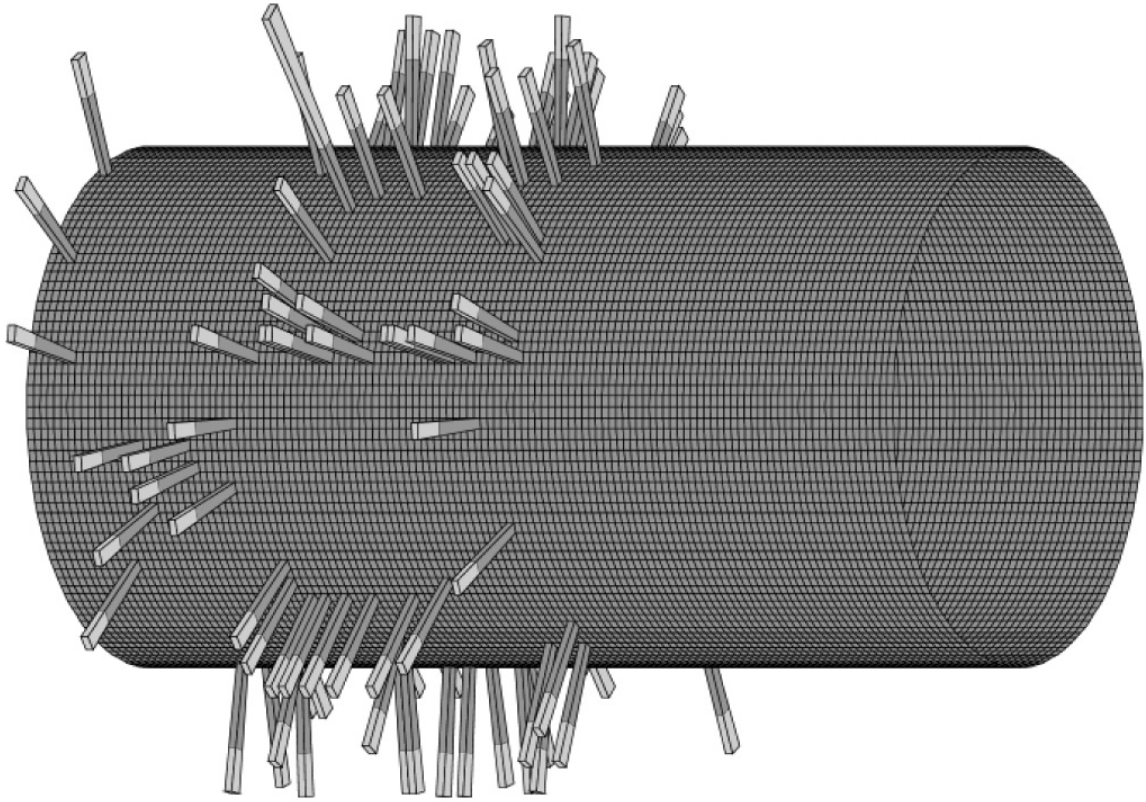


Figure 4.10: This is a plot of where the fiber detector was hit during a dump of antiprotons.

For each generated trigger signal the data acquisition system reads out and stores all the available detector signals the $3 \times 128 = 384$ fibers, the scintillators and all the BGO crystals. This processing takes about 1 millisecond and thus is only useful for observing annihilation rates below 1000 per second. These data are analyzed offline to extract useful informations like the hit position of a charged particles at the fibre tube (Fig. 4.10).

4.5 Using the Fiber Detector While Loading Antiprotons

The fast detection mode of the fiber detectors and scintillators are a very valuable real time resource. Figure 4.11 shows an example of the fiber detector display accessible

during the operation of the experiment. The program, referred to as ".xh", puts a real time chart of several information channels, selected by the user, such as the number of coincidence events in two fibers on the computer monitor. An annotated screen capture is shown in figure 4.11. This plot was chosen because it shows many of the different situations an experimenter would encounter during a run.

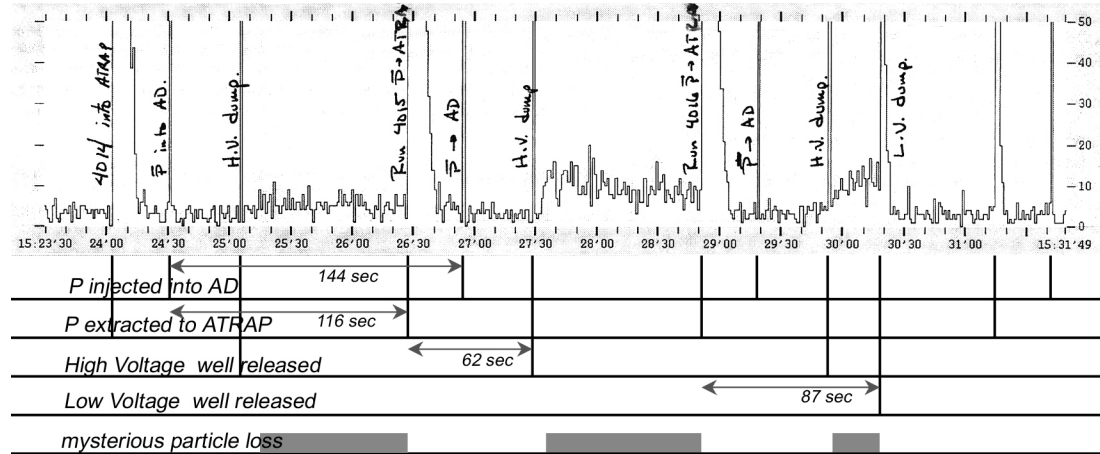


Figure 4.11: An annotated plot of the realtime output from the fiber detector.

Every 144 seconds anti-protons are injected into the AD decelerator. The number 144 depends on the cycle time of the Proton Synchrotron, the accelerator that feeds the AD. The antiprotons arrive at 3.56 GeV/c and are cooled down to 100 MeV/c via stochastic cooling and are then delivered to ATRAP, 116 seconds later⁴. This cycle of injection into the AD and extraction to the ATRAP continues during a beam run. The first peak on the plot is an extraction to ATRAP, the associated AD injection came before this peak and is not on the plot. The first three extractions to ATRAP loaded antiprotons. The pulse thickness indicates that antiprotons are escaping the trap over time. During the final extraction in the figure, the gate valve that isolates ATRAP from the AD was closed so no

⁴ This time has become shorter as CERN improved the AD.

particles were loaded. This is detected by the fibers due to the secondary particles produced when antiprotons collide with the gate valve (which is located about a meter away from the detector).

To load antiprotons into ATRAP we first accept the particles into a high voltage well approximately 4 kilovolts deep. Interior to the 4 kV trap is a 10 volt well containing electrons. The antiprotons lose their energy to the electrons and sink down in the trap [23]. After waiting a predetermined time, set by the experimenter, one end of the high voltage well is dropped and the antiprotons, which have not cooled into the electron plasma, are released. This process is called a high voltage dump. In Fig. 4.11 we have chosen to wait 62 seconds before dumping out the hot antiprotons.

After receiving a number of antiprotons shots (in this case 3) we can then dump the well containing the electron plasma and count the number of antiprotons which have accumulated there. We call this process the low voltage dump and it occurs 87 seconds after the third antiproton load in Fig. 4.11. There is nothing special about 87 seconds, we could have waited an hour.

In conclusion, Fig. 4.11 also contains some mysteries. In between the time when we let the high voltage well down and when the next antiprotons arrive we seem to have particles leaking out from our trap, this has been labeled mysterious loss. The character of this loss changes over time. It could be related to radio frequency heating of antiproton plasmas. Or maybe as antiprotons cool into the electron plasma some trajectories become unstable

Chapter 5

Trapping Antihydrogen

We would like to contain the atoms of antihydrogen we create long enough to do spectroscopic measurements. Neutral particles, such as atoms, can be held in magnetic gradient trap [41]. The most obvious technique for holding the antihydrogen atoms would be put a magnetic gradient trap around the Penning trap to capture any antihydrogen that is formed. But then one has to wonder if the large gradients will destroy the containment properties of the Penning trap. The charged particles in the Penning trap must remain confined long enough for neutral particles atoms to form. This chapter discusses many of the results that are in the paper that resulted from this research [42].

A spherical quadrupole particle trap might seem like the best choice for the neutral particle trap to superimpose on the Penning trap. This gradient trap is axially symmetric and hence would not disturb the axial symmetry of the Penning trap. But at the center of a spherical quadrupole trap magnetic field is zero. Aligning the center of the Penning trap with the center of the anti-helmholtz trap would require the Penning trap to operate with a zero magnetic field. This is clearly impossible. Adding a magnetic bias field just shifts the zero field (and the center of the neutral trap) to a new point. So our investigation centered on another common neutral particle trap, an Ioffe trap, which allows a nonzero bias field.

We approached the study of charged particles in a combined Penning Ioffe trap using four separate techniques. We use the guiding center approximation to form a qualitative image of the particle motion. Then we used a multiple scales series solution to

investigate where the particle orbit will go unstable do to resonate coupling between modes. The third method of analysis was adiabatic invariants which enabled us investigate what will happen when an Ioffe field is slowly added to a Penning trap. Finally, we used numerical computations to guide and check the analytical results. Our investigation was only for one particle. It remains to be seen how these results will extend to a dense plasma of particles.

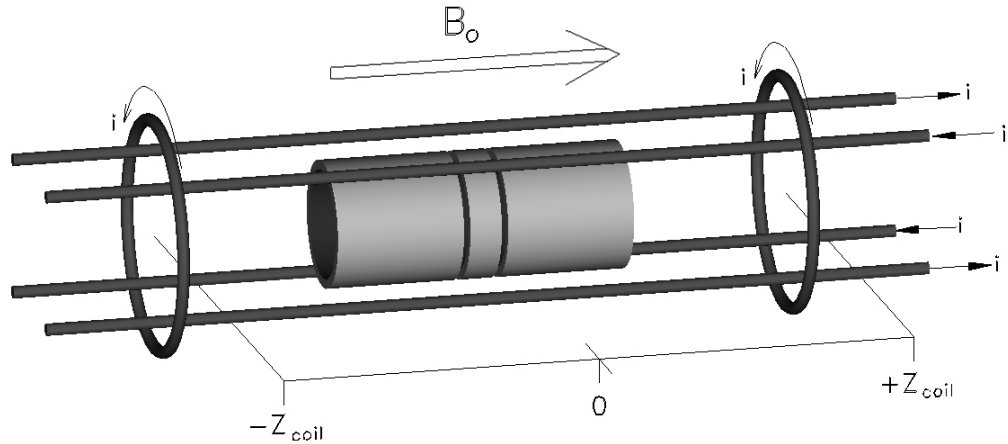


Figure 5.1: A open access Penning trap electrodes, with horizontal current bars and pinch coils of an Ioffe trap.

5.1 The Equations of Motion

A Ioffe trap consist of two coils with parallel current and four straight conductors with current in alternation directions (Fig. 5.1). To arrive at an approximation for the magnetic field I will following the development of Bergerman et al., [41]. Take four infinitely long, straight wires at $\rho = S$, $\phi_i = \pm\pi/4, \pm3\pi/4$ carrying current I and ϕ_i . Add together the magnetic field expression from each wire, then expand in a series about zero. The lowest order term is $\vec{B} = C_1(x\hat{x} - y\hat{y})$ where $C_1 = 2\mu I / \pi S^2$. We have decided not to include the magnetic field from the so called pinch coils of the Ioffe trap since in an experimental realization they could be placed far enough away from the Penning trap, so as

to not significantly effect the particle's dynamics. For the Penning trap we use the first order electrostatic potential

$$V = V_o \frac{z^2 - (x^2 + y^2)/2}{2d^2}, \quad (5)$$

and the magnetic field, $\vec{B} = B_o \hat{z}$.

The Lorentz force law gives the equations of motion for a particle in a Penning-Ioffe trap,

$$m\ddot{x} = \frac{qV_o}{2d^2}x - qB_o\dot{y} + qC_1y\dot{z}, \quad (6)$$

$$m\ddot{y} = \frac{qV_o}{2d^2}y + qB_o\dot{x} + qC_1x\dot{z}, \quad (7)$$

$$m\ddot{z} = -\frac{qV_o}{d^2}z - qC_1(x\dot{y} + y\dot{x}) \quad (8)$$

If you remove that last term on the right hand side of Eqs. 6-8 you will arrive at the equations for a particle inside a simple Penning trap. The solutions to the linear equations are simple sinusoidal orbits with the angular frequencies

$$\omega_{\pm} = \frac{b \pm \sqrt{b^2 - 4k}}{2}, \quad (9)$$

$$\omega_z = \sqrt{2k}.$$

where

$$k = qV_o / 2d^2m,$$

$$b = qB_o / m.$$

This simple orbit consist of three separate motions each on a dramatically different timescale (Fig. 5.2). The fastest time scale is that of the cyclotron orbit, ω_+ , which is a tight

circular motion in a plane perpendicular to the local magnetic field. The intermediate time scale is the axial motion, ω_z , which is the motion parallel to the magnetic field line. The slowest time scale is the magnetron orbit, ω_- , which is a “drift” motion that results from the E cross B force, $\vec{F} = \vec{E} \times \vec{B} / B^2$. The ExB force is the most dominate drift present, given the typical operating conditions in the Penning-Ioffe trap. These three separate motions, and time scales, are not particular to Penning traps but rather a characteristic of all charged particles traveling in electromagnetic fields. Another well studied example is charged particles trapped in the magnetic field of the earth. The general nature of these motions has resulted a set of analysis techniques, the guiding center approximation and the theory of adiabatic invariants, both of which we will use to investigate the Ioffe-Penning trap motion.

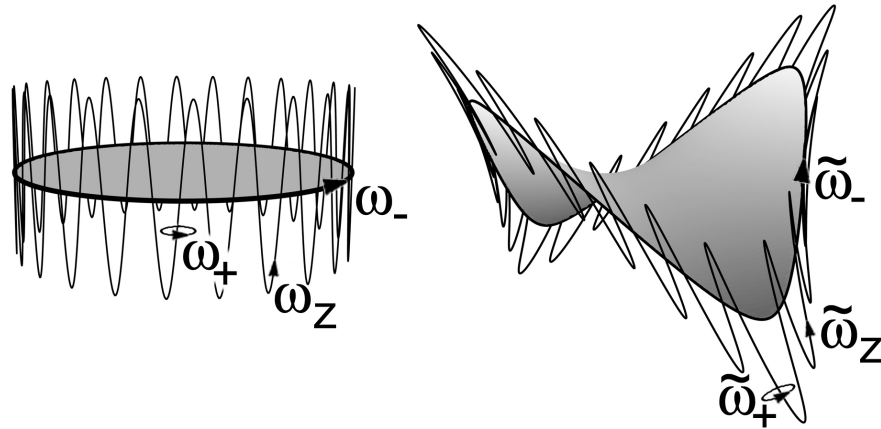


Figure 5.2: On the left is a the orbit for a charged particle in a Penning trap. The right shows an orbit in a Penning-Ioffe trap. The surface in each figure shows the surface which will be used to calculate the magnetic flux through the orbit.

The dramatically different time scales makes it difficult to solve these equations on a computer. For example, if we use the Eq. 9, for an electron in a 6 Tesla magnetic field ($V_0=10$ Volts, $d=.00335 \times 2$) has a cyclotron frequency of 1.06×10^{12} Hz, and a magnetron frequency of 18.6 kHz. The difference between the two frequencies is almost 8 orders of magnitude. If it takes 50 time steps to accurately track the cyclotron frequency then it would

require 2.8×10^9 steps to compute the 1 magnetron orbit. And hence to compute 1 second of the particle's dynamics would take 5.2×10^{13} time steps. So we must use the “guiding center” approximation for computations as well.

The notation of the Eqs. 6-8 can be simplified by introducing the definition

$$\varepsilon = \frac{\left(\frac{qV_o}{2md^2} \right)^{1/2}}{\left(\frac{qB_o}{m} \right)} = \frac{\omega_z}{\omega_+ \sqrt{2}}. \quad (10)$$

Both the analytical analysis and the numerical simulations are greatly simplified by removing all the dimensions from the equations. First set the period of the cyclotron orbit to 1 using $\tau = \omega_+ t$. This is very useful for the numerical simulations because then time step is always relative to 1, regardless of the actual physical parameters. Distance is scaled by $R_o = B_o / C_1$. Scaling by R_o has the effect of making all Ioffe/Penning trap configurations look the same. Furthermore R_o has the special property of being the exterior limit of stable traps (see below). Making these substitutions into Eqs. 6-8 brings us to the non-dimensional equations for the Penning-Ioffe trap.

$$\begin{aligned} \ddot{x} &= \varepsilon^2 x - \dot{y} + y\dot{z}, \\ \ddot{y} &= \varepsilon^2 y + \dot{x} + x\dot{z}, \\ \ddot{z} &= -2\varepsilon^2 z - (x\dot{y} + y\dot{x}). \end{aligned} \quad (11)$$

Notice that the dynamics of the charged particle in a Penning-Ioffe trap is characterized by a single parameter ε . In these scaled units the angular frequencies for the Penning trap are approximately,

$$\begin{aligned} \omega_+ &\approx 1 \\ \omega_z &\approx \varepsilon \sqrt{2} \\ \omega_- &\approx \varepsilon^2. \end{aligned}$$

5.2 Guiding Center Approximation

The guiding center approximation [43] averages out the cyclotron orbit and replaces it with a magnetic moment $\vec{M}$, located at the center of the original cyclotron orbit. This provides a tremendous computational savings because we can now do computations on the time scale of the axial motion. As we will see below the new equations do not look much different from the original equations and the difference is how the equations are interpreted.

In order to make the guiding center approximation the magnetic field near the particle cannot change appreciably over the time scale of one cyclotron orbit, so that

$$a \left| \frac{dB_j}{dx_k} \right| \left| B_j \right| \ll 1, \quad (12)$$

$$\frac{1}{\omega_+} \left| \frac{dB_j}{dt} \right| \left| B_j \right| \ll 1. \quad (13)$$

In words these equations say the change in each component of the magnetic field is small on the scale of the cyclotron radius a , and the change in the magnetic components during a cyclotron period, ω_+ , due to particle motion or time varying fields, are also small.

Assuming these two conditions are met the guiding center approximation proceeds with the following steps. First the cyclotron motion, $\vec{a}$, is explicitly separated from the slow motion of the particle, $\vec{C}$,

$$\vec{r}(t) = \vec{C}(t) + [\vec{a}_1 \cos(\omega_+ t) + \vec{a}_2 \sin(\omega_+ t)] = \vec{C} + \vec{a}.$$

Second the magnetic and electric fields are expanded in a series about the guiding center,

$$\vec{B} = \vec{B}_c + \sum_{n=1}^{\infty} \frac{1}{n!} (a \cdot \nabla)^n \vec{B}_c,$$

$$\vec{E} = \vec{E}_c + \sum_{n=1}^{\infty} \frac{1}{n!} (a \cdot \nabla)^n \vec{E}_c.$$

The third and difficult step is to substitute the last 3 equations into the Lorentz force law and then average over one cyclotron orbit, which replaces the cyclotron orbit with a magnetic moment $\vec{M} = m\omega_+/2B$. The new force law is,

$$m \frac{d\vec{u}}{dt} = q\vec{E}_c + q(\vec{u} \times \vec{B}_c) + \nabla(\vec{M} \cdot \vec{B}) \quad (14)$$

Here $\vec{u}$ is velocity of the virtual particle, $d\vec{C}/dt$. The subscript c on the two field quantities in Eq. 14 indicates the fields are to be evaluated at the center of the cyclotron orbit and not at the position of the original particle, $\vec{r}$.

Equation 14 looks a lot like Lorentz force law with a new term which accounts for the force felt on the virtual particle due to the interaction of it's magnetic moment, $\vec{M}$, with the gradient of the magnetic field. This, incidentally, is the same force, which is used to trap neutral (or even charged) particles in Ioffe traps. We will drop this term from our analysis because it leads to forces that are much smaller than those given by the other two terms and because the magnetic gradient forces only helps to keep the charged particle trapped. The Ioffe trap was originally conceived as a trap for charged particles undergoing thermal fusion [44] (unfortunately Ioffe's original paper is in Russian). Without the final $\nabla(\vec{M} \cdot \vec{B})$ equation 14 is exactly the Lorentz force law; the only difference is in how you apply it.

5.3 Applying the Guiding Center Approximation

Using the guiding center approximation we can do a very detailed analysis of the particles motion and frequencies. The guiding center approximation assumes that the particle is undergoing cyclotron motion, thus by assumption the cyclotron frequency is

$$\tilde{\omega}_+ \approx \sqrt{1 + x^2 + y^2}, \quad (15)$$

which is just the magnitude of the magnetic field. The other motions of the particle break into two categories, motions along the magnetic field lines and motions perpendicular to the magnetic field.

To find the motion along the field line take the scalar product of both sides of Eq. 14 with $\hat{B}$,

$$F_{\parallel} = \hat{B} \cdot m \frac{d\vec{u}}{dt} = q \hat{B} \cdot \vec{E}_c. \quad (16)$$

For our particular case,

$$F_{\parallel} = \frac{\epsilon^2 (x^2 - y^2 - 2z)}{|\vec{B}|} \quad (17)$$

Motion along the field lines is known as axial motion and is equivalent to the axial motion in a simple Penning trap. The force parallel, $F_{\parallel}$, is the component of the electric field along

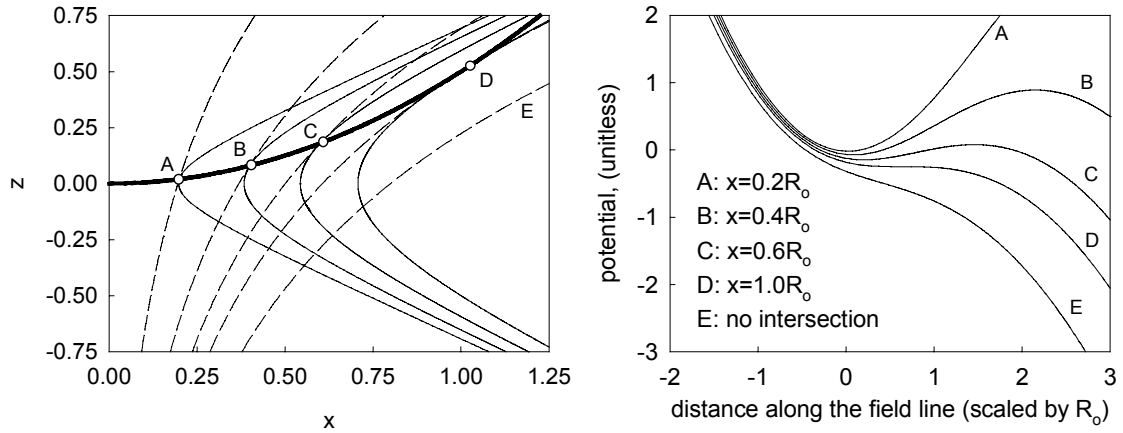


Figure 5.3: In the left figure the magnetron orbits (the dots) lie on the intersection of the force-free sheet (solid line) and the equipotentials of the electrostatic quadrupole (dotted line). Through each point is the magnetic field line (dashed line) on which the particle would travel during its axial orbit. The right figure shows the effective axial well depth along the magnetic field line. The well depth decreases with increasing magnetron radius.

the magnetic field lines. If we integrate along the field line we get the effective axial potential for the particle. Another way to get at the same result is to evaluate the electro-

static potential along a magnetic field line (Fig. 5.3) shows the axial wells for a selection of field lines.

If we set $F_{\parallel}=0$ then we get a sheet on which a particle would feel no axial force; we call this surface,

$$z = (x^2 - y^2)/2. \quad (18)$$

the force free sheet. The frequency of small axial oscillations about a point on the force free sheet is

$$\tilde{\omega}_z = \omega_z \frac{\sqrt{1-x^2-y^2}}{\sqrt{1+x^2+y^2}} \quad (19)$$

Motion perpendicular to the magnetic field lines is found by computing the cross

Velocity Name	expression	relevance
external force drift	$\vec{u}_F = \vec{F}_e \times \vec{B} / qB^2$	is zero because we have no external forces, (like gravity for example)
magnetic gradient drift	$\vec{u}_B = \left[\frac{M(1 + 2u_{\parallel}^2/W^2)}{qB^2} \right] \vec{B} \times \nabla B$	is zero because we've set M, the magnetic moment to zero
transverse inertia drift	$\vec{u}_m = \left(\frac{m}{qB^2} \right) \vec{B} \times \frac{d\vec{u}_{\perp}}{dt}$	becomes important for larger ϵ due to increase in $u_{\perp}$. See text.
electric drift	$\vec{u}_E = \vec{E} \times \vec{B} / B^2$	the dominate velocity in our problem
polarization drift	$\vec{u}_p = \left(\frac{m}{qB^4} \right) \vec{B} \times \left(\frac{\partial \vec{E}}{\partial t} \times \vec{B} \right)$	zero because $\partial \vec{E} / \partial t = 0$.

Table 5.1: A summary of the guiding center drift velocities. Part of this table are from ref. [43]

product of Eq. 14 with $\hat{B}$. Some algebra results in an expression for the transverse velocity, this has five terms, all of which have standard names and are listed in Table 5.1.

The two most important drift terms for the Penning-Ioffe trap are the electric drift and the transverse inertia drift.

We use these drift expressions to calculate the velocity of the slowest orbit (the magnetron motion). The transverse velocity has to be calculated iteratively, since its definition contains the total transverse velocity. To begin a numerical calculation of the particle's motion is necessary to know the transverse velocity very accurately. All the simulations were started in the xz plane, which orientates the transverse velocity in the y direction; in the dimensionless units the first two terms are

$$u_{\perp} = \dot{y} = -x\epsilon^2 - \frac{\epsilon^4 x}{(1+x^2)}. \quad (20)$$

The first term of Eq. 20 is the $\vec{E} \times \vec{B}$ drift and the second term is the first correction due to transverse inertia drift.

The magnetron orbit must lie in the force free sheet, Eq. 18, because this surface links together the equilibrium points of all the axial orbits. And the orbit must be on a surface of constant energy. In the guiding center picture the Hamiltonian for the Penning-Ioffe trap (neglecting the magnetic moment term) is

$$H = qV + \frac{m}{2}(u_{\perp}^2 + u_{\parallel}^2).$$

Here V is the familiar Penning trap electro-static potential, given in Eq. 5. Hence a surface of constant energy is a surface with constant V . Thus the magnetron orbit is the intersection of the force free sheet and a surface of constant electrostatic potential (Fig. 5.4). Fig. 5.5 shows the surface intersections projected in the XY plane. Near the center of the trap, $\mathbf{p}=0$ (the pure Penning trap limit) the orbits are circular. Their projections into the XY plane become more diamond like as $\mathbf{p}$ goes to 1.

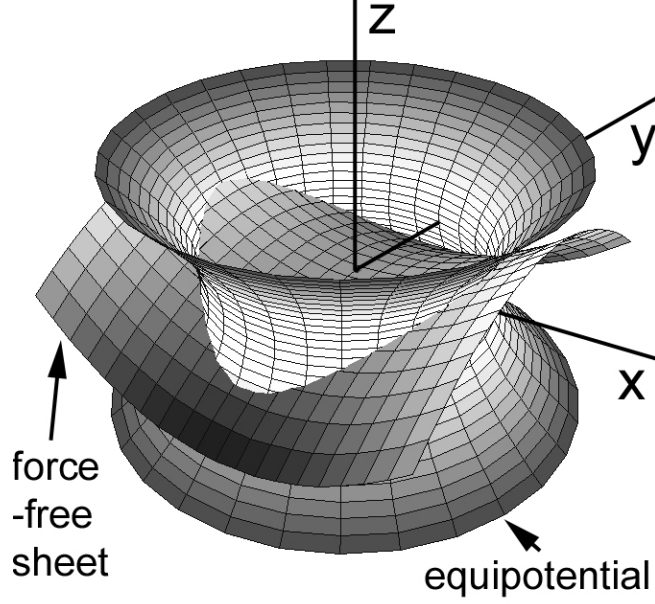


Figure 5.4: The magnetron motion is the intersection of the force-free sheet and an equipotential surface.

Something truly amazing can now be seen. Beyond $x=1$ there ain't no more stable magnetron orbits, because the force free sheet no longer intersects the electrostatic potential. All particles placed beyond $x=1$ will leave the trap. That this limit of just happens to be at $x=1$ (at $x=B_0/C_1$ in real units) surprised me.

Using the geometrical magnetron orbit found by surface intersections and the $\vec{E} \times \vec{B}$ velocity we can derive an approximate expression for the magnetron period,

$$\frac{1}{\tilde{\omega}_-} = \frac{4}{\pi \epsilon^2} \int_0^{\sqrt{2x_0^2 - x_0^4}/2} \frac{1 - 2v^2 / \sqrt{1 - 2x_0^2 + x_0^4 + 4v^2}}{\sqrt{1 + v^2} - \sqrt{1 - 2x_0^2 + x_0^4 + 4v^2}} dv.$$

This integral is easy to evaluated numerically. In Fig. 5.4 we have plotted the predictions of this expression against numerically calculated points.

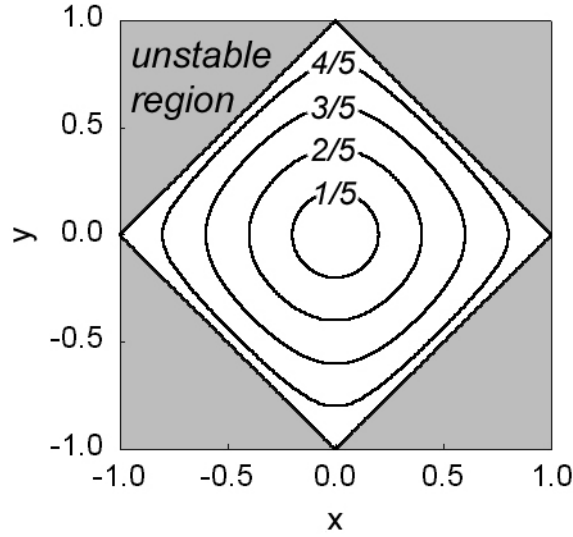


Figure 5.5: This plot shows the XY projection of the force-free sheet's intersection with the equipotential surface, for 5 equally spaced values of x .

In summary the guiding center gives a qualitative picture of the overall Penning-Ioffe trap motion. The orbits have three distinct time scales, which are analogous to the Penning trap. We are able to make predictions about the orbit's frequencies. With this knowledge we are able to classify a large range of behavior, which at first looks, surprisingly different. If Fig. 5.3 the axial energy is varied while the magnetron orbit remains the same. The figures on the right hand side show the XZ projections of the motion.

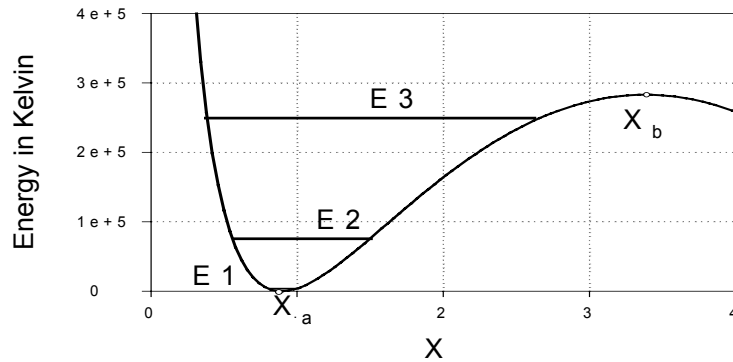


Figure 5.2: This figure shows an axial well along a field line in the XZ plane. Three energy levels are marked. These are the energies of the particle in figure 5.3.

Here we see that the motion of the particle is indeed on the field line. As the amount of axial energy is increased the particles travel further out along the field line, which gives the XY projection of the orbit, the left hand figure, a star shape.

For many reasons the predictions from the guiding center approximation become less accurate as the velocity of the magnetron orbit increases. Eq. 20 for $u_{\perp}$, loses its accuracy because as transverse inertia drift velocity grows. As ϵ increases the cyclotron frequency and magnetron frequency approach each other, so the conditions on the magnetic field (Eq. 13) no longer holds. In fact, cyclotron frequency and magnetron frequency become equal when $\epsilon = 1/2$. Thus we need another method of analyzing the Penning-Ioffe trap equations for large ϵ .

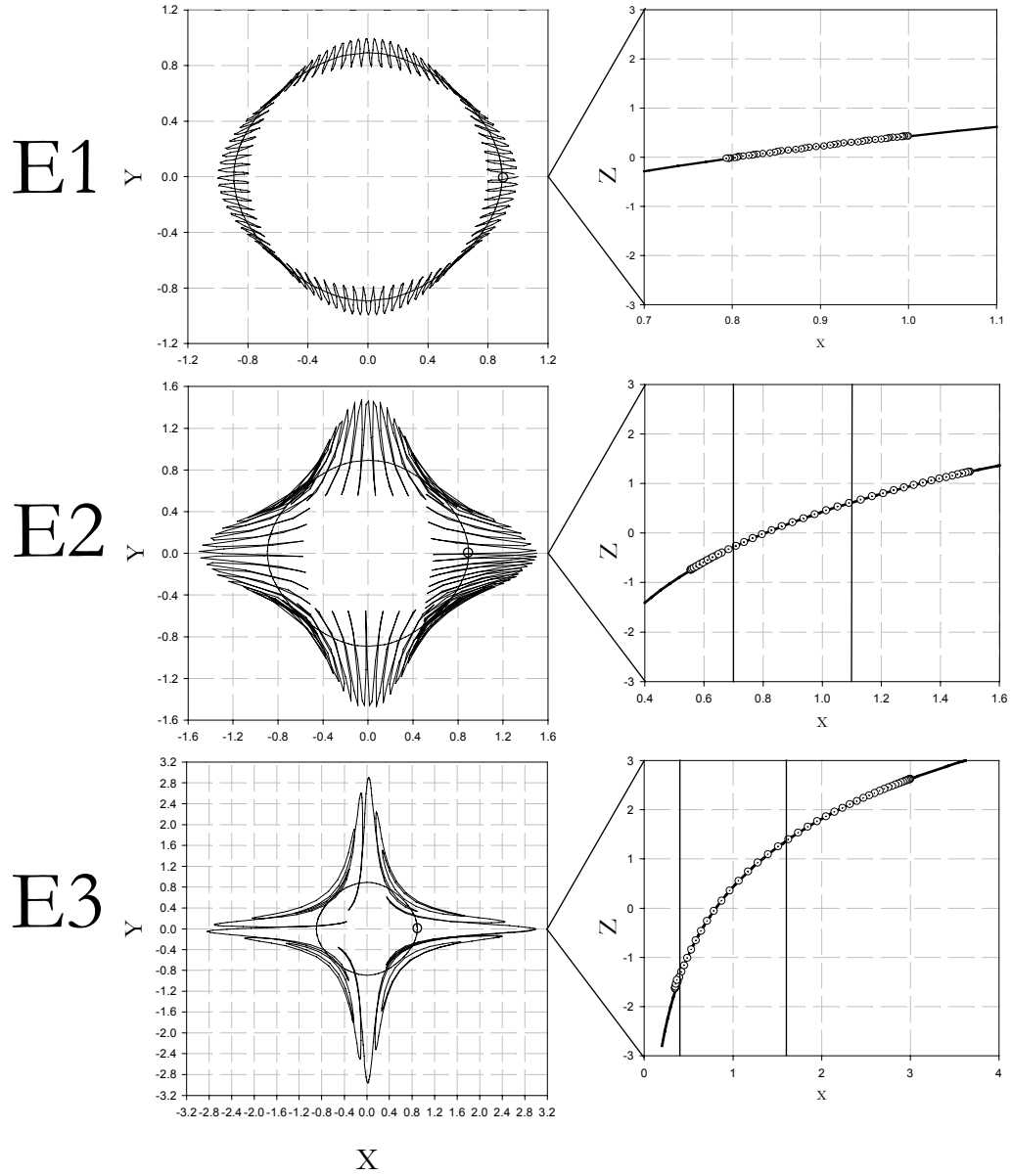


Figure 5.3: Putting together the assumptions of the guiding center approximation we can arrive at a complete picture of the particle dynamics. Here we show a particle with an increasing amount of axial energy $E_1 < E_2 < E_3$. Each trajectory has the same magnetron orbit. The XY projection of the orbit becomes star shaped as the axial energy is increased due to the larger distance traveled along the magnetic fieldlines. In the XZ plots we see the magnetic field line on which the particle was started. The vertical lines in XZ plots represent the space included in the preceding plot. Also see Fig. 5.2.

5.4 The Series Expansion

A well-known technique for solving differential equations is to assume the answer is in the form of a Taylor series. One plugs in the series to the differential equations and groups terms of like orders to solve for the coefficients of the series. For differential equations that have oscillatory behavior this technique often has to be extended because the equations for the coefficients have secular resonances - terms which are undefined because they have zero denominators. The method of solving these equations is called Multiple-Scale Analysis [45]. This technique begins by assuming that the coefficients of the series are not constant but rather slowly varying functions of time. In my literature search I was unable to find any examples of this method being applied to find solutions beyond 3rd order. The lack of published examples was no obstacle to Todd Squires who calculated a 7th order solution to the Ioffe-Penning trap. We checked his solutions by comparing the predictions to numerical calculations, as illustrated in Fig. 5.4 and Fig. 5.5.

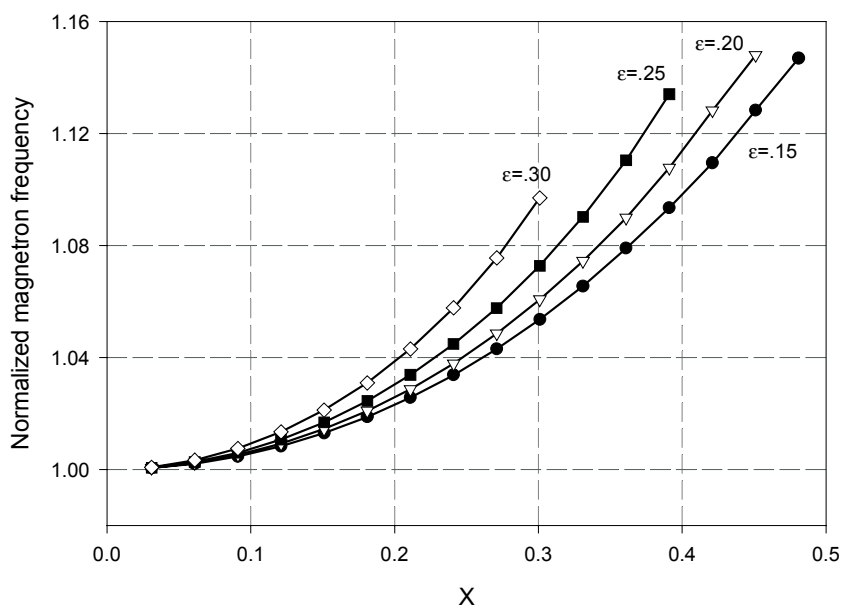


Figure 5.4: A plot of the magnetron frequency, as a function of x , for several values of ϵ . The points represent values calculated numerically.

The series solution begins by making the substitutions $u = x + iy$ and $v = x - iy$ into the equations of motion Eq. 11. This reduces the system to two variables instead of three,

$$\begin{aligned} \left(\frac{d^2}{dt^2} + i \frac{d}{dt} - \varepsilon^2 \right) u &= iv \frac{dz}{dt} \\ \left(\frac{d^2}{dt^2} + 2\varepsilon^2 \right) z &= \frac{i}{4} \frac{d}{dt} (u^2 - v^2). \end{aligned} \quad (21)$$

Notice that there is no “small” parameter explicitly written in the equations. We will expand in terms of the magnetron radius a . As a goes to zero the equations become more like the simple Penning trap. By solving each order in a exactly in ε we are assuming a cyclic orbit. One could rescale distances by a so the small parameter would appear explicitly in the equations of motion.

The first order solution to Eq. 21 is simply the pure Penning trap motion,

$$\begin{aligned} u_0 &= A e^{-i(\omega_- t + \beta_-)} + B e^{-i\omega_+ t} \\ z_0 &= C \cos \omega_z t \end{aligned}$$

where the solution angular frequencies are given by,

$$\omega_{\pm} = \frac{1 \pm \sqrt{1 - 4\varepsilon^2}}{2}$$

and $\omega_z = \sqrt{2\varepsilon}$.

The actual calculation of the series is quite complicated and so I will just present the results. If we define $f(\omega) \equiv \omega^2 + \omega + 2\varepsilon^2$, $g(\omega) \equiv 2\varepsilon^2 - \omega^2$ and $h(\omega) \equiv 1 - 2\omega$ then to 4th order in a the Penning-Ioffe trap frequencies are:

$$\begin{aligned} \tilde{\omega}_- &= \omega_- + \frac{\omega_-^2}{g(2\omega_-)h(\omega_-)} a^2 + \\ &\frac{\omega_-^3}{g(2\omega_-)^2 h(\omega_-)} \left[\frac{2}{h(\omega_-)} + \frac{\omega_-}{h(\omega_-)^2} - \frac{3\omega_-}{f(3\omega_-)} + \frac{8\omega_-^2}{g(2\omega_-)h(\omega_-)} \right] a^4 + \dots \end{aligned} \quad (22)$$

$$\begin{aligned}\tilde{\omega}_z = \omega_z + \frac{\omega_z}{4} \left[\frac{1}{f(\omega_- - \omega_z)} + \frac{1}{f(\omega_- + \omega_z)} \right] a^2 + \frac{\omega_z}{2} \left[1 + \frac{14\epsilon^2}{2} + \dots \right] a^4 \\ - \frac{\omega_z}{4} \left[1 + \frac{30\epsilon^2}{4} + \dots \right] a^4 \cos(4\tilde{\omega}_- t) + \dots,\end{aligned}\quad (23)$$

$$\begin{aligned}\tilde{\omega}_+ = \omega_+ + \frac{(\omega_+ + \omega_-)^2}{2g(\omega_+ + \omega_-)h(\omega_+)} a^2 - \frac{1}{8} [1 + 26\epsilon^2 + \dots] a^4 \\ + \frac{1}{8} [1 + 7\epsilon^2 + \dots] a^4 \cos(4\tilde{\omega}_m t) + \dots\end{aligned}\quad (24)$$

The coefficients for $\tilde{\omega}_z$ and $\tilde{\omega}_+$, of order a^4 , are given as expansions in ϵ . The frequency modulation in these terms arises from the magnetron motion, which takes the particle through different magnetic fields.

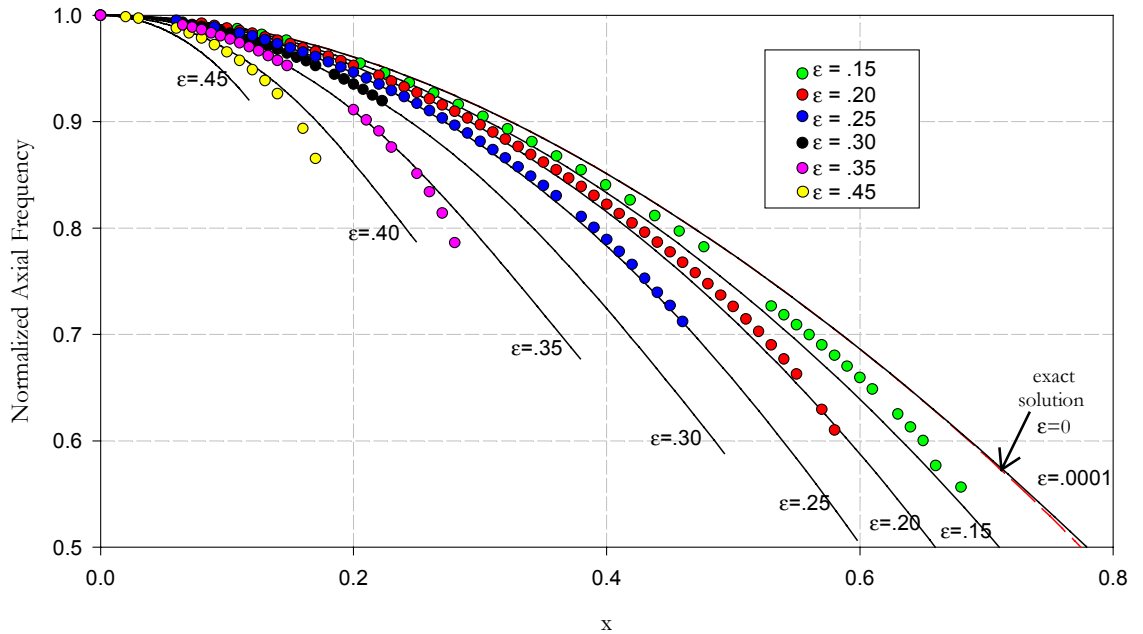


Figure 5.5: The solid lines are the axial frequency as calculated by the series. The dots are the numerically calculated values of the axial frequency. The dashed line is the exact solution for $\epsilon=0$, but it is difficult to

The orbits are Fourier series in harmonics of the eigenfrequencies. For pure magnetron motion and small ϵ ,

$$u_- = ae^{i\omega_- t} + \left[\frac{a^3}{8} + \frac{3a^5}{64} \right] e^{-3i\tilde{\omega}_- t} + \frac{a^5}{64} e^{5i\tilde{\omega}_- t}, \quad (25)$$

$$z_- = \left[\frac{a^2}{2} + \frac{a^4}{8} + \frac{3a^6}{64} \right] e^{2i\tilde{\omega}_- t} + \frac{3a^6}{128} e^{6i\tilde{\omega}_- t} \quad (26)$$

to 6th order in a . Substitution into Eqs. (5) and (18) explicitly confirms that this orbit lies on the electrostatic quadrupole and the force-free sheet. A small cyclotron oscillation adds

$$\begin{aligned} u_+ &= \beta e^{i\omega_+ t} - \frac{a^2 \beta}{4} e^{-i(\tilde{\omega}_+ + 2\tilde{\omega}_-)t} + \frac{a^4 \beta}{8} \left[e^{-i(\tilde{\omega}_+ + 2\tilde{\omega}_-)t} - \frac{1}{2} e^{-i(\tilde{\omega}_+ + 2\tilde{\omega}_-)t} \right], \\ z_+ &= -a\beta e^{i(\tilde{\omega}_+ + \tilde{\omega}_-)t} + \frac{a^3 \beta}{4} \left[e^{i(\tilde{\omega}_+ + \tilde{\omega}_-)t} - \frac{1}{2} e^{i(\tilde{\omega}_+ - 3\tilde{\omega}_-)t} \right], \\ \beta &= be^{-\frac{ia^4}{8} \cos 4\tilde{\omega}_- t}. \end{aligned}$$

A small axial oscillation adds

$$u_z = \frac{ac}{2} \left[e^{-i(\tilde{\omega}_- - \tilde{\omega}_z)t} + e^{-i(\tilde{\omega}_- + \tilde{\omega}_z)t} \right] + \frac{a^3 c}{16} \left[e^{-i(3\tilde{\omega}_- - \tilde{\omega}_z)t} + e^{-i(3\tilde{\omega}_- + \tilde{\omega}_z)t} \right], \quad (27)$$

$$z_z = ce^{i\tilde{\omega}_z t}. \quad (28)$$

Substitution in the leading terms for the adiabatic invariants M , J and Φ (see section 5.7) also verifies that these are invariants through order a^4 for small ϵ .

$$\begin{aligned} u_z &= \frac{ac}{2} \left[e^{-i(\tilde{\omega}_- - \tilde{\omega}_z)t} + e^{-i(\tilde{\omega}_- + \tilde{\omega}_z)t} \right] + \frac{a^3 c}{16} \left[e^{-i(3\tilde{\omega}_- - \tilde{\omega}_z)t} + e^{-i(3\tilde{\omega}_- + \tilde{\omega}_z)t} \right], \\ z_z &= ce^{i\tilde{\omega}_z t} \end{aligned}$$

During our investigation we see many examples of resonantly unstable orbits (Fig. 5.6). These trajectories become unstable because energy is passed from the magnetron mode to the axial mode. As energy leaves the magnetron orbit the radius of this orbit increases. Eventually, as shown in Fig. 5.6, the expanding orbit crosses the diamond

boundary of axial stability, figure 5.5, and then leaves the trap along a field line. It is interesting that the particle can make many orbits before its radius starts to expand. It seems as if the transfer of energy is exponential in nature; unnoticeable at first but then growing quickly and taking the particle out of the trap. The resonate transfer of energy happens when $\tilde{\omega}_z = 2\tilde{\omega}_-$, because then one magnetron orbit takes the particle up and down in the z direction twice (see Fig. 5.2) effectively “driving” the axial motion at angular frequency $2\tilde{\omega}_-$.

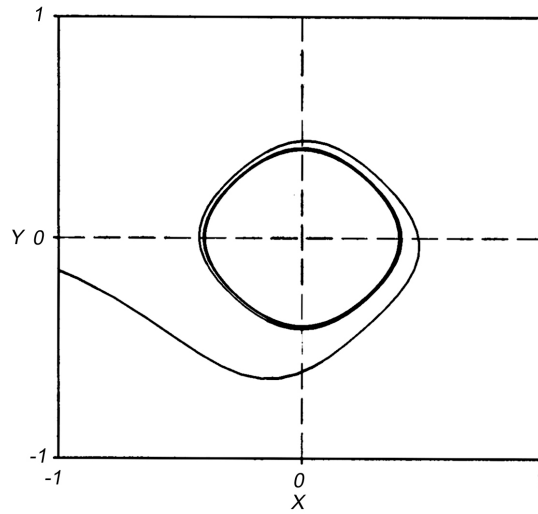


Figure 5.6: This is a plot of an resonatly unstable orbit in a Penning-Ioffe trap.

Fig. 5.7 shows the magnetron radius at which the destabilizing resonance occurs for a given ϵ . This plot is for small cyclotron and axial energies, as the axial energy grows the resonance boundary moves down to lower values of ϵ . The line derived from the series expansion is valid for orbits with a small radius, while the guiding-center approximation is valid for larger orbits and small ϵ . The dots are resonance orbits that were found numerically (see Sec. 5.6). As Fig. 5.7 indicates, the resonate transfer of energy does not happen at very small x , because near $x = 0$ the magnetron motion and axial motion

decouple. The magnetron orbit stays in the XY plane, like a pure Penning trap orbit, orthogonal to the axial motion.

There are secondary resonances due to magnetron orbits away from the trap center having Fourier components at $\tilde{\omega}_z = 2N\tilde{\omega}_-$, where N is an odd integer greater than 1. Higher order resonances also pass energy from the magnetron motion to the axial motion causing the magnetron radius to grow. But a larger magnetron radius changes the orbital frequency, pushing the particle out of resonance, then energy transfer stops and the orbit remains stable.

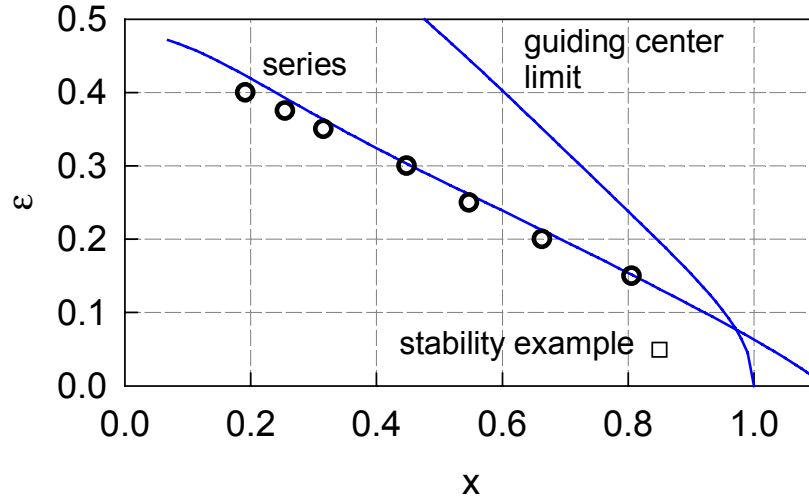


Figure 5.7: This plot shows the region of $\{x, \epsilon\}$ parameter space where the magnetron motion becomes resonate with the axial motion. The series line was calculated using the series solution valid for small x . The guiding center limit is valid for very small ϵ . The circular dots a resonate points which were found numerically. The square is the point in parameter space where we numerically integrated an orbit for a long period of time.

5.5 Numerical Calculations

All of the predictions from the series solution and the guiding center approximation were checked against numerical solutions. Equations 11 were integrated using a standard Runge-Kutta integrator from Numerical Recipes [46]. In general I did not use any adaptive

step size control since most calculations were for circular, periodic orbits in which the time scale of the dynamics does not change appreciably. So the additional computation involved in optimizing the step size only slows down the over all integration.

The challenge of simulating charged particles in magnetic fields is to avoid spending time calculating the cyclotron orbit. Thus we implement the guiding center approximation. Because the full equations and the guiding center equations are identical (when $\vec{M} = 0$) in practice implementing the guiding center approximation for a single particle comes down to picking the initial conditions so that there is no cyclotron energy. For plasma simulations implementing the guiding center approximation is more involved. To begin a numerical simulation one needs the value of six variables, $\{x, y, z, \dot{x}, \dot{y}, \dot{z}\}$. If we start in the XZ plane then we get three parameters for free, $\{y = 0, \dot{x} = 0, \dot{z} = 0\}$. If we pick x as a parameter of the simulation then only $\{z, \dot{y}\}$ have to be determined. For small ϵ one can use equation 20 or the predictions from the series solution, but for large ϵ this is not accurate and the calculated orbits will have some cyclotron energy, which means much longer integration times. So before the program begins the integration it has to do a 2 dimensional search of $\{z, \dot{y}\}$ parameter space to find the optimum starting conditions.

A lot of time was wasted trying to find the resonance instability boundary (Fig. 5.7) by trying to march up to its edge in parameter space. Our thinking was that we would start by picking a value of x then slowly turn up epsilon until the computed orbits were no longer stable. The transition between stable and unstable would happen gradually in the sense that as epsilon increased the orbits would be stable for less time. There was some evidence that this is true. But the problem with this approach is the optimum starting values of $\{z, \dot{y}\}$ become increasingly more difficult to find as ϵ approaches the stability boundary. This is

because region of parameter space around the optimum value of $\{z, \dot{y}\}$, which is stable, becomes smaller and smaller. After wasting a year refining algorithms to search increasingly smaller regions of parameter I abandon this approach for an extrapolation algorithm, which could find the instability points within minutes. Diagram 5.1 included at the end of this chapter shows how the extrapolation algorithm works.

5.6 Adiabatic Invariants

Adiabatic invariants are approximate constants of the particle motion. These quantities are defined in the same way as action angle variables [47]. The Penning-Ioffe trap has three adiabatic invariants; $\vec{M}$ the magnetic moment, $\vec{J}$ the longitudinal invariant and $\vec{\Phi}$ the flux invariant, see table 5.2. An adiabatic invariant quantity I changes on a time scale, which is exponential in ratio frequencies [48],

$$\frac{\Delta I}{I} \propto \exp(-\omega_z/\omega_-).$$

For reasonable values $\Delta I/I \sim 110$ minutes, in other words the flux through the magnetron orbit, Φ , may change on the order of 110 minutes. The expression for adiabatic invariant quantity, such as the magnetic moment, $\vec{M}$, is a series in which the familiar expression, $\vec{M} = mv_c^2/(2B)$ is only the first term.

Invariant	Associated velocity	Associated period	Approximation holds when
magnetic moment $\vec{M}$	cyclotron $\vec{W}$	$W\tau^2 \approx \text{constant}$	$\tau_c \ll \tau_f$
longitudinal invariant $\vec{J}$	axial $u_{\parallel}$	$\langle u_{\parallel}^2 \rangle \tau_{\parallel} \approx \text{constant}$	$\tau_c \ll \tau_{\parallel} \ll \tau_f$ $\vec{M} \approx \text{constant}$
flux invariant $\vec{\Phi}$	magnetron $u_{\perp}$	$\langle u_{\perp}^2 \rangle \tau_{\perp} \approx \text{constant}$	$\tau_c \ll \tau_{\parallel} \ll \tau_{\perp} \ll \tau_f$ $\vec{M} \approx \text{constant}$ $\vec{J} \approx \text{constant}$

Table 5.2: A summary of the adiabatic invariant quantities.

The flux invariant is useful for predicting what would happen if we have a particle in a simple Penning trap and then add the magnetic field of the Ioffe trap, which indeed is the first experiment we plan to do. The magnetic flux enclosed by the magnetron orbit is

$$\Phi = \iint \vec{B} \cdot \hat{n} \, d\sigma$$

where we take the force free sheet, Eq. 18, to be the surface of integration, see the shaded surfaces in figure 5.2. We define $\hat{n}$ to be the unit normal vector to the force free sheet,

$$\hat{n} = \frac{C_1(-xx + yy) + B_o z}{\sqrt{C_1^2 x^2 + C_1^2 y^2 + B_o^2}}.$$

Since

$$B \cdot n = B_o \frac{1 - \tilde{x}^2 - \tilde{y}^2}{\sqrt{1 + \tilde{x}^2 + \tilde{y}^2}}$$

and

$$d\sigma = \frac{B_o^2}{C_1^2} \sqrt{1 + \tilde{x}^2 + \tilde{y}^2} d\tilde{x} d\tilde{y}$$

the magnetic flux is given by

$$\Phi = \frac{B_o^3}{C_1^2} F(\tilde{x}_o),$$

where $F(\tilde{x}_o)$ is a dimensionless flux function given by

$$F(\tilde{x}_o) = \int_{-\tilde{x}_o}^{\tilde{x}_o} \int_{-y_m}^{y_m} (1 - \tilde{x}^2 - \tilde{y}^2) d\tilde{x} d\tilde{y}.$$

Here we use $\tilde{x}$ and $\tilde{y}$ to denote dimensionless variables as section 5.2. If we use the intersection of the force free sheet and the equipotential surface to define the magnetron orbit (section 5.4) then we can calculate the limits of integration,

$$y_m = \sqrt{1 + \tilde{x}^2} - \sqrt{(1 - \tilde{x}_o^2)^2 + 4\tilde{x}^2}.$$

Where $\tilde{x}_o$ is the maximum extent of the magnetron orbit. Now given the trap constants B_o and C_1 we have determined the magnetic flux enclosed by a magnetron orbit which crosses the YZ plane at $\tilde{x}_o$.

Let's say we start with a particle in a pure Penning trap; no Ioffe field. If $\tilde{x}_{o,i}$ is the radius of its initial orbit, then the flux enclosed by the initial orbit is

$$\Phi_i = \frac{B_o^3}{C_1^2} \pi \tilde{x}_{o,i}^2,$$

all distances are normalized by B_o/C_1 where C_1 is the final Ioffe field. Now we ramp up the Ioffe field to its final field C_1 .

To conserve the flux invariant the radius of the magnetron orbit will change to $\tilde{x}_{o,f}$, the flux through the new orbit is

$$\Phi_f = \frac{B_o^3}{C_1^2} F(\tilde{x}_{o,f}).$$

Due to the adiabatic invariance of the flux we can set the initial value equal to the final value, $\Phi_i = \Phi_f$. Figure 5.8 shows how all orbits will expand as the Ioffe field is turned on. If you know the radius of the particles orbit before turning on Ioffe field you can look up what the final radius will be by looking at the figure. Furthermore, we can determine the maximum magnetron radius of particles which will remain trapped after the Ioffe field is turned on. The dimensionless flux, $F(1)$ for the last stable orbit is $4/3$. So any particle whose flux, in the pure Penning trap, starts out greater than this are lost from the trap. This corresponds to an initial magnetron radius

$$\tilde{x}_{o,i,\max} = \sqrt{\frac{4}{3\pi}} \approx .65$$

For a trap whose bias field is $B_0=2$ Tesla and an Ioffe trap with gradient $C_1 = 150\text{T/m}$, the final trap size is $B_0/C_1=1.33$ cm. Therefore particles starting out more than about 0.87 cm from the center of the Penning trap are lost when the Ioffe field is ramped up.

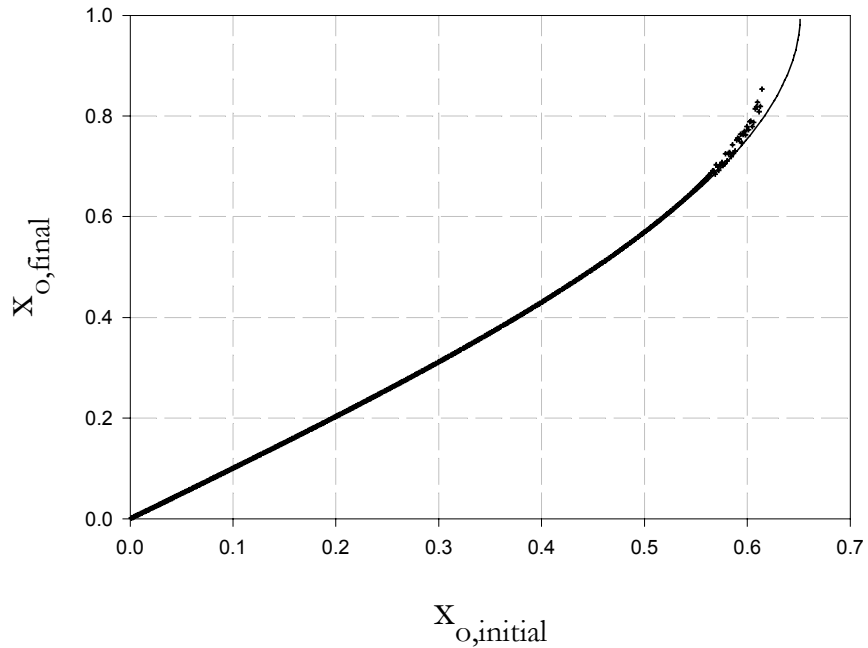


Figure 5.8: The solid curve shows the predicted magnetron radius expansion when a Ioffe field is added to a Penning trap. The small dots are values that were calculated numerically.

Figure 5.8 also includes the numerical test of flux invariant prediction. We integrated the equations of motion while changing the value of C_1 at a rate much slower than the magnetron period. As you can see the numerical results agree almost perfectly with the adiabatic theory. Towards $x_{o,f}$ it becomes difficult to calculate the orbit numerically because this is very close the edge of the stable trap and the small numerical errors cause the particle to leave the trap. To my surprise it was absolutely necessary to include the extremely small electric field caused by the changing Ioffe field (due to $\nabla \times \vec{E} = -\partial B/\partial t$). Sometimes it is just amazing how the theory of physics holds together.

5.7 Experimental Quantities

Nathaniel Bowden is assembling a Penning-Ioffe trap system, so we will soon be able to test the predictions of the Penning-Ioffe trap theory. The most obvious test is whether a particle will actually remain stable in a Penning-Ioffe trap for a long period of time. We could also measure the relationship between $\tilde{\omega}_z$ and $\tilde{\omega}_+$ and compare it with Eqs. 23 and (24) see figure 5.9. These measurements would determine a and with a one can know the magnetron radius using equation 25.

It should also be possible to measure the frequency modulation in the spectra of $\tilde{\omega}_z$ and $\tilde{\omega}_+$. The series solution for the axial motion gives rise to an equation for z motion

$$z = z_m + C \cos \left(\omega_z t - \frac{a^4 \sqrt{2}}{16\epsilon} \sin(4\omega_m t) \right), \quad (29)$$

this looks a bit different than equation (28) because I have included terms of order a^4 . I will start by dropping the z_m term, which is the z part of the magnetron motion, because this is

at lower frequencies that we are interested in. I will also set $C=1$ because this just scales the final results. For simplicity let's set, $B = \sqrt{2}a^4 / (16\epsilon)$ and $\phi = 4\omega_m t$. Then we expand the first equation using the trig identity,

$$\cos(\omega_z t) \cos(B \sin \phi) + \sin(\omega_z t) \sin(B \sin \phi)$$

We can combine the trig expansion with the following Bessel function identities [49],

$$\begin{aligned} \cos(B \sin \phi) &= J_0(B) + 2 \sum_{k=1}^{\infty} J_{2k}(B) \cos(2k\phi), \\ \sin(B \sin \phi) &= 2 \sum_{k=1}^{\infty} J_{2k-1}(B) \sin(2k-1)\phi, \end{aligned}$$

to get the amplitudes of the frequencies in the power spectrum. Hence the high frequency power spectrum of Eq. 29 has a peak at ω_z with the amplitude $|J_0(\omega_z)|$ and sidebands

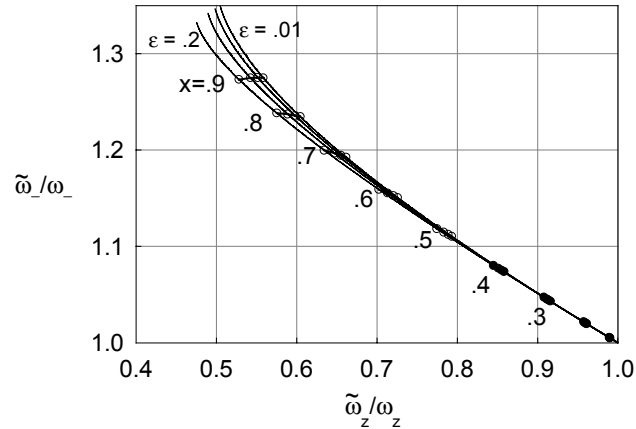


Figure 5.9: The predicted relationship of the axial and the magnetron frequencies could be tested experimentally and used to determine the magnetron radius at $y=0$.

spaced at intervals $\pm 4k\omega_m$ with the amplitude of each sideband given by $2|J_k(\sqrt{2}a^4/16\epsilon)|$.

Figure 5.10, shows what the axial signal of a proton would look like for various value of magnetron radius. The sidebands, which are spaced $4\omega_m$ away from the axial peak, enable

the experimenter to measure the magnetron frequency right from the spectrum analyzer (at least for particles with an appreciable magnetron radius). The calculations for Fig. 5.10 were done with Mathematica and Eq. 29 not by simulating the particle motion. The sidebands grow in number and size as the particle moves further out in the trap.

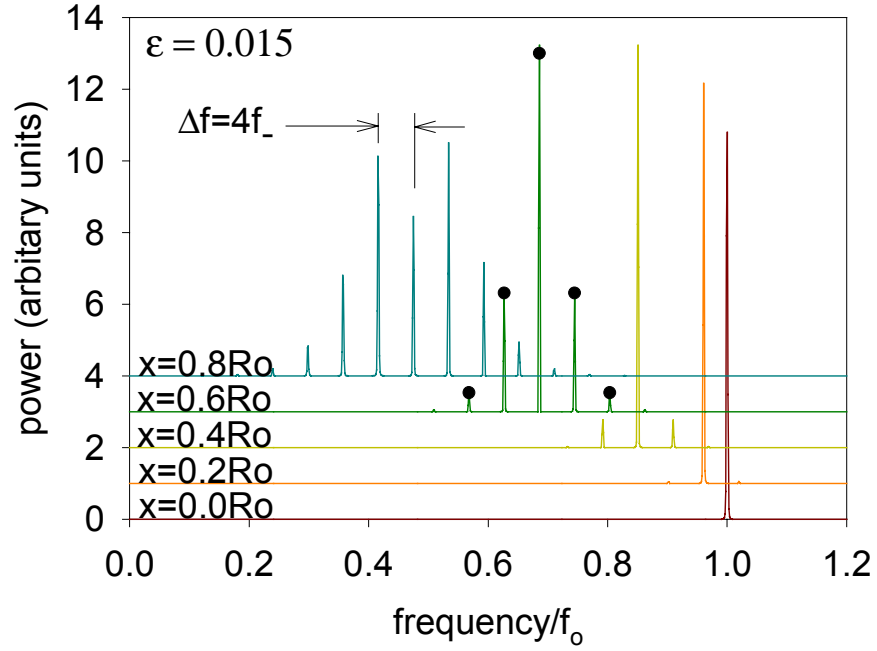
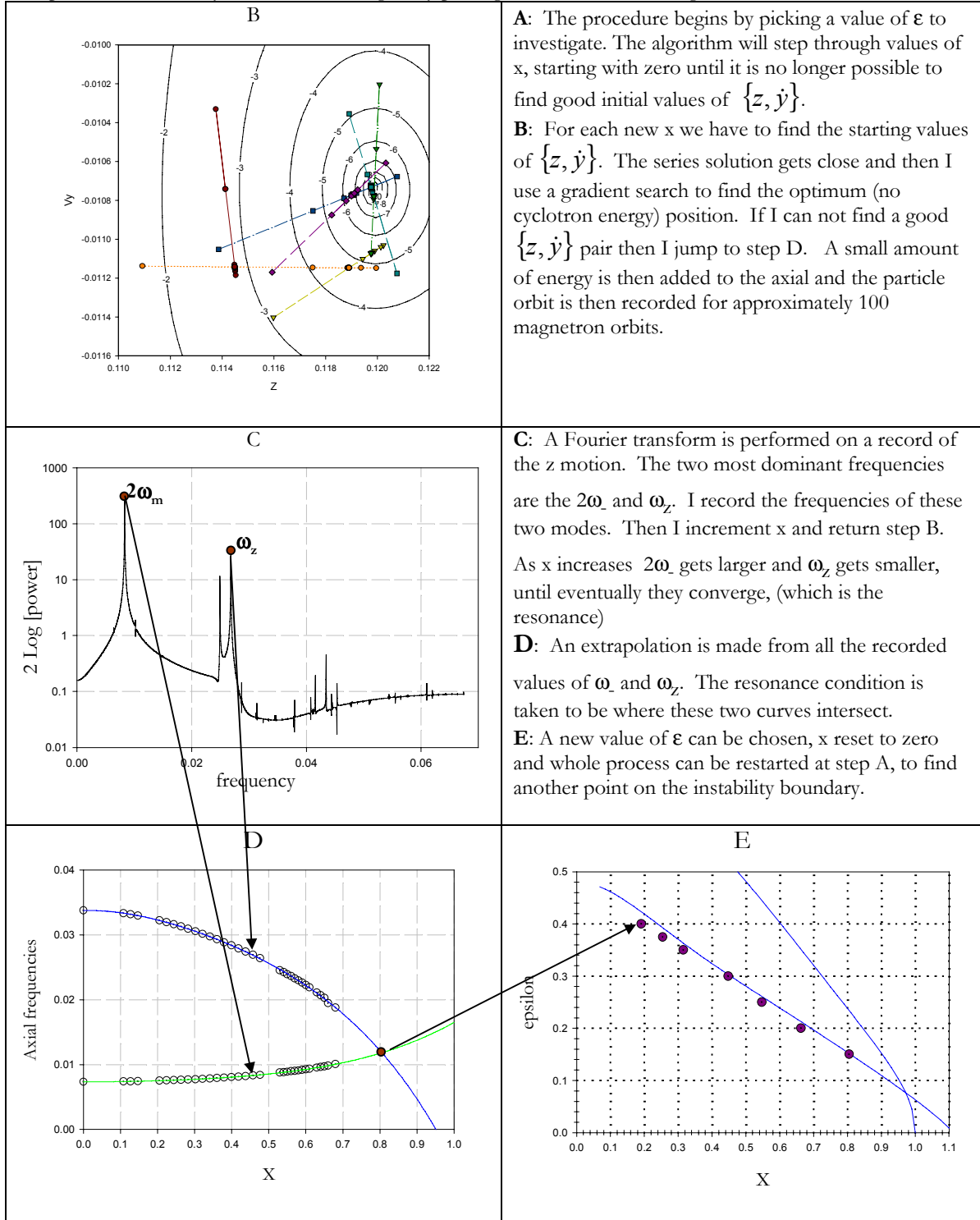


Figure 5.10: As the magnetron radius grows sidebands will appear on the axial frequency.

Diagram 5.1

Visual outline of an algorithm to find resonance instabilities

This diagram illustrates how to numerically find the border between the regions of phase space where the particle is resonantly unstable. One begins by picking a value of ϵ to investigate.



Chapter 6

Future Directions

The work of this thesis has brought us close to observing cold antihydrogen, the most immediate ATRAP goal. The production of antihydrogen will almost certainly take place in the apparatus built during my doctoral work. In fact, we may have already made antihydrogen, even though we have yet to observe it.

One possible improvement will be the incorporation of the BGO positron detector. This detector is now installed, but it is not yet clear how clean a positron annihilation signal it will give. Another promising addition to the experiment will be a CO₂ laser down the axis of the trap to enable us to try stimulated radiative recombination to the $n = 10$ level of antihydrogen. The predicted rates for stimulated radiative recombination are similar to three body recombination. But the laser would be a "controllable switch". The experiment could be done with the laser at slightly different frequencies, which should reveal an associated tuning in the number of antihydrogen atoms produced. A correlation between laser detuning and the rate of antiproton annihilations would be irrefutable proof of antihydrogen production.

I will give a brief outline of two possible future directions, which seem particularly important to me. By applying a time varying quadrupole field to particle plasmas we can compress its density. Also, during the run of 2001 we collected a very nice data set, which

raised many questions about the process of antiproton cooling in positron plasmas. I will discuss both of these projects below.

Finally, there is one project on the horizon, which I find particularly exciting, the experimental measurement of a particle's stability in a combined Penning-Trap. All the parts we need are converging at CERN. Professor Doyle has lent us an Ioffe trap. A large solenoid is soon to be delivered by American Magnetics, and Nathaniel Bowden has built a Penning trap to fit inside.

6.1 Simulating the Cooling of Antiprotons with Positrons

During the 2001 accelerator run ATRAP repeated the antiproton cooling experiments discussed in Ch. 4. These data have raised many questions. Antiprotons appear to cool to a level that is not easily explained by sympathetic cooling. The act of measuring the particle energy distribution of a trapped plasma modifies the distribution. I think it would be interesting to see if it is possible to recover numerically what the original energy distribution is.

It is computationally difficult to simulate the full system of particles contained in our trap (2×10^6 positrons together with 1×10^5 antiprotons). So simulations should center around computing the trajectory of a single particle, and then approximating the effect of the other particles on that single trajectory. The first step in setting up such a simulation would be to solve for electric and magnetic fields of the plasma containing Penning trap. An iterative algorithm for calculating the finite length thermal equilibrium of a pure electron plasma has been published by O'Neil [50].

The simulation could then estimate the effects of the plasmas on the simulated

particle by using a damping term [51] in the equations governing motion of the particle.

This term would only apply when the test particle was inside the positron plasma. Another approach is to simulate random collisions when the antiproton is traveling through the positron plasma.

6.2 Rotating Wall and Plasma Density

Another novelty is the electron trap compensation electrode, which is split into 6 parts. The purpose of this arrangement is to compress the electron plasma by applying a rotating electric field, a “rotating wall”. During the last decade the new technique of a rotating wall has been used by two groups [52] to compress an ion plasma to within 20% of the Brillouin density limit [53] and plasmas of electrons by a factor of 20 [54]. The Brillouin density, n_B , in Eq. 30 is the density at which the rotation of the charged plasma cancels out the effect of the externally applied magnetic field,

$$n_B = \frac{\epsilon_o m \Omega^2}{2q^2} \quad (30)$$

where $\Omega = qB/m$ is the cyclotron frequency. For a 6 Tesla field $n_B = 9.54 \times 10^{10}$ for protons and $n_B = 1.75 \times 10^{14}$ for electrons. Beyond this density it is not possible even in principle to store more particles because there is no magnetic field to confine them.

The rotating wall compresses the density by adding angular momentum to the plasma. Spinning the plasma faster reduces its radius. To develop this a bit further we make the requirement that

$$\left(\begin{array}{c} \text{length scale of} \\ \text{the plasma} \end{array} \right) \ll \left(\begin{array}{c} \text{total size of} \\ \text{the plasma} \end{array} \right) \ll \left(\begin{array}{c} \text{inner diameter of} \\ \text{the Penning trap} \end{array} \right)$$

The typical length scale of the plasma is the Debye length, λ_D , which is the distance over which a charge of a single particle is shielded by its neighbors, see Table 6.1 for realistic values [51],

$$\lambda_D = \left(\frac{\epsilon_o kT}{n_o q^2} \right)^{1/2} \quad (31)$$

A plasma in a Penning trap has a spheroid shape with uniform density, n_o . The axial length of the spheroid is $2z_o$ and its diameter is $2r_o$. The aspect ratio of the cloud is defined as $\alpha = z_o/r_o$. At the plasma edge the density drops from n_o to 0 over a distance on the order of λ_D . This is considered a sharp boundary since λ_D is much smaller than r_o . The spheroid rotates around the axis of the magnetic field (the z axis by convention) with the angular frequency ω_r , which is *not* the magnetron frequency ω_m . The plasma rotation frequency is related to the density of the plasma n_o and the cyclotron frequency $\Omega = qB/m$ by,

$$n_o = \frac{2\epsilon_o m \omega_r (\Omega - \omega_r)}{q^2}. \quad (32)$$

The rotational frequency ω_r is also related to the plasma frequency,

$$\omega_p^2 \equiv \frac{q^2 n_o}{\epsilon_o m} = 2\omega_r (\Omega - \omega_r). \quad (33)$$

The plasma frequency is a basic property of all plasmas, which characterizes the lowest order breathing mode of the plasma. Finally one can relate the aspect ratio of the spheroid to the

ratio of the axial frequency, which we measure routinely, ω_z and the plasma frequency ω_p

(when $\alpha < 1$)

$$\frac{\omega_z^2}{\omega_p^2} = \frac{1}{\alpha^2 - 1} \left(u_o \arctan\left(\frac{1}{u_o}\right) - 1 \right), \quad u_o \equiv \frac{\alpha}{(1 - \alpha^2)^{1/2}}. \quad (34)$$

Looking over equations 32-34 we can see that if we were able to determine ω_r we would be able to calculate two very important plasma parameters: the density n_o and the aspect ratio α . Knowing the density of our plasmas would enable us to calculate the antihydrogen recombination rates, and knowing α would enable us to determine how much of the antiproton plasma will overlap with the positron plasma. Although these two quantities are of fundamental importance for making antihydrogen, they remain largely undetermined in our experiments.

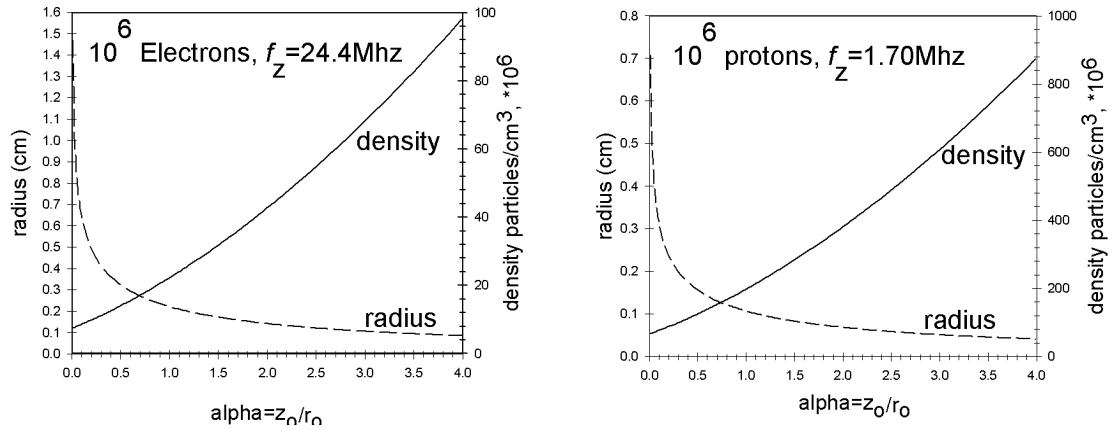


Figure 6.1: The radius and density of a plasma in a Penning trap plotted vs. the aspect ratio of the cloud

The vibration modes of the plasma spheroid were solved for in a paper by Dubin et al. [55]. The indices of the spherical harmonics (l,m) are used to label these modes. The lowest order modes (1,0) and (1,±1) correspond to the modes that we often measure. The

axial harmonic oscillation of the center of mass is the (0,1) mode. The cyclotron and magnetron motions are the (1, ± 1) modes. The (1,0) mode is easy to picture as a breathing along the z axis. The (2,0) mode of the plasma is the plasma frequency, ω_p .

	$\lambda_D(\text{cm})$	$n_o(\text{particles}/\text{cm}^3)$	$r(\text{cm})$	$\omega_p(\text{Mhz})$ rad/sec	$\omega_r(\text{Khz})$ rad/sec	$\Omega(\text{Mhz})$ rad/sec	$f_z(\text{MHz})$
electrons	.045	9.88×10^6	.494	177	16.6	94.5×10^6	24.4
protons	.015	48.0×10^3	.238	12.4	148	515	1.703

Table 6.1: Some values for 1,000,000 particle plasma in a Penning trap at 5.38Tesla, 4.2 Kelvin and $\alpha=0.2$.

By measuring higher order modes one can determine α and n_o . Wineland et.al [56] measured a number of the higher order modes electronically as peaks on the axial amplifier. From this they were able to track how the aspect ratio of the plasma changed in time. They found that side band cooling was able to compress the plasmas to $\alpha = 0.02$, which was the most compressed value they ever obtained. Unfortunately this work was done at a much smaller magnetic field of 0.11 Tesla. The measurements were more difficult at higher magnetic fields because the high order frequencies took longer to appear (15800 seconds after loading the cloud in a 1.4 Tesla field). Electrically measuring the frequencies of the higher order plasma modes ought to be possible in our traps, although this has yet to be done.

In Fig. 6.1 I have plotted n_o and the radius vs. α for both electrons and protons using 1 million particles and the typical ATRAP parameters. These plots give the size and density of our largest clouds. In Wineland [56] work the largest aspect ratio ever obtained with sideband cooling was $\alpha=0.02$. Our aspect ratios must be larger than 0.02 because if

they were not, a plasma of 1 million would hit the electrode wall. It seems reasonable to think our aspect ratios can be larger than 0.02 since our magnetic field is almost 60 times the field used to collect the data in Wineland [56]. Finally, to add some concrete numbers to this discussion, I have included Table 2, which has all the numerical values assuming $\alpha=0.2$.

Other techniques have been proposed to measure the radial distribution and density of the cloud. Several strips of metal attached to a charge integrating amplifier might work but at best would provide the minimum resolution. One possibility [22] is to use a position sensitive backing for a micro channel plate, but this method can only image the plasma one particle at a time, which would enforce a slow measurement process. The most promising technique although difficult to fit in place at CERN, would be to let the charged particles to hit a phosphor screen that was then imaged with a camera. While these proposals should all be relatively easy to implement they are all, unfortunately, destructive techniques.

6.3 Conclusion

This thesis demonstrated a new physical method for loading positrons, the only demonstrated way to efficiently load positrons into an ultrahigh vacuum, 4.2 K environment. The most intricate Penning trap the world has ever seen was built and now operates at CERN. We demonstrated the cooling of antiprotons in a positron plasma. A complete theory of a particle in the combined fields of a Penning-Ioffe trap was developed. This thesis has traveled a long way down the road towards antihydrogen but at the end it seems to hang like a symphony without its last note. The conclusion of the stability test and the demonstration of antihydrogen production, hopefully both soon to be realized, will serve as the final conclusion to this thesis work

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