A Precision Comparison of the $\bar{p} - p$ Charge-to-Mass Ratios

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Abstract

A new comparison of the antiproton-proton charge-to-mass ratios has been completed. The measured ratio of charge-to-mass ratios for the antiproton and proton is $1.000\ 000\ 001\ 5\pm 0.000\ 000\ 001\ 1$. Comparing the cyclotron frequencies of a single \bar{p} and p in a Penning trap improves upon the accuracy of earlier techniques by a factor of 45,000. This comparison is the most accurate mass spectroscopy of particles of opposite charge in a Penning trap and the most accurate test of the CPT theorem with baryons. Because of the high precision of the measurement, relativistic shifts in the cyclotron frequency provide a clean demonstration of the "relativistic" mass shift for typical cyclotron energies of 10-100 eV.

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Publications

- "Antiprotons, Positrons and Antihydrogen",
 G. Gabrielse, W. Jhe, D. Phillips, R Kaiser, H. Kalinowsky, J. Gröbner; Mat. Sci. Forum 105-110, 75 (1992).
- "Cylindrical Penning Traps and Self-Shielding Superconducting Solenoids for High Precision Experiments",
 W. Jhe, D. Phillips, L. Haarsma, J. Tan, G. Gabrielse; Physica Scripta 46, 264 (1992); RPS 20, 44 (1993).
- "Precision Mass Measurements of Antiprotons in a Penning Trap",
 W. Jhe, D. Phillips, L. Haarsma, J. Tan, G. Gabrielse; Physica Scripta 46, 268 (1992); RPS 20, 48 (1993).
- "Extremely Cold Antiprotons for Mass Measurements and Antihydrogen",
 G. Gabrielse, W. Jhe, D. Phillips, W. Quint, C. Tseng, L. Haarsma, K. Abdullah, J. Gröbner, H. Kalinowsky; in *Atomic Physics 13, Thirteenth International Conference on Atomic Physics*, (American Institute of Physics, New York, NY), 85 (1993).
- "Extremely Cold Antiprotons for Antihydrogen Production",
 G. Gabrielse W. Jhe, D. Phillips, W. Quint, C. Tseng, L. Haarsma, K. Abdullah, J. Gröbner and H. Kalinowsy; Hyper. Int. 76, 81 (1993).
- "Observing a Single Trapped Antiproton",
 G. Gabrielse, W. Jhe, D. Phillips, W. Quint, H. Kalinowsky, J. Gröbner; Nuclear Physics A 558, 701c (1993).
- 7. "A Single Trapped Antiproton and Antiprotons for Antihydrogen Production",

G. Gabrielse, W. Jhe, D. Phillips, W. Quint, L. Haarsma, K. Abdullah, H. Kalinowsky, J. Gröbner; Hyper. Int. 81 5 (1993).

- Special Relativity and the Single Antiproton: Forty-fold Improved Comparison of p
 p and *p* Charge-to-Mass Ratios",
 G. Gabrielse, D. Phillips, W. Quint, H. Kalinowsky, G. Rouleau, W. Jhe; Phys. Rev. Lett., 74, 3544 (1995).
- 9. "New Comparison of p̄ and p Charge-to-Mass Ratios,
 G. Gabrielse, D. Phillips, W. Quint, H. Kalinowsky, G. Rouleau, W. Jhe; in Proc. of LEAP '94 Conference (in press).
- 10. "Improved Comparison of p̄ and p Charge-to-Mass Ratios",
 D. Phillips, W. Quint, G. Gabrielse; Physicsa Scripta (in press).

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Chapter 1

Introduction

A new comparison of the antiproton and proton charge-to-mass ratios has recently been completed [1]. The cyclotron frequencies of single antiprotons and protons stored in a Penning trap are compared to a fractional precision of 1 part in 10⁹ (1 ppb). Because of this high precision, shifts in the cyclotron frequency due to the "relativistic mass shift" are observed for energies down to a few eV. These shifts provide a powerful tool for determining the motion of the particle. This comparison is the most accurate mass spectroscopy of particles of opposite sign and provides the most accurate test of the CPT theorem with baryons, a factor of 45,000 improvement over previous techniques.

CPT, a fundamental symmetry of quantum field theory, states that under the inversion of space and time coordinates along with charge conjugation, the physics of the theory remain invariant. Consequences of this symmetry include the equality of particle and antiparticle masses, lifetimes and charge and magnetic moment magnitudes. Few precision measurements have been performed to test CPT, however (Fig. 1.1). The neutral kaon system, with a fractional mass difference $(m_{K^0} - m_{\bar{K}^0})/m_{K^0}$ less than 10^{-18} [2], has been studied the most precisely. The electron–positron mag-

netic moments [3] and charge-to-mass ratios are equal to within 2×10^{-12} and 10^{-8} respectively. Prior to the work of our collaboration, however, no baryon antibaryon pair had been compared to a fractional precision better than 5×10^{-5} . The availability of antiprotons at the Low Energy Antiproton Ring (LEAR) and the sensitivity to detect a single antiproton in the precision environment of our Penning trap, create the opportunity for a greatly improved $\bar{p} - p$ measurement.

1.1 History

One of the great successes of Dirac theory was in interpreting the positron (observed in cosmic ray events as having the same mass and opposite charge as the electron), as the antiparticle of the electron. The large anomalous magnetic moment of the proton led some to question its status as a Dirac particle [4] and the existence of its antiparticle, the antiproton, would be important favorable evidence. Thus, the search for the \bar{p} was a key physics goal in the design of the Bevatron at Berkeley.

The antiproton was discovered at the Bevatron [4] by identifying its charge-tomass ratio as equal but of opposite sign to that of the proton. By measuring both its momentum (actually momentum over charge) in a bending magnet, and its velocity via a Čerenkov detector (whose threshold for producing photons depends only on a particle's velocity) the antiproton was recognized. By scanning the magnetic field, the antiproton's charge-to-mass ratio was ascertained to be the same as that of the proton to 5% (Fig. 1.2).

Increasingly accurate comparisons were performed using exotic atoms [5, 6, 7, 8] beginning fifteen years later. An exotic atom is formed when an antiproton is captured into an "atomic" orbital (having the same quantum numbers as an electron orbital but a much higher energy and smaller mean radius due to the much larger



Figure 1.1: Fractional uncertainties in tests of CPT through comparisons of mass, mean life and magnetic moment for particle – antiparticle pairs of leptons, mesons, bosons and baryons.



Figure 1.2: $\bar{p} - p$ comparisons of mass and charge-to-mass ratios over the 40 years since the discovery of the antiproton.

mass of the antiproton). "Atomic" transitions energies were measured from X-rays emitted as the \bar{p} cascaded towards its "electronic" ground state and annihilation with a proton from the nucleus. The energies of these states are given to first order by the Bohr formula

$$\epsilon_n = -\frac{1}{2} \frac{(Zq_p q_{\bar{p}}/\hbar c)^2}{n^2} \frac{M_{\bar{p}} M_N}{M_{\bar{p}} + M_N} c^2, \qquad (1.1)$$

where n is the principle quantum level and Z and M_N the nuclear charge and mass. The energy levels of this exotic atom are, hence, sensitive to the product $M_{\bar{p}} q_{\bar{p}}^2$ (after correcting for the reduced mass of the system). As greater precision from this technique was desired, more and more corrections including relativistic effects, radiative corrections, electron screening, finite nuclear size and nuclear recoil became necessary [8], making interpretation of experimental results more and more difficult. The final result determined by this technique was a fractional mass difference¹ of less than 5×10^{-5} [7].

The availability of low energy antiprotons (at the Low Energy Antiproton Ring (LEAR)), and the techniques of precision frequency measurements using Penning traps [9], made possible a much more precise comparison of the charge-to-mass ratios. Current best ion mass comparisons are roughly 10^{-10} [10]. (Note that these comparisons are for particles of the same sign of charge and that there are no published results at any comparable precision for particles of opposite signed charge.) Antiprotons were first trapped [11] and cooled from 6 MeV beam energies to thermal equilibrium with a 4 K environment (over 10 orders of magnitude in energy) [12, 13, 14]. This allowed an initial mass measurement (using hundreds of trapped antiprotons) which set a limit of 4×10^{-8} on any potential $\bar{p} - p$ charge-to-mass ratio difference [14]. By performing the measurement with a single \bar{p} in an improved environment, the measurement has now been improved by a factor of 40 [1]. In this thesis we show that the antiproton and proton charge-to-mass ratios differ by less than 1 ppb (1×10^{-9}) , the most precise test of CPT in a baryon system.

1.2 Discrete Symmetries and CPT

Studies of the symmetry properties lead to much progress in recent physics. Translation invariance (that the choice of origin should not affect the physics described by a theory) is such a symmetry. While in translation invariance, the origin can be

¹This analysis assumed the charges of \bar{p} and p were of equal magnitude and that the measurements could thus be interpreted as a mass measurement.

shifted to any value, there exist discrete symmetries with a finite set of transformations. Three important discrete symmetries [15] are parity $(\vec{x} \rightarrow -\vec{x})$, time reversal $(t \rightarrow -t)$, and charge conjugation (classically $q \rightarrow -q$, but in a field theory, particle \rightarrow antiparticle).

Classically, these symmetries are conserved. The equations of motion for a classical multiparticle system are

$$\frac{\mathrm{d}\mathbf{p}_{i}}{\mathrm{d}t} = \sum_{j} \mathbf{f}(\mathbf{r}_{ij}) \tag{1.2}$$

where $\mathbf{p_i}$ is the momentum of particle *i*, $\mathbf{r_{ij}}$ is the separation of particles *i* and *j* and $\mathbf{f}(\mathbf{r}_{ij})$ is the force between particles *i* and *j*. Eq. 1.2 is invariant under time reversal because momenta change signs under time reversal while the forces do not. Provided that the forces transform as a vector, $(\mathbf{f}(\mathbf{r}) = -\mathbf{f}(-\mathbf{r}))$, parity will also be conserved. Charge conjugation similarly leaves the system unchanged as the force is proportional to the product of the charges, q_iq_j and changing the signs of both charges leaves Eq. 1.2 unchanged. Thus the three discrete symmetries are independently conserved.

It was originally believed that quantum field theories would also be invariant under these three discrete symmetries independently. However, it was realized by Lee and Yang and in 1956 [16] that parity violation had not been experimentally tested in the weak interaction. Parity violation was quickly observed in β decay of Co⁵⁷. This parity violation arises because the weak interaction only describes interactions with neutrinos of left-handed helicity (in which the spin of the neutrino points in the direction opposite to that of its velocity). As parity transforms a left-handed neutrino into a right-handed neutrino, which is not observed, the weak interaction maximally violates parity. The charge conjugate particle of the neutrino, the antineutrino, however, has only right-handed helicity. Thus, while parity is violated by the existence of only one helicity of neutrino, the product of parity and charge conjugation (CP) is conserved.

After parity violation was discovered, CP was quickly proposed as a replacement "good symmetry" [17]. However, a small violation of CP symmetry was observed in decays of the K^0 by Cronin, Fitch and coworkers [18]. The eigenstates of the weak decay for the K^0 , K_S and K_L , are not eigenstates of CP. A mixing of a few parts in 1000 is observed as the K_S which usually decays to three pions which have odd CP, decays instead to two pions, a final state with even CP. This mixing of CP states in the weak eigenstates of the K^0 is thus a violation of CP.

In studying the K^0 system, the masses of the K^0 and \bar{K}^0 have been compared to a fractional precision better than 10^{-18} [19] and a test is being performed to check that there is a corresponding T violation to match the observed CP violation. Despite this confirmation of the CPT symmetry, its fundamental nature makes more probes of CPT desirable.

Despite the observation of P and CP violations, conservation of CPT is still expected on theoretical grounds. A proof of CPT invariance may be constructed along similar lines to classical theory. A typical proof of this theorem [20, 21, 22] consists of writing all possible terms in the Lagrangian describing the theory, and showing that none change in the process of a CPT inversion. More rigorous proofs of CPT are derived from an axiomatic formulation of quantum field theories [23]. Basic postulates including Lorentz invariance and unitarity are sufficient for a proof of the CPT theorem for general quantum field theories. Thus, a violation of the CPT symmetry would imply a breakdown in some of the fundamental assumptions leading to quantum field theories.

String theories, which may underlie quantum field theories at very small distance

scales may fail to satisfy these postulates. While these postulates appear to be reinstated on experimentally accessible scales, recent speculations suggest the possibility of very small CPT violations as low energy remnants of stringy physics. It has been suggested [24] that there may be quantum gravity driven mass differences in the K^0 system of order

$$\frac{m_{\bar{K}^0} - m_{K^0}}{m_{K^0}} = \frac{m_{K^0}}{M_{\text{PLANCK}}} \approx 10^{-19}$$
(1.3)

where M_{PLANCK} is the Planck mass of 10^{19} GeV. This fractional mass difference is one order of magnitude below current experimental limits for the kaon system.

1.3 Long Range Interactions

To perform a CPT inversion of a proton in a cyclotron orbit, not only must the proton be inverted to an antiproton, but the *mostly* matter universe must be inverted as well. As this rather difficult feat is not performed, long range interactions could shift the frequency of the \bar{p} relative to the p without a CPT violation. While strong constraints have been placed upon gravitation–like long range forces by "fifth force" experiments [25], direct tests of the weak equivalence principle (which demands that the inertial and gravitational masses of a particle are identical) have not been performed on antimatter.

As the cyclotron motion of a particle is a clock, it will undergo a gravitational redshift in a gravitational field. If a \bar{p} has an anomalous coupling to gravity αg (where the p has only g), then the fractional difference in cyclotron frequency will be given by [26]

$$\frac{\nu_c(\bar{p}) - \nu_c(p)}{\nu_c} = \frac{3(\alpha - 1)U}{mc^2}$$
(1.4)

where U is the gravitational energy. That this relation depends upon the total

potential and not on a relative difference is a direct effect of a violation of the weak equivalence principle, and not an artifact of the model used [26]. Taking the potential to vanish at infinity and only including the mass of the local supercluster the potential is estimated as $|U/mc^2| \approx 3 \times 10^{-5}$. The 1 ppb limit on any frequency difference between the \bar{p} and p charge-to-mass ratios combined with the assumption of no CPT violations, therefore, sets a limit on α of

$$|\alpha - 1| < 1 \times 10^{-5}. \tag{1.5}$$

Scalar models of gravity and massive Yukawa couplings which interact differently with matter and antimatter can be constrained as well.

While this analysis provides a very stringent limit on antimatter couplings to gravity, it is indirect and model dependent. A direct measurement of the gravitational acceleration of antimatter would be a much more direct study of such effects. A direct measurement of the gravitational acceleration on antiprotons has been proposed [27], but is very difficult due to the very weak nature of gravity compared to electromagnetic forces [28]. A test of the gravitational acceleration on electrically neutral antihydrogen would eliminate this difficulty, but awaits the production and confinement of antihydrogen [29].

1.4 Conclusions

The 1 ppb comparison of antiproton and proton charge-to-mass ratios provides the most accurate test of the CPT theorem (and anomalous long range forces) with baryons and the most accurate mass spectroscopy on particles of opposite sign. The Penning trap, with its well characterized potential, is an ideal environment for performing such a measurement. A single antiproton may be confined for extended periods and its cyclotron frequency measured with extremely high resolution.

Chapter 2

Particle Motions

An ideal Penning trap consists of an electric quadrupole field superimposed on a spatially homogeneous magnetic field (Fig. 2.1). A charged particle bound in such a trap has three independent, oscillatory degrees of freedom: the modified cyclotron motion in the plane perpendicular to the magnetic field, the axial motion parallel to the magnetic field and the magnetron motion also in the plane perpendicular to the magnetic field (Fig. 2.2). The three motions are harmonic with frequencies independent of the energies in the motions. This is a natural environment in which to perform an accurate measurement of the cyclotron frequencies of the antiproton and proton for a comparison of their charge-to-mass ratios. A detailed analysis of these motions, including the effects of many perturbations and imperfections, may be found in [9]. A summary of the motions and the geometry of electrodes necessary to produce the potentials and admit antiprotons will be presented here.



Figure 2.1: The ring and two endcap electrodes of hyperbolic traps lie on equipotential lines of a quadrupole potential and generate the field lines shown. The magnetic field is directed vertically in this picture and the trap is rotationally symmetric about the central vertical axis.

2.1 Particle Motions and the Invariance Theorem

A particle of mass M and charge e in a magnetic field **B** oscillates in a circular cyclotron motion in a plane perpendicular to the magnetic field (Fig. 2.2) with a frequency,

$$\nu_c = \frac{|e\mathbf{B}|}{2\pi M c},\tag{2.1}$$

of approximately 89 MHz for an antiproton in a 5.85 Tesla field. As this frequency is proportional to the charge-to-mass ratio, measuring it for both the antiproton and proton in the same **B**, allows their charge-to-mass ratios to be compared. The magnetic field also confines a particle along field lines. In the direction parallel to the field (\hat{z}), confinement is provided by the electrostatic field (Fig. 2.1) which leads to harmonic axial oscillations with a frequency ν_z , independent of the amplitude of the motion in an ideal quadrupole potential. The electric field slightly reduces the cyclotron frequency, however, to the modified cyclotron frequency ν'_c . The third degree of freedom, the magnetron motion, is a motion through crossed electric and magnetic fields for which the Lorentz force on the particle vanishes. Like a velocity filter, this motion is independent of the charge-to-mass ratio with the magnetron frequency, ν_m , determined by the electric and magnetic field strengths.

The three measured frequencies ν'_c , ν_z and ν_m are related to the free space cyclotron frequency ν_c by an invariance theorem [30]

$$(\nu_c)^2 = (\nu'_c)^2 + (\nu_z)^2 + (\nu_m)^2.$$
(2.2)

This relation includes effects due to offset angles between the trap axis of symmetry and the **B** field direction and the quadratic imperfections in the electrostatic properties of the trap. Because $\nu'_c \approx 89$ MHz is much larger than $\nu_z \approx 1$ MHz which is

| $\nu_{c}' = 89.3 \text{ MHz}$ | B = 5.9 Tesla |
|-------------------------------|------------------|
| $\nu_z = 954 \text{ kHz}$ | $V_0 = 18$ Volts |
| $\nu_m = 5 \text{ kHz}$ | |

Table 2.1: The frequencies and voltages of antiprotons in the trap

much larger than $\nu_m \approx 5$ kHz, ν_z and ν_m need not be measured as well as ν'_c for a desired accuracy in ν_c . To measure ν_c to 1 ppb (or 0.1 Hz out of 89 MHz), ν_z must be measured to 10 ppm (10 Hz out of 1 MHz), and ν_m to only 20% (1 kHz out of 5 kHz).

Other shifts are not, however, accounted for in the invariance theorem. Relativity affects ν'_c through the "relativistic mass" shift. We shall see in chapter 4 that this extremely clean systematic effect can be used to great advantage to count the number of antiprotons as well as to determine the energy in the cyclotron motion, but must be accounted for in comparing cyclotron frequencies. To measure ν_z of a single particle to sufficient accuracy (Ch. 5) requires a high sensitivity detector and well controlled trap potential, sufficiently close to an electrostatic quadrupole so that ν_z does not shift when the motion is excited to sufficient amplitude to measure it. The very low frequency magnetron motion is of such low velocity that it is not easily measured by direct means (as with ν'_c and ν_z) but also small enough that it need only be known to 20% (Ch. 6).

2.2 Open-Access Traps

The potential in a Penning trap (Fig. 2.1) is produced by electrodes surrounding the trapping region. The beam size of antiprotons delivered from the CERN antiproton complex (Ch. 7) is several mm^2 requiring a large entrance into this region. Our



Figure 2.2: The three independent motions in an ideal Penning trap: the axial oscillation parallel to the magnetic field, the slow $\mathbf{E} \times \mathbf{B}$ magnetron drift in the plane perpendicular to the field and the fast cyclotron motion (shown as a small circle) in the same plane.

trap is, therefore, composed of stacked cylindrical rings (Fig. 2.3). A careful choice of electrode lengths [31] and tuning of applied voltages produces the high quality electrostatic quadrupole needed to produce harmonic motions, with frequencies independent of excitation energy and provides the access needed to admit antiprotons before cooling. This open-access trap is in marked contrast to the electrodes of more traditional traps which follow equipotentials of the quadrupole potential and are shaped as hyperbolae of revolution. In addition to providing the necessary access for antiprotons, the cylindrical geometry allows the properties of the trap to be calculated analytically.

If voltages $\pm V_0/2$ are applied to the ring and both endcap electrodes respectively and a voltage V_c is applied to compensation electrodes, the potential near the center



Figure 2.3: Scaled drawing of the inner surfaces of the electrodes used to generate the harmonic trapping potential. Note that the ring is split in four quadrants and the compensation electrodes into two segments. Tuned circuits are coupled to segments of the ring and compensation electrodes to detect the motion of particles.

of the trap may be expanded in spherical coordinates (r, θ, ϕ) as

$$V = \frac{1}{2} \sum_{\substack{k=2\\even}}^{\infty} \left(V_0 C_k^{(0)} + V_c D_k \right) \left(\frac{r}{d}\right)^k P_k(\cos \theta),$$
(2.3)

where d is a trap dimension defined in terms of the spacing between endcaps, $2z_0$, and the ring radius, ρ_0 , by $d^2 = z_0^2 + \frac{1}{2}\rho_0^2$. Only even k terms are included in the expansion due to reflection symmetry through the z = 0 plane, while symmetry about the \hat{z} axis eliminates any ϕ dependence. Since $r \leq 0.1d$ for even our largest excitations, the series converges rapidly. When the voltages are adjusted such that the ratio of V_0 and V_c remains constant, the coefficients C_k defined as

$$C_k = C_k^{(0)} + \frac{V_c}{V_0} D_k \tag{2.4}$$

are frequently used. The expansion coefficients C_k and D_k are calculated for our electrode geometry [31] and displayed in Table 2.2.

An ideal electrostatic quadrupole potential yields a harmonic axial motion with frequency ν_z , which is independent of the axial energy E_z . The only non-zero term in the expansion of such an ideal potential is C_2 leading to an axial frequency of

$$\nu_z = \frac{1}{2\pi} \sqrt{\frac{C_2 e V_0}{M d^2}}.$$
(2.5)

In a real trap, however, higher order terms shift the axial frequency by $\Delta \nu_z$. A nonzero C_4 yields a shift proportional to the energy [31]

$$\frac{\Delta\nu_z}{\nu_z} = \frac{3}{2} \frac{C_4}{C_2} \frac{E_z}{eV_0 C_2}.$$
(2.6)

 C_4 must, therefore, be sufficiently small that shifts in the axial frequency do not

| | | calculation | measurement |
|-------------------------|-------|----------------------|-------------|
| design | dime | ensions | |
| $ ho_0$ | = | $0.600 \mathrm{~cm}$ | |
| z_0 | = | $0.586~\mathrm{cm}$ | |
| d | = | $0.5116~\mathrm{cm}$ | |
| V_c / V_0 | = | 0.381 | 0.3816(1) |
| symmet | tric | expansion coe | efficients |
| C_2 | = | 0.5449 | 0.548(2) |
| D_2 | = | 0 | -0.0053(2) |
| C_4 | = | 0 | $< 10^{-5}$ |
| D_4 | = | -0.556 | |
| C_6 | = | 0 | $< 10^{-3}$ |
| asymm | etric | expansion co | oefficients |
| c_1 | = | 0.335 | 0.31(3) |
| c_3 | = | 0.220 | 0.23(2) |
| κ_1 | = | 0.252 | 0.23(2) |
| λ_1 | = | 0.713 | 0.74(8) |
| d_1 | = | 0.9 | |
| d_1^* | = | 0.45 | |

Table 2.2: A comparison of calculated [31] and measured electrostatic properties of the trap. The expansion coefficients are defined in the text.

affect the cyclotron frequency ν_c and limit the accuracy of the measurement. C_4 may be tuned to zero by adjusting V_c (Eq. 2.3) as discussed in section 5.4. Higher order coefficients (e.g. C_6) will cause the apparent tuning point where $C_4 = 0$ to change as the energy in the axial motion increases. The trap was designed, however, such that both C_4 and C_6 vanish at nearly the same value of V_c . Finally, when V_c is changed, ν_z will also change if $D_2 \neq 0$. To eliminate these shifts the open access trap has also been "orthogonalized" [32] to reduce D_2 to zero. A measurement of D_2 is also discussed in section 5.4.

2.3 Shifting the Equilibrium Location of a Trapped Particle

An asymmetric potential applied across the trap moves the equilibrium position of a trapped \bar{p} or p away from the geometric center of the trap, allowing a direct measurement of the magnetic gradients (Ch. 9). A differential potential between the two usually grounded endcaps moves the particle vertically in the trap (along \hat{z}). The axial translation of a trapped particle has been analyzed [31] and only the results to be compared to experiment will be summarized here. A particle may also be moved radially by adjusting the potentials on two of the four segments of the central ring of the trap (Fig. 2.4) which otherwise would be at the same potential. Since the radial translation of particles has not been previously analyzed, the relevant electrostatic potential and its consequences are calculated in section 2.4.

If a potential $V_A/2$ is applied to one endcap electrode, and $-V_A/2$ to the other, with all other electrodes grounded, the potential near the center of the trap may be expanded in spherical coordinates (r, θ, ϕ) as

$$V = \frac{V_A}{2} \sum_{\substack{k=1\\odd}}^{\infty} c_k \left(\frac{r}{z_0}\right)^k P_k(\cos\theta).$$
(2.7)

Only odd k terms are included in the expansion owing to reflection antisymmetry $(V \rightarrow -V \text{ when } z \rightarrow -z)$, and any ϕ dependence is eliminated by the symmetry about the \hat{z} axis. The expansion coefficients c_n , calculated for our electrode geometry [31], are displayed in Table 2.2. To lowest order in r/z_0 , the shift in equilibrium position is given by [31]

$$\Delta z = -\frac{c_1}{C_2} \frac{d^2}{2z_0} \frac{V_A}{V_0}.$$
(2.8)

At the new equilibrium position, the effective trapping potential is shifted from V_0 to $V_0 + \Delta V_0$ where

$$\frac{\Delta V_0}{V_0} = -\frac{3}{2} \left(\frac{d}{z_0}\right)^4 \frac{c_1 c_3}{(C_2)^2} \left(\frac{V_A}{V_0}\right)^2.$$
(2.9)

We measure ΔV_0 directly as the potential added to the ring to keep the axial frequency constant at each V_A . The product $c_1c_3 = 0.0732 \pm 0.0001$ is then extracted from a fit to the voltages. The measured c_1 may be independently determined using knowledge of magnetic gradients and is discussed in chapter 9.

A similar set of coefficients, d_n , may be determined for asymmetric potentials applied to the compensation electrodes. While we do not use the compensation electrodes to move particles, the coupling of a tuned circuit (Fig. 2.3 and Sec. 3.3) is determined by the linear term in the asymmetric potential. Because the compensation electrodes are split to allow drives with proper symmetries to be applied, only half the electrode is used for detection. While the properties of the split ring are studied below, the compensation has not, and d^* , the linear coefficient of half of a compensation electrode will be approximated as $d_1/2$ in estimates of the detection sensitivity and damping constants in section 3.3.

2.4 The Radial Asymmetric Potential

A particle may be moved radially by adding voltages $V_A/2$ and $-V_A/2$ to two opposing segments of the four piece ring electrode (Fig. 2.4b). To determine the effects of this voltage, we calculate the potential near the center of the trap with the above voltages on these two segments and all other electrodes grounded. (This asymmetric potential is then added to the usual trapping component of Eq. 2.3 to determine



Figure 2.4: Asymmetric potential added to the endcaps (a) to move the particles vertically and to the ring (b) to move particles radially. The unshown electrodes remain at zero volts.

the total potential.) In cylindrical coordinates (ρ, ϕ, z) , the potential has the form¹

$$V(\rho,\phi,z) = \sum_{m,n=0}^{\infty} A_{mn} \cos(k_n z) \cos(m\phi) I_m(k_n \rho), \qquad (2.10)$$

where the ${\cal I}_m$ are modified Bessel functions and

$$k_n = (n + \frac{1}{2})\frac{\pi}{z_e}.$$
(2.11)

 $^{^1\}mathrm{I}$ am indebted to Anton Khabbaz for demonstrating that potentials from split electrodes can be calculated analytically.
Note that unlike the previous two expansions, the ϕ dependence may not be neglected here. By matching the potential on the trap electrodes, the constants A_{mn} may be evaluated, yielding

$$V_{r}(\rho,\phi,z) = V_{A} \frac{2\sqrt{2}}{\pi z_{e}} \sum_{\substack{n,m=0\\m \ odd}}^{\infty} \frac{\sin(k_{n}z_{r})}{k_{n}m} \cos(k_{n}z) \cos(m\phi) \frac{I_{m}(k_{n}\rho)}{I_{m}(k_{n}\rho_{0})} \times \begin{cases} 1 & \text{for } m = 1, 3 + 8p \\ -1 & \text{for } m = 5, 7 + 8p \end{cases}$$
(2.12)

where p is a nonnegative integer. By expanding the potential along the \hat{x} axis $(\phi = 0)$ as a Taylor series,

$$V_r(x,0,0) = \frac{V_A}{2} \sum_{\substack{n=1\\odd}}^{\infty} \kappa_n \left(\frac{x}{\rho_0}\right)^n,$$
(2.13)

and combining it with the usual trapping field (Eq. 2.3), the shift of the equilibrium position,

$$\Delta x = -\frac{\kappa_1}{C_2} \frac{d^2}{\rho_0} \frac{V_A}{V_0},$$
(2.14)

is determined (provided that $\Delta x \ll \rho_0$ and κ_3 may be neglected). Solving Eq. 2.12 for κ_1 leads to

$$\kappa_1 = \frac{2\sqrt{2}}{\pi} \frac{\rho_0}{z_e} \sum_{n=0}^{\infty} \frac{\sin(k_n z_r)}{I_1(k_n \rho_0)},$$
(2.15)

which is evaluated in table 2.2. Similarly, the shift in the axial frequency may be expressed as

$$\Delta(\omega_z^2)(x,0,0) = \frac{e}{M} \frac{\partial^2 V_r}{\partial z^2} = \frac{eV_A}{M\rho_0^2} \sum_{n=1}^\infty \lambda_n \left(\frac{x}{\rho_0}\right)^n.$$
(2.16)

Differentiating V_r in Eq. 2.12 twice and retaining only lowest order in x/ρ_0 yields

$$\lambda_1 = -\frac{\sqrt{2}\rho_0^3}{\pi z_e} \sum_{n=0}^{\infty} k_n^2 \frac{\sin(k_n z_r)}{I_1(k_n \rho_0)}.$$
(2.17)

The corresponding change in trap depth from V_0 to $V_0 + \Delta V_0$ when an asymmetric potential V_A is applied is then

$$\frac{\Delta V_0}{V_0} = 2 \frac{\Delta \omega_z}{\omega_z} = -\left(\frac{d}{\rho_0}\right)^4 \frac{\lambda_1 \kappa_1}{(C_2)^2} \left(\frac{V_A}{V_0}\right)^2.$$
(2.18)

We extract the product $\lambda_1 \kappa_1 = 0.17 \pm 0.01$ (like $c_1 c_3$ for the axial case) from the change in the applied voltage needed to keep the axial frequency constant at each value of V_A . Like the axial coefficient c_1 , the constant λ_1 can be determined using knowledge of the magnetic gradients as discussed in chapter 9. Table 2.2 gives both the calculated and measured values for our electrodes.

2.5 Conclusion

The properties of the potentials in the open endcap trap have been calculated as described here. The electrodes, shaped to admit antiprotons, are optimized in their aspect ratios and lengths to produce a quadrupole field, allowing a precision comparison of the charge-to-mass ratios of the antiproton and proton. The cylindrical geometry also allows the properties of the trap to be calculated in analytic forms.

Chapter 3

The Apparatus

The cylindrical electrodes described in the previous chapter are housed in a vacuum enclosure cooled to near 4 K by thermal contact to a dewar filled with helium. Amplifiers consisting of *LRC* tuned circuits and FETs detect the motions of a single antiproton or proton in the trapping region. This assembly is placed in a high field superconducting solenoid. An RF shielded control room beside the solenoid houses power supplies and synthesizers to supply DC and RF voltages and a microcomputer to control the experiment.

3.1 Trap Electrodes

The trap comprises nine hollow copper cylinders of inner diameter 1.2 cm. OFHC (Oxygen Free High Conductivity) copper improves the electrical and thermal conductivities and reduces residual magnetic impurities of the copper. The electrodes have also been plated with roughly 2 μ m of gold to minimize oxidation and reduce surface charging. Five electrodes compose the precision trap described in the previous chapter. Four additional electrodes, located above the precision trap, increase



Figure 3.1: Cryogenic apparatus showing helium dewar, cold electronics section, and the trap vacuum enclosure. Not shown are thermal radiation shields which cover the apparatus, keeping 77 K thermal radiation off the 4 K pieces.

the length available for loading antiprotons (Ch. 7).

The three central electrodes of the precision trap are split vertically (Fig. 2.3) to allow for the proper symmetries for drives as well as isolation between the detectors. The ring is split into four quadrants, labeled 0°, 90°, 180° and 270° (Fig. 3.4). The compensation electrodes, split in half, are labeled 0° and 180° as their alignment is such that they are closest to the ring segment of the corresponding name. In later chapters, radial directions will be referred to as x and y, where the positive x direction runs from the center of the 180° electrode towards the 0° electrode and the y direction from the 270° towards the 90° electrode.

A copper vacuum enclosure houses the electrodes. The enclosure is connected to the bottom of a liquid helium dewar (Fig. 3.1) and inserted in the bore of a 5.85 Tesla superconducting solenoid. Titanium windows 10 μ m thick on the top of the beamline from the accelerator and the bottom of the enclosure admit the 5.9 MeV antiprotons to the trap (Ch. 7). (Details of the design and construction of the trap and the supporting hardware (as well as the previous mass measurement) are discussed in detail in [33, 13].) When the cryostat is filled with liquid helium, the trap cools to near 4K by conduction. The tuned circuits and FETs used to detect the motion of the particles (Sec. 3.3) reside between the dewar and the trap enclosure and are also cooled to near 4K by conduction through the copper. (The FETs typically dissipate several mW of power which can heat the tuned circuits above 4K, however.) At these low temperatures, cryopumping provides an outstanding vacuum. Antiprotons have been held for 59 days indicating a lifetime greater than 103 days. Using calculated cross low energy cross sections for capture of antiprotons by helium or hydrogen, this leads to a pressure less than 5×10^{-17} Torr at 4 K (100 He atoms/ cm^3) [14, 34] in the trapping region.

3.2 Trap Circuitry

3.2.1 DC Circuits

A stable voltage source is critical for observing the axial motion. Fluctuations in the voltage applied to the electrodes increase the linewidth and reduce the signal amplitude. This voltage has often been applied using standard cells (low noise, high stability batteries kept at constant temperature). These cells require a stable environment to operate which is difficult to attain in the accelerator hall. They also supply only one fixed voltage and supply little current — to the extent that large capacitors are usually charged using a secondary supply before the standard cells are connected. As antiprotons, protons and electrons are more conveniently studied at different voltages, and low voltages are used in removing electrons and ions from the trap during the loading process (Sec. 7.1.2), an adjustable solid state supply is a flexible alternative to the standard cells.

A Fluke 5440A Direct Voltage Calibrator, therefore, provides the trapping voltage. The voltage applied to the compensation electrodes is divided from the main supply using a precision voltage divider (a 10 k Ω Kelvin – Varley precision voltage divider). As it is powered from the AC mains, the 5440A is intrinsically tied to local ground. While 1 M Ω resistors which tie the low output of the supply to ground can be removed, the capacitance of the coils of the transformer to local ground cannot be eliminated. Therefore, the "ground" of the system is chosen to be the local ground of the power supply. This should be contrasted to the usual way in which standard cells are wired in which the voltage of a plate on the top of the trap enclosure is defined as "ground".

The power supply – with internal microprocessors and GPIB control – generates a great deal of RF noise. Low pass RC filters with a 10 second time constant reduce



Figure 3.2: When separate grounds are used for the power supply and filter (a), line frequency ground loops (up to 10 mV differences at 50 Hz between the power supply and filter local grounds) create voltage drops across the resistor. Using one ground (b) eliminates this source of noise.

this noise before the voltage lines enter the cryogenic apparatus (Fig. 3.3). The high impedance magnifies the voltage fluctuations caused by small ground loop currents. Chassis ground at the filters typically differs from that at the power supply by more than 10 mV at line frequency (50 Hz) as well as at DC. At 50 Hz, the 10 μ F capacitor in the *RC* filter will have an impedance of 300 Ω , much less than the 1 M Ω resistor. Therefore, if the low side of the capacitor is connected to local ground, a current will flow at 50 Hz (Fig. 3.2a), creating voltage noise which will disturb the axial frequency of the particle. If instead, the filters are grounded to the low of the power supply via a cable returning to the supply from the filters (Fig. 3.2b), this current loop is broken. Both the ring and compensation voltages are filtered with ten second filters, but the impedance of the endcaps must be kept small as it is the same voltage as the low of the 10 second filters. Instead, an *LC* low pass filter (Fig. 3.3) connected to local ground reduces any high frequency noise picked up on this line. Similarly on the ring and compensation lines, small valued *RC* filters tied to local ground reduce any common mode high frequency noise. The voltages then enter the cryogenic section of the apparatus where an additional set of filters (Fig. 3.4) further reduce any high frequency noise before the voltages are applied to the trap electrodes.

Additionally, asymmetric DC voltages may be applied to shift the equilibrium position of a particle. For an axial asymmetric voltage, the endcap voltages are shifted from zero to $\pm V_A/2$. A single current through a balanced set of 1 M Ω resistors (Fig. 3.3) provides equal and opposite voltages on the electrodes. A battery supplies the current to avoid grounding problems like those described for the supply which provides V_0 . By balancing the resistances of the four resistors, matched voltages may be applied to the endcaps without having to match two power supplies. Radially, asymmetric voltages are applied across opposite quadrants of the ring (Fig. 3.5). The voltage dividers are located in the cryogenic electronics section (Fig. 3.5). In the 90°/270° direction (as in the axial case), one current (supplied by batteries) flows from $V_A(90^\circ)$ to $V_A(270^\circ)$ to offset the opposing segments. In the 0°/180° direction, however, the necessary currents are complicated by the need to correct the shift in the 90°/270° segments.

3.2.2 AC Circuits

Radio-frequency drives resonantly excite and couple the motions of the particles. These drives must be applied with the proper frequency and symmetry corresponding to the motion to be driven. A twisted pair of constantan wires delivers the signal from room temperature to the cryogenic section so that it is not broadcast to the other lines. In the cryogenic electronics region, it is capacitively coupled to the DC potential for the electrode (Figs. 3.4 and 3.5). The thin twisted pair has a resistance



Figure 3.3: The trapping and compensation potentials are supplied by a high precision voltage calibrator represented above by the battery on the left. The filters shown are critical. The connections shown as dotted lines in the figure can be seen in more detail in figures 3.4 and 3.5. Note that the long time constant filters are grounded near the voltage calibrator, not in the filter box. The filter components for the modulation (ν_{mod}) input are c = 20 nF and l = 0.15 mH.



Figure 3.4: Circuits for the harmonic trap electrodes. Note that the ring and the amplifiers are shown very schematically. See figures 3.5, 3.7 and 3.8 for more detail.



Figure 3.5: Circuits for the ring electrode. Note that the amplifier is shown very schematically. See figure 3.7 for more detail.

of approximately 100 Ω which along with a 1000 pF capacitor coupling the drive to the DC component produces a low pass filter with a corner frequency of 1.6 MHz. The 954 kHz axial drive for the antiproton or proton is not attenuated by this filter. Extra power is supplied to the electron axial drive at 72 MHz and the \bar{p} cyclotron drive at 89 MHz to compensate for attenuation by the filter.

The \bar{p} and e^- axial drives are applied on the upper and lower endcaps to produce the z symmetry required of the axial motion. Tuned circuits on the compensation electrodes opposite the corresponding endcaps are tuned to detect the axial motion (Sec. 3.3). The small response signal would not be detectable over the large drive signal. Therefore, the drive is detuned from the axial frequency and the trapping potential is modulated at this detuning frequency. This FM driving scheme separates the driving frequencies from the detection frequency and enables small signals to be detected by filtering the driving frequencies. The modulation drive applied to the ring is typically 90 kHz for antiprotons and 1 to 5 MHz for electrons. The modulation is added to the ring voltage after the large filters but before the signal enters the cryostat (on the line labeled ν_{mod} in Fig. 3.3).

Magnetron cooling drives (Sec. 6.1) for antiprotons and electrons are applied on the 0° segment of the upper and lower compensation electrodes to produce the xzsymmetry needed to couple these two motions. The cyclotron and axial motions also couple with an xz symmetry and as $\nu'_c - \nu_z$ is roughly 88 MHz, it is applied to one segment of the lower compensation electrode along with the electron magnetron sideband drive. The drive line on the ring electrode is constructed slightly differently than the others (Fig. 3.5). A balanced drive scheme was wired to reduce direct feedthrough from the drive onto the proton cyclotron amplifier. This wiring was never optimized and for the measurements described here, a short in the drive line led to cyclotron drives being applied to the lower compensation electrode (which has a component with x symmetry as well as with xz) along with electron magnetron cooling and antiproton $\nu'_c - \nu_z$ drives.

3.3 Amplifiers

A charged particle near a conducting surface induces an image charge in the conductor. In a trap, the oscillatory motions lead to time-varying image charges and thus, image currents [35]. The image currents induce a voltage across an external resistor R_{eff} which is amplified and detected. The power dissipated in the resistor comes from the particle's motion which exponentially damps (with a time constant $\tau \propto R_{eff}^{-1}$ into thermal equilibrium with the resistor. The current from the motion of a single particle is very small, requiring a large resistance and sensitive amplifier to detect the signal.

Three amplifiers used to detect the antiproton and proton axial and cyclotron motion and the electron axial motion are located between the helium dewar and the trap vacuum enclosure (Fig. 3.1). The amplifiers are connected to electrodes with the proper symmetry for the motion to induce a signal. Thus, the cyclotron amplifier must be on an electrode which breaks rotational symmetry while the two axial amplifiers must not be in the xy plane. Also the three amplifiers must be isolated from one another to avoid couplings which degrade the Q. (At 954 kHz, 89 MHz and 72 MHz, they are already isolated in the frequency domain.) Finally, pickup from drives at frequencies near the resonance of the amplifier should not saturate the amplifier. The amplifiers, therefore, should be as far as possible from the corresponding drive electrodes.

The axial amplifiers are placed on one segment of the compensation electrode on the opposite side of the trap from their corresponding drive. (One segment of a compensation electrode has z symmetry as well as xz symmetry.) Because of the screening by the compensation electrodes, the endcaps would detect roughly the same signal as half a compensation electrode (comparing $d_1/2$ to c_1). As was discussed previously, the cyclotron amplifier is on the 0° segment of the ring. The orientation of the compensation electrodes relative to the ring was chosen so that the tuned circuits of the three amplifiers would not be coupled. While there are several pF of capacitance between the 0° segments of the ring and 180° of the compensation electrodes where the axial amplifiers are.

The antiproton cyclotron amplifier must be located close to the trap. Its 90

MHz resonant frequency has a quarter wavelength of less than one meter, roughly the distance to the top of the helium dewar (Fig. 3.1). The tuned circuit for the antiproton axial motion is constructed from superconducting materials to obtain a high enough Q to allow detection of a single antiproton. It is therefore constructed of type II superconducting material so that it also operates in the high magnetic field near the trap, allowing higher frequency operation than would be easily obtainable one meter from the trap in lower fields.

3.3.1 Detecting a Single Particle

The induced current from a single particle,

$$i_s = \kappa_1 \frac{e\dot{x}}{2\rho_0},\tag{3.1}$$

is very small (see Table 2.2 for values of κ_1 and ρ_0). For example, 1 \bar{p} with 1 eV of kinetic energy in its cyclotron motion induces approximately 10^{-13} amps across the ring. The highest possible detector impedance is required to make a detectable voltage from this small current (and to damp the cyclotron motion in a reasonably short time).

A tuned circuit provides the very large resistance $R_{eff} = Q\omega L$, on resonance, necessary where L is the inductance and $Q = \omega/\Delta\omega$ is the quality factor of the tuned circuit. Because the signal voltage $(i_s R_{eff})$ and the damping rate from a particle are both proportional to this resistance, the highest possible Q and L are sought. The silver plated copper tuned circuit of the cyclotron amplifier with a Q of 800, an inductance of 0.3 μ H and frequency of 89 MHz has an effective resistance of 130 k Ω and the NbTi superconducting tuned circuit with a Q of 3000, an inductance of 1 mH and a frequency of 954 kHz has a resistance of 15 M Ω . Whether made



Figure 3.6: The inductive coil and FET in their shielding can (a) and a schematic of the tuned circuit coupled to a particle in the trap (b). The particle in a quadrupole potential is represented as a mass on a spring.

from silver plated copper wire for the high frequency, small inductance coils or from NbTi superconducting wire for low frequencies, Q values of 10^3 are achieved. The inductors are housed in shielding cans to reduce coupling to other electrical elements. The geometry of the can must be chosen correctly [36], however, not to limit the Qor increase the self capacitance of the circuit. Losses external to the resonator, such as the feedthroughs, MACOR or grounding paths as well as anomalous losses such as magnetoresistance [37] and residual resistance in the shields and wire, set a limit on the Q of a well constructed coil and shielding can.

Maximizing the inductance also increases the effective resistance. Therefore, a premium is put on minimizing stray capacitance. In addition to winding low self capacitance coils, the tuned circuit is close to the trap to reduce transmission line capacitance. This also reduces the pickup of stray noise onto the input of the detector.

The voltage across the tuned circuit is detected with an amplifier "tapped" across the coil. This amplifier must have a high input impedance so as not to compromise the Q of the tuned circuit. An FET in a cascode configuration is used. Roughly 1/4of the turns from ground to the high side of the coil, a pickup wire is soldered to the coil. This is capacitively coupled to the input of the amplifier. The "tapping" reduces the signal voltage by the tap ratio, but also decreases the loading of the tuned circuit by the FET by the tap ratio squared. Additionally, it reduces the noise that the FET produces in the tuned circuit.

Standard silicon transistors at very low temperature have no charge carriers with enough energy to populate the conduction band. We use gallium arsenide (GaAs) transistors for which heavy doping of GaAs materials guarantees that even at zero temperature, there will still be charge carriers available [38, 39]. GaAs FETs at cryogenic temperatures have been widely used for such varied detection purposes as exotic particle searches [40, 41], NMR [42] and space physics [43]. We use a commercially available dual gate MESFET, the Mitsubishi MGF 1100. It is intended for operation near 1 GHz and can hence have a very high 1/f noise corner. Below this corner frequency the noise power per unit bandwidth grows as 1/f rather than being constant (as for Johnson noise). We have found with the MGF 1100 that the corner frequency varies from batch to batch, being greater than 1 MHz (and hence ν_z) on occasion. Careful choices of FETs [44, 41] can eliminate this problem, however.

The 10 k Ω output impedance of the FET must be matched to the 50 Ω , microcoax cable used to bring the signal from the cryogenic region of the apparatus to



Figure 3.7: Cyclotron motion detection. The cold tuned circuit detector and the room temperature electronics are shown.



Figure 3.8: Axial motion detection. The cold tuned circuit detector and the room temperature electronics are shown.

room temperature and 1 atmosphere. For the cyclotron amplifier, the length of the microcoax is a substantial fraction of a wavelength, and hence impedance matching is critical to effective power transfer. A π network, composed of two capacitors and an inductor, is used to transform the output impedance of the FET. The π network is tuned to the proper frequency and has a Q around 10, providing additional frequency selectivity as well as impedance matching.

The amplifier does not provide voltage gain. The FET operating in cascode has a transconductance, the ratio of output current i_{ds} to input voltage v_{gs} , given by

$$g_t = \frac{i_{ds}}{v_{gs}} \approx 0.01 \Omega^{-1}. \tag{3.2}$$

The voltage gain after the π network (using values for the 954 kHz axial amplifier) is

$$g = \frac{v_{out}}{v_{in}} = g_t n_t \sqrt{\frac{R_{out} \ 50\Omega}{2}} = 1.25 \tag{3.3}$$

where n_t is the "tapping ratio" which is 1/4 for the axial amplifier and R_{out} is the output impedance of the FET of approximately 10 k Ω . When the π network is properly impedance matched to the approximately 2 k Ω output impedances of our FETs (which varies between batches of FETs), the gain is near unity. The power gain, on the other hand, will be roughly the ratio of the input and output impedances (1 M Ω / 50 Ω) or 2000.

After passing through a vacuum feedthrough to atmospheric pressure, (Figs. 3.7 and 3.8), additional gain is needed before the signal is mixed down below 100 kHz for detection using the FFT dynamic signal analyzer. Filters are used both to keep strong but off-resonant drives from saturating the amplifiers and to eliminate any noise at the lower sideband from the signal before it is mixed down. (When ν_z at 954 kHz is mixed down to 91 kHz using an 863 kHz local oscillator, noise at 772 kHz must be filtered out.) After mixing down to low frequencies, additional gain and filtering is used to condition the signal for the signal analyzer. The final stage of filtering is provided by the signal analyzer itself. An FFT can be used as an extremely narrow band filter. The smallest bandwidth available on our signal analyzer is below 1 mHz. Similarly, the lock-in amplifier can be set to very narrow band (long integration times) to eliminate as much of the broadband noise as possible.

3.3.2 The Axial Coil

A new feature of this measurement is an axial amplifier constructed from superconducting materials. The low frequency of the axial motion requires a very large inductor to resonate at ν_z without adding additional capacitance. The Q of a coil wound from copper wire is limited by DC resistance of the wire. To increase the Q and make possible the detection of a single particle, the coil is wound from superconducting wire. Made from NbTi, a type II superconductor, it operates in the high-field region, close to the trap allowing a higher frequency than would be possible if the amplifier were in the lower field.

Others [45, 46, 47] have constructed superconducting tuned circuits. They are fabricated from type I superconductor, however, which looses its superconducting properties in magnetic fields much lower than our 6 T field. In the references above, therefore, the coils are kept a meter from the trap. As our superconducting amplifier is in the high field region, it uses type II (NbTi) superconducting wire which maintains its superconducting properties in a 6 Tesla field.

The coil is 50 μ m formvar-coated NbTi wire wound on a Teflon form. Copper clad NbTi wire with the copper etched from one end are electron beam welded to



Figure 3.9: The Johnson noise power and Lorentzian fit for the NbTi (Type II) superconducting tuned circuit. The high Q of this circuit (Q = 3400) allows efficient detection and damping of the axial motion of a single antiproton.

the bare NbTi wire. Copper wire could then be soldered to the ends of the coil. To "tap" the coil, the formvar was removed from a short portion of the coil 1/4 of the distance between the bottom and top of the coil and another copper clad wire with the copper etched off one end was electron beam welded in place. This provided the signal which was sent to the rest of the amplifier.

The can in which the coil was housed was also made of superconductor to avoid losses in the can as the capacitive coupling between the coil and wall induced currents in the can. As with the coil, copper straps were electron beam welded to the can so that connections could be made using solder. (First NbTi straps were welded to the copper straps. These were then welded to the NbTi can.) When coupled to the trap and placed in the high field, the superconducting coil produces a tuned circuit with a resonant frequency of 954 kHz and Q = 3400 (Fig. 3.9). This corresponds to a effective parallel resistance $R_{eff} = 15M\Omega$. Studies of high frequency losses in superconductors have typically been for high power RF cavities [48, 49], while, losses associated with high static fields are typically for low frequency power generation devices [50]. Some of the basic physics of resistance in static magnetic fields has also been examined [49]. However, none of these describe the effects of a high Q cavity in a large magnetic field. To further understand the loss mechanisms in the superconducting coil, the behavior of our circuit must be studied further in both low and high fields.

3.3.3 Cyclotron Signal-to-Noise Ratio

The relativistic shift in cyclotron frequency provides a direct calibration of E_c from $\Delta\nu'_c$. A comparison of expected and measured signal-to-noise ratios may then provide information about the effective temperature of the tuned circuit. An antiproton induces a voltage

$$V_S = \kappa_1 \frac{e}{2\rho_0} \dot{x} R_{eff} \tag{3.4}$$

in the tuned circuit [9] and the intrinsic Johnson noise is given by

$$V_N^2 = 2(kTR_{eff} + V_B^2)\Delta\nu \tag{3.5}$$

where $\Delta \nu$ is the measurement bandwidth. The first term is the Johnson noise of the effective resistance of the tuned circuit and the second the background noise independent of the tuned circuit. (A factor of two multiplies the sum because the signal is mixed down from ν_s to $\nu_s - \nu_{lo}$ (≈ 2 kHz). The noise at $\nu_{lo} - \nu_s$ mixes to the same frequency and is not filtered before the mixing. Hence there is twice the noise power.)

Comparing a 140 eV peak (peak 1 of Fig. 4.1a) and its associated noise floor, we observe a signal noise ratio of roughly 7 and calculate from the above formulae a value of 10 for the FFT bandwidth of 0.1 Hz. The good agreement indicates that the noise temperature is near that expected from the 4 K bath. Excess noise from a commercial radio station and nearby accelerator cavities (though greatly reduced through RF shielding) limits the minimum detectable signal to a few eV of excitation.

3.3.4 \bar{p} Cyclotron Damping Rate

The signal size and damping rate decrease as the resonant frequency (ν'_c) is detuned from the resonant frequency of the amplifier. With the axial motion, ν_z may be centered on the resonance by adjusting the trapping voltage. For the cyclotron motion, the persistent currents in the superconducting solenoid must be adjusted to change ν_c . To avoid this (as it takes weeks for the field to stabilize after this small adjustment), the amplifier has been constructed so that its resonant frequency may be adjusted, instead.

The capacitance of the FET gate, C_{gs} depends upon the gate voltage V_{gs} . This capacitance combines with the electrode capacitance (as well as stray capacitance) in forming the tuned circuit used to detect the cyclotron motion. The resonant frequency of the tuned circuit is adjusted by changing V_{gs} . For the axial amplifier the tapping ratio is set very low (roughly 1/4 of the way from ground to the signal) so that the effects of C_{gs} are very small. On the cyclotron amplifier, however, after observing that the Q was not compromised by a large tapping ratio, the tap



Figure 3.10: Tuned circuit center frequency as the voltage applied to the first gate (V_{gs}) is adjusted.

ratio was set at 2/3, making a larger fraction of C_{gs} contribute to the tuned circuit capacitance. As the gate voltage is adjusted over a 60 mV range, the resonant frequency changes by roughly 100 kHz (Fig. 3.10) corresponding to a change ΔC_{gs} of 0.1 pF. This tuning range is broad enough that the tuned circuit may be adjusted (mechanically) at room temperature such that when it shifts by slightly more than 1 MHz, as it cools to 4 K, it can be tuned exactly onto resonance using the gate voltage.

To study the effects of amplifier tuning on the cyclotron motion, the time constant was measured for various detunings (Fig. 3.11). The coupling to the amplifier causes an exponential damping of an initial energy, E_i in the cyclotron motion



Figure 3.11: Measured cyclotron time constants (τ_c) vs. values calculated from amplifier detuning. The solid line is the fit and the dashed lines are one sigma errors from uncertainties in the calculation of the damping rate (See Text).

 $(E_c = E_i e^{-t/\tau_c})$ with a time constant (half the time constant with which the amplitude damps) [9]

$$\tau_c = (\gamma_c)^{-1} = \frac{1}{2} \left(\frac{e\kappa_1}{2\rho_0}\right)^{-2} M/R.$$
 (3.6)

A calculation of κ_1 is in Ch. 2. The resistance R for arbitrary detuning of the tuned circuit center frequency away from the cyclotron frequency is

$$R = \frac{Q\omega_c L}{1 + (\Delta\omega)^2 / (\omega_c/2Q)^2},\tag{3.7}$$

where L is the inductance of the tuned circuit and $\Delta \omega$ is the detuning between the

center frequency of the tuned circuit and the cyclotron frequency. By measuring the detuning and the Q for the amplifier for each cyclotron decay, a calculated time constant can be determined as well as a measured one.

An overall uncertainty in the calculated time constants remains due to uncertainties in L and κ_1 . L is determined by direct measurements at low frequency of both the inductor itself and the resonating capacitance. The inductance of only 0.3 μ H is only known to 30 %. Measurements and calculation give $\kappa_1 = 0.24(1)$. These factors lead to the relatively large uncertainties of the calculated time constant and will systematically shift the calculated time constant relative to the measured value. The agreement at slightly over the one sigma level between the calculated and measured values is therefore quite reasonable agreement (Fig. 3.11).

There is one dangerous side effect of tuning the cyclotron amplifier. The cyclotron frequency of the antiproton is sensitive to sudden changes in the gate voltage (Fig. 3.12). Reducing the gate voltage from -2 volts (the normal operating point) to -5 volts (at which no current flows from drain to source) and then returning it to -2 volts shifts the cyclotron frequency by 2.5 Hz (28 ppb). After 30 minutes, ν'_c has returned to its original value. The mechanism for this effect is not fully understood. When ν_c shifts, the axial frequency does not. Therefore the trap depth is not shifting. Changing the drain voltage (and hence the drain current) does not produce a shift. Thus, it is not caused by the temperature of the FET. To avoid systematics associated with this difficulty, the tuned circuit was always allowed to "settle" for more than 30 minutes after the amplifier was turned on before measuring the cyclotron frequency.



Figure 3.12: Systematic shift in the cyclotron frequency caused by shutting off the FET of the cyclotron amplifier. (a) The cyclotron decay. (b) residuals to fitting only the data before the FET was turned off.

3.4 Conclusions

To detect the motion of a single antiproton, several amplifier innovations have been developed. A tunable cyclotron amplifier with well understood time constants allows the center frequency to be tuned to resonance with the cyclotron frequency. This gives the shortest time constants and thus minimizes the measurement time of a particle. The axial amplifier, constructed of superconducting materials which operate at 6 Tesla, has a Q of 3400, which allows detection of a single antiproton. These amplifiers are placed in high field region immediately above the trap (Fig. 3.1) which is wired to allow the excitation and detection of the motions of a single antiproton or proton as well as the loading of antiprotons (Ch. 7).

Chapter 4

Relativistic Cyclotron Motion

A particle of mass M and charge e oscillates in a magnetic field **B** in a circular cyclotron motion in a plane perpendicular to the magnetic field (Fig. 2.2) with a frequency,

$$\nu_c = \frac{|e\mathbf{B}|}{2\pi M c},\tag{4.1}$$

of approximately 90 MHz for an antiproton (or proton) in a 6 Tesla field. The only effect of the electric field that is added to axially confine the particle is to slightly reduce the cyclotron frequency to ν'_c . An invariance theorem (Eq. 2.2) relates the desired ν_c to the three observed frequencies for a particle in the trap.

Table 4.1: Energies of various cyclotron excitations of a \bar{p} .

| Cyclotron motion in a 5.9 Tesla field | | | |
|---------------------------------------|--------------------|----------------|---|
| $\nu_c' = 89.3 \text{ MHz}$ | | | |
| $\Delta \nu_c'(\mathrm{Hz})$ | $E_c(eV)$ | $ ho_c(\mu m)$ | |
| 3×10^{-5} | 4×10^{-4} | 0.5 | Equilibrium with detector $(T = 4.2 \text{ K})$ |
| 0.5 | 5 | 60 | Cyclotron measurement ends |
| 20 | 210 | 400 | Cyclotron measurement begins |
| 500 | 5250 | 2000 | Excitation during loading process |

4.1 Special Relativity and Damping

When the Dirac equation for the cyclotron motion is solved, the previous equation for the cyclotron frequency pertains, provided the mass is replaced by the "relativistic mass"

$$M = \gamma M_0 \tag{4.2}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the usual relativistic factor. While typical energies observed in our experiment (Table 4.1) of less than a few hundred eV are usually considered extremely nonrelativistic, the high precision to which the cyclotron frequency is measured ($\langle 2 \times 10^{-10} \rangle$), makes relativity a critical effect in determining the cyclotron frequency.

A kinetic energy E_c in the cyclotron motion shifts the cyclotron frequency from ν_c to $\nu_c + \Delta \nu_c$ with

$$\frac{\Delta\nu_c}{\nu_c} = -\frac{E_c}{M_0 c^2}.\tag{4.3}$$

Thus, a 10 eV cyclotron excitation shifts the cyclotron frequency of a \bar{p} ($M_0 c^2 \approx 938$ MeV) by 10.6 ppb. This shift is at least a factor of 50 larger than any other we know of, such as the "magnetic bottle" (Ch. 9) and electrostatic anharmonicity (Ch. 5).

As discussed in section 3.3, the cyclotron motion induces an image current in the trap electrodes and hence a voltage drop across a resistor that is proportional to its velocity. This not only is used to detect the particle, but also removes energy from the particle to bring it into thermal equilibrium with the (roughly 4 K) tuned circuit. The equation of motion for the cyclotron motion (ignoring weak, nonresonant couplings to the magnetron motion) in terms of the x component of the velocity v_x [9] is

$$\left[\frac{d^2}{dt^2} + \frac{1}{2\tau_c}\frac{d}{dt} + (\omega_c')^2\right]v_x = 0$$
(4.4)

where $\omega_c' = 2\pi\nu_c'$ and

$$\tau_c = \frac{1}{2} \left(\frac{2\rho_0}{e\kappa_1}\right)^2 \frac{M}{R} \approx 10 \text{ min.}$$
(4.5)

for our trap and amplifier.

This is the equation of motion of a damped harmonic oscillator of frequency ν'_c whose energy damps exponentially as

$$E_c = E_0 e^{-t/\tau_c}.\tag{4.6}$$

As the energy damps, however, the oscillator's frequency shifts in proportion to the kinetic energy (Eq. 4.3). The cyclotron frequency exponentially approaches the limiting value, $\omega'_{c0} = eB/M_oc$ needed in the invariance theorem (Eq. 2.2) to determine ν_c and the charge-to-mass ratio. Special relativity and damping, therefore, yield a cyclotron frequency $\nu'_c(t)$ which shifts in time by

$$\nu_{c}'(t) = \nu_{c}' - \Delta \nu_{c}' e^{-t/\tau_{c}}.$$
(4.7)

where $\Delta \nu'_c$ is the initial cyclotron excitation. The parameters ν'_c , $\Delta \nu'_c$ and τ_c are extracted by fitting a time series of measured frequencies (Fig. 4.1).

4.2 Measuring the Cyclotron Frequency

The signal from the cyclotron motion of a single \bar{p} is very small. Approximately 7 nV is generated across the tuned circuit from a 10 eV cyclotron excitation, while the intrinsic Johnson noise across the bandwidth of the tuned circuit is several μ V. However, the cyclotron signal has a very well defined frequency. An ideal linewidth of 0.3 mHz is due to the damping time of 10 minutes, with magnetic field fluctuations



Figure 4.1: The cyclotron frequency (a) at 3 different times followed by the cyclotron frequency as a function of time (b) and the residuals to a fit to an exponential decay (c).

increasing the smallest observed line widths to 20 mHz. Since this is many orders of magnitude smaller than the 100 kHz bandwidth of the tuned circuit, a very narrow band filter dramatically reduces the broadband noise. A fast Fourier Transform (FFT), in addition to passive filtering, provides both the narrow band filtering and frequency measurement for the cyclotron motion.

The FFT, performed by an HP3561A dynamic signal analyzer (Fig. 4.2), is read out by computer and the frequency channel with the maximum amplitude is interpreted as the cyclotron frequency. Because the cyclotron frequency shifts as the energy in the particle damps (Fig. 4.1a), the computer adjusts the center frequency of the FFT so that the cyclotron frequency remains near the center of the FFT range. The amplitude of the signal also decreases along with the rate of change in



Figure 4.2: A simplified schematic of the wiring of the ring electrode showing only the drive and detection lines for the cyclotron motion.

the frequency as the particle damps by more than an order of magnitude in energy, requiring more averaging to maintain an acceptable signal to noise ratio. To balance these two effects, the span of the FFT is decreased as the cyclotron motion damps. The cyclotron frequencies as a function of time (Fig. 4.1b) are recorded until the excitation of the particle drops below a few eV.

4.3 Uncertainties

As a check of the cyclotron measurements, a frequency synthesizer was adjusted to mimic the cyclotron signal induced in the detector by a single p or \bar{p} . The computer controlled the frequency and amplitude of an HP3325B function generator (Fig. 4.3) at approximately 2 kHz which was then mixed to 89 MHz and then fed into the trap on the cyclotron drive line (Fig. 3.5). The attenuation was then adjusted to match



Figure 4.3: Schematic of drive and detection scheme used for simulation.

that of a particle with the equivalent excitation. The computer both acquired the frequency measured with the FFT and adjusted the function generator emulating the particle.

To duplicate the measurement of an antiproton cyclotron frequency, the simulated time constant was 8 minutes and the measurement continued until the frequency was within 0.5 Hz of the "zero energy" frequency. Thus, the measurement took just as long as a normal measurement (Fig. 4.4). The amplitude (Fig. 4.4b) fit with the time constant fixed to twice the value measured from the frequency $(v \propto \sqrt{E})$. Due to slight nonlinearities in the mixer, the amplitude does not precisely follow the expected shape of $A_0 \exp(-t/2\tau_c)$.

Measured "zero energy" frequencies of the simulated cyclotron measurements agree with the expected value to better than 0.1 ppb (Fig. 4.5). Histogramming the endpoints from one night's simulated cyclotron measurements (Fig. 4.5) shows a



Figure 4.4: A simulated cyclotron measurement (see text) agrees to better than 0.3 ppb with the expected parameters (a). The fit to the amplitude (b) uses the time constant from (a). Slight non-linearities in the mixer lead to small systematic deviations in the amplitude.



Figure 4.5: Histogram of the results of simulating the cyclotron signal from a \bar{p} using a drive.

systematic offset between the measured and expected cyclotron frequencies of 0.08 ppb with a width of 0.03 ppb. Compared to the 1 ppb desired accuracy of our measurement these are insignificant errors.

Additionally, statistical errors on the fitted parameters of a cyclotron measurement are determined by Monte Carlo analysis [51]. First, synthetic data sets { t_i , ν_i^{syn} , σ_i } are generated by adding Gaussian noise ($\Gamma(\sigma_i)$) with a width σ_i given by the uncertainty in the measured frequency to the fitted function

$$\nu_i^{syn} = \nu_c'(true) - \Delta\nu_c'(true)e^{-t_i/\tau(true)} + \Gamma(\sigma_i), \qquad (4.8)$$

evaluated at the measurement times t_i using $\nu'_c(true)$, $\Delta\nu'_c(true)$ and $\tau(true)$, the


Figure 4.6: Histogram of Monte Carlo results of ν'_c for a typical cyclotron decay, with a Gaussian width, $\sigma = 0.015$ Hz.

parameters of the measured cyclotron decay extracted from the fit. These new data sets are then fit and synthetic parameter sets $(\nu_c^{\prime syn}, \Delta \nu_c^{\prime syn}, \tau^{syn})$ extracted. Histogramming the values of ν_c^{\prime} (Fig. 4.6), error bars for the parameter (as well as potential systematic deviation of the mean from the true value) may be determined.

A sample measurement has a statistical error (Fig. 4.6) on ν'_c of $\sigma_{\nu} = 0.015$ Hz (less than 0.2 ppb). Since in a typical cyclotron decay, the final measurement point is roughly 0.5 Hz (5 ppb) from the endpoint, the fitting routine extrapolates to the endpoint value. The Monte Carlo shows that no bias is introduced by this extrapolation. The input value of ν'_c and the mean agree at the 0.005 Hz size of the bin. Errors of approximately 0.1 ppb are also consistent with the scatter observed



Figure 4.7: Histogram of normalized residuals (residuals / measurement error) of all measured cyclotron decays with a Gaussian width $\sigma = 0.86$ where normally distributed errors should be Gaussian with a width of 1.

in the residuals of a cyclotron decay fit (Fig. 4.1c). Over times of the 30 minutes it takes to measure a cyclotron decay the cyclotron frequency is thus known to a fraction of 1 ppb. Over longer time scales, however, the magnetic field, and hence the cyclotron frequency, drifts over much larger ranges (Ch. 8).

Accurate error bars from the Monte Carlo analysis require that the random noise, Γ , added to the synthetic data, properly model the noise in the system. The measurement errors of the cyclotron frequency are not exactly Gaussian because of the discrete nature of the FFT. Histogramming the residuals from fits of all the cyclotron decays normalized to their respective error bars (which for Gaussian errors should be Gaussian with a width of one) in the measurement set (Fig. 4.7), however shows a close approximation to a Gaussian errors with a normalized width of 0.86.

4.4 Conclusion

We have seen in this chapter that we can measure the cyclotron frequency of a single particle in the trap to fractional accuracies better than 1 ppb (10^{-9}) despite relativistic shifts that are initially greater than 200 ppb. While for periods greater than one hour, the magnetic field drifts by many ppb, for times less than one hour, accuracies approaching 0.1 ppb are possible. As can be seen in the residuals, Monte Carlo and simulation results, systematic biases in the measurement of a single cyclotron decay are also well below the 1 ppb level. Systematics studies (Ch. 9) take advantage of the sub-ppb resolution of a single cyclotron decay even when a full charge-to-mass comparison is limited by other effects to the 1 ppb level.

Chapter 5

Axial Motion

An electrostatic quadrupole potential (Fig. 2.1) confines the particle axially. With our trap dimensions (Table 2.2) and a trapping potential of 17.9 volts, an antiproton oscillates harmonically with a frequency $\nu_z = 954$ kHz which is independent of the amplitude of the motion. The axial frequency, 100 times smaller than the cyclotron frequency, need only be measured to a fractional accuracy of 10^{-5} (10 Hz) to determine ν_c to 1 ppb (Sec. 2.1). Using a high Q superconducting tuned circuit (Sec. 3.3), the axial frequency of a single \bar{p} or p may be measured to better than this accuracy when higher order terms in the potential are sufficiently small.

5.1 Equation of Motion

A particle of charge e and mass M in a quadrupole potential has the equation of motion

$$\ddot{z} + \gamma_z \dot{z} + \omega_z^2 z = f(t)/M, \tag{5.1}$$



Figure 5.1: A schematic of the tuned circuit and a \bar{p} in which the restoring force of the electric field is represented as a spring (a) and the noise resonance of the tuned circuit (b).

of a damped harmonic oscillator with angular frequency

$$\omega_z = \sqrt{\frac{C_2 e V_0}{m d^2}} \tag{5.2}$$

and Lorentzian linewidth $\gamma_z/2\pi$. Slight deviations from a quadrupole potential will cause energy dependent shifts in the frequency (Sec. 5.4). The force term $f(t) = V_d \cos \omega_d t + v_n(t)$ includes both noise from the tuned circuit and any applied drive which excites the motion.

An axial excitation damps with a rate proportional to $\exp(-\gamma_z t)$ (note that the damping rate in Ch. 4 was the energy damping rate and hence twice as large) where

$$\gamma_z = \left(\frac{ed_1^*}{2z_0}\right)^2 R_{eff}/M.$$
(5.3)

 $R_{e\!f\!f}$ is the impedance of the tuned circuit (Sec. 3.3) and d_1^* is a geometrical factor

| Axial motion in an 18 Volt well. | | | |
|---|---------------------|-----------------------|-----------------------------------|
| $\nu_z = 954 \text{ kHz}$ | | | |
| | E_z | z_{max} | $\Delta \nu_c'$ (magnetic bottle) |
| thermal equilibrium $(T = 4.2 \text{ K})$ | 0.3 meV | $33 \ \mu \mathrm{m}$ | 0.04 ppb |
| precision axial measurement | $0.1 \ \mathrm{eV}$ | $0.6 \mathrm{mm}$ | 20 ppb |
| power to drive out of trap | $7 \mathrm{eV}$ | $6 \mathrm{mm}$ | 2 ppm |

Table 5.1: Energies of various axial excitations of a \bar{p}

which would be unity if the detection electrodes were infinite parallel plates separated by a distance $2z_0$. The axial tuned circuit detects the current induced in half a split compensation electrode relative to the other electrodes (the other half is used to apply a magnetron cooling drive (Ch. 3)). Screening of the endcap by the compensation electrode reduces the current induced in the endcap to less than that induced in half a compensation electrode. The geometrical factor for half a compensation electrode, d_1^* , will be approximated as $d_1/2$ (Table 2.2 and Ref. [31]). Evaluating Eq. (5.3) leads to a linewidth of $\gamma_z/2\pi = 50$ mHz for a single \bar{p} or p. As the width is proportional to e^2/M , N_p trapped particles will have a width $N_p\gamma_z$ provided that $N_p\gamma_z \ll \Delta\omega$, the width of the noise resonance, and R_{eff} may be approximated as constant over the frequency range. The natural linewidth is much less than the 10 Hz resolution needed to determine ν_c to 1 ppb.

5.2 Undriven Axial Signal

The axial motion of antiprotons in thermal equilibrium with the (nearly) 4K tuned circuit may be detected. The Johnson noise from the *LRC* tuned circuit is $v_n = (4k_BTR_{eff}\Delta\nu)^{1/2}$ in a bandwidth $\Delta\nu$ at the resonant frequency of the tuned circuit.



Figure 5.2: The equivalent electrical circuit for the tuned circuit and the axial motion of antiprotons. The inductor and capacitor l_p and c_p correspond to the antiprotons and L, R and C are the components of the tuned circuit. The voltage source v_J represents the Johnson noise from the resistor, R.

Off resonance, the effective resistance is

$$R_{eff} = \frac{R}{1 + Q^2 (1 - \omega/\omega_0)^2},$$
(5.4)

where $\omega_0 = (LC)^{-1/2}$ and $Q = R/\omega L$. When no particles are present, the noise resonance has a Lorentzian profile proportional to the effective resistance as a function of frequency (Fig. 5.1b).

An equivalent electrical circuit for a harmonic oscillator like the axial motion of an antiproton is an inductor and capacitor in series. This allows the particles and tuned circuit to be modeled as the usual *LRC* of the tuned circuit and a second l_p and c_p of the antiprotons (Fig. 5.2). When ν_z is resonant with the tuned circuit, the impedance of l_p and c_p will cancel at the same frequency the effective resistance of the original tuned circuit was at a maximum. Therefore, the Johnson noise from the tuned circuit will be "shorted" to ground by the particles [52] and a "dip" will appear in the noise resonance of the tuned circuit (Fig. 5.3).

For large numbers of particles, the separation of the two peaks is proportional to the square root of the number of particles [52] (Fig. 5.3a is approximately 4000 antiprotons). For smaller numbers, the width of the "dip" is $N_p \gamma_z$, the number of particles times the linewidth of a single particle (Fig. 5.3b is 2 \bar{p}). In this linear regime, the Fourier transform of the voltage developed across the tuned circuit is (Eq. 3.13 of Ref. [9])

$$V_S^2(\omega) = \frac{4(\omega - \omega_z)^2}{4(\omega - \omega_z)^2 + (N_p \gamma_z)^2} v_n^2,$$
(5.5)

showing the Lorentzian line shape of the dip. Thus the number of particles may be determined by measuring the linewidth. For very small numbers of particles, however, fluctuations in the power supply broaden the width of the dip above the natural linewidth and reduce its depth. When working with small numbers of antiprotons the "dip" is a rapid technique to measure ν_z at very low energies which is proportional to the number of particles.

5.3 Detecting the Axial Motion of a Single \bar{p}

Resonantly driving the axial motion allows detection of a single antiproton. The motion must be driven such that the direct response from the drive does not mask that of the particle. Unlike the cyclotron motion, in which the drive is turned off before the measurement begins, the axial frequency is measured using a continuous drive. To avoid direct feedthrough, a two drive modulation scheme is used. The trap depth is modulated at $\nu_{mod} = 90909$ Hz and an endcap is driven at a frequency near $\nu_z - \nu_{mod}$ (Fig. 5.4). The \bar{p} responds at the sum frequency near ν_z , 91 kHz lower than the drive and hence easily separable from the drive with filtering and a FFT (Fig. 3.8). The induced voltage, which is proportional to the amplitude of the axial motion, may then be measured as a function of drive frequency (Fig. 5.5).



Figure 5.3: The damped motion of particles is observed as a 'dip' in the noise resonance in which the Johnson noise of the LRC tuned circuit is shorted by the particles. (A) is a large cloud of roughly 4000 antiprotons, which is most easily measured by looking for the 'dip'. The two \bar{p} 'dip' (b) is more easily measured using driven techniques.

Endcap Drive



Figure 5.4: Dual drives used to excite the axial motion of a \bar{p} or p.

Initially, when ν_z is not known to within one linewidth of the tuned circuit (~ 100 Hz), the voltage is swept rather than the drive frequency. As the signal voltage is proportional to the effective resistance of the tuned circuit at the response frequency, adjusting ν_z by sweeping the trap depth allows the maximum signal-to-noise ratio near the center frequency of the tuned circuit to be used through the entire measurement. For such a sweep, the drives are fixed such that their sum is the center frequency of the tuned circuit. The voltage is then swept using the adjustable voltage calibrator (Sec. 3.2.1), to change the resonant frequency of the antiproton. A much larger frequency span may then be covered than would have been possible at fixed voltage. However, the resolution of such a sweep is typically less than a frequency sweep. Therefore, once ν_z has been roughly measured, V_0 is fixed at the correct value and a high resolution measurement of ν_z .

5.4 Anharmonicity

Slight deviations from a quadrupole potential lead to amplitude dependent shifts in the axial frequency (Sec. 2.2). The leading correction to the potential is proportional to z^4 (Eq. 2.3) and (along the z axis) may be included in the equation of motion (Eq. (5.1) with the replacement $\omega_z^2 \rightarrow \omega_z^2 (1 + 2C_4 z^2/d^2)$. Therefore, the axial frequency of a \bar{p} depends upon the energy in the motion. The shift $\Delta \nu_z$ in ν_z is

$$\frac{\Delta\nu_z}{\nu_z} = \frac{3C_4E_z}{2(C_2)^2 eV_0}$$
(5.6)



Figure 5.5: Driven axial measurements for various compensation voltage settings. The drive frequency is swept both from low to high (black circles) and from high to low (white circles). For $C_4 < 0$ (a), ν_z shifts down as the energy increases and a large response is observed when the drive is swept down. In a well compensated trap (b), ν_z is independent of E_z and both curves are the same. For $C_4 > 0$ (c), a large response is observed sweeping up.

for a \bar{p} of axial energy E_z . C_4 has contributions both from the ring electrode and the compensation electrode and may be written as

$$C_4 = C_4^{(0)} + \frac{V_c}{V_0} D_4. ag{5.7}$$

By adjusting V_c , the voltage on the compensation electrodes, C_4 may be reduced to zero.

The axial frequency will also shift as the compensation voltage is adjusted to reduce C_4 . It must then be redetermined as the compensation voltage is changed, making the process of tuning the trap tedious. The open endcap trap has been "orthogonalized" [32] such that ν_z is independent of the voltage applied to the compensation electrodes (Sec. 2.2). In an orthogonal trap, $D_2 = 0$ and only $C_2^{(0)}$ (Eq. 2.3) contributes to the frequency. Measuring ν_z as the compensation voltage and trap depth are adjusted allows a measurement of D_2 as [31]

$$D_2 = C_2 \left(\frac{\partial \nu_z}{\partial V_c}\right) \left/ \left(\frac{\partial \nu_z}{\partial V_0}\right) = -0.0053 \pm 0.0002, \tag{5.8}$$

corresponding to slight shifts in ν_z over the normal range of tuning (Fig. 5.5). The compensation voltage may be adjusted over 10% of the trap potential before ν_z shifts out of range of the tuned circuit.

The amplitude of a harmonic oscillator at frequencies near its resonant frequency has a Lorentzian profile. In an anharmonic resonance, ν_z is "pulled" when the drive is swept in the direction ν_z shifts as its energy increases [53, 9]. If the drive is swept the other direction, only a small signal will appear before the axial frequency shifts out of resonance (Fig. 5.5a). Because the "pull" is asymmetric, the zero energy axial frequency is near one end of the resonance rather than at the center as in a Lorentzian line.

Uncertainty in the axial frequency must be less than 10 Hz to determine ν_c to 1 ppb by applying the invariance theorem (Eq. 2.2). Therefore, C_4 must be small enough that shifts in ν_z are less than 10 Hz. With the axial energy necessary for a good signal-to-noise ratio, the linewidth of ν_z is less than 10 Hz when $C_4 < 10^{-4}$ (Fig. 5.5). With care C_4 and corresponding shifts in ν_z may be reduced and order of magnitude less than this allowing a 1 Hz measurement of ν_z .

5.5 Conclusion

Several techniques are available for measuring the axial frequency. Large numbers of antiprotons may be measured while they remain in thermal equilibrium with the detector, while a single antiproton or proton is driven above the noise and detected using a narrow band FFT. The open endcap trap allows the potential to be compensated such that $C_4 < 10^{-5}$. Tuning can be performed with very slight changes in the axial frequency itself. Thus the axial frequency of a single antiproton or proton may be measured to a fractional accuracy of 10^{-6} , an order of magnitude better than is necessary to perform a 1 ppb measurement of the cyclotron frequency.

Chapter 6

Magnetron Motion

The magnetron motion (the large circle in Fig. 2.2) is a motion through crossed electric and magnetic fields such that the Lorentz force on the particle vanishes. Like a velocity filter, this motion is independent of the charge-to-mass ratio. The Lorentz force, $F = q(\vec{E} + \vec{v} \times \vec{B})$ vanishes when v = -E/B and is directed in a circle about the center of the trap, perpendicular to the radial electric field lines.

The magnetron frequency must be measured to determine the free space cyclotron frequency ν_c . The three measured frequencies ν'_c , ν_z and ν_m summed in quadrature using the Invariance theorem (Eq. 2.2). determine ν_c . Because ν_m is much smaller than ν_c , the magnetron frequency may be measured to a much lower precision. For our trap parameters, the magnetron frequency is

$$\nu_m = \frac{\nu_z^2}{2\nu_c'} \approx 5 \text{ kHz} \tag{6.1}$$

and need only be measured to a 30% fractional accuracy to determine ν_c to 1 ppb.

Because the electric field is directed outwards, a particle centered in the trap has its potential energy maximized. Increasing the magnetron radius corresponds to decreasing the kinetic energy in the magnetron motion. Thus the magnetron motion is unstable in the sense that any damping causes the magnetron radius to grow without bound. For sufficiently large trapping potentials (such that stray potentials are relatively small), the magnetron radius increases very slowly and a particle can be kept for days without magnetron cooling. The low velocity of the magnetron motion makes it difficult to measure by the tuned-circuit techniques applied to the cyclotron and axial frequencies. (An unacceptably large 1 mm magnetron radius has a velocity of 3×10^4 cm/s, 100 times smaller than the typically detected axial excitation.) If such a technique were used, it would also remove energy from the particle, increase its radius and eventually expel the particle from the trap. An alternate technique must, therefore, be used to determine ν_m and to reduce its radius.

6.1 Magnetron Cooling

A drive applied at $\nu_z + \nu_m$, known as a "sideband drive" [54] couples the magnetron and axial motions. When the magnetron radius is large the drive reduces the magnetron radius as it increases the axial amplitude. The increased axial energy then damps through its tuned circuit. This rate of this "magnetron cooling" is limited by the axial damping rate and the detuning from the resonance. The linewidth of the cooling resonance increases with the strength of the drive and can be quite broad at large powers. The magnetron frequency may be determined by comparing the difference between the drive frequency and an observed response near the axial frequency. In the process of loading a single \bar{p} or p, the magnetron frequency is routinely determined to better than 1 kHz, more than sufficient for this measurement.

Magnetron cooling has a limit related to the energy E_z in the axial motion by

$$E_m = -\frac{\nu_m}{\nu_z} E_z. \tag{6.2}$$

The expected minimum radius for our trap parameters is 4 μ m. This is a factor of 20 smaller than we have been able to observe directly through shifts in the cyclotron frequency (See below).

In an ideal trap in which the magnetic field lines are exactly parallel to the z axis and the electrodes have exact cylindrical symmetry, the magnetron frequency is given by

$$\tilde{\nu}_m = \frac{\nu_z^2}{2\nu_c'} = 5104.3 \text{ Hz}$$
 (6.3)

for our trapping parameters. The actual magnetron frequency ν_m differs slightly from $\tilde{\nu}_m$ by [9]

$$\nu_m \approx \tilde{\nu}_m \left(1 + \frac{9}{4} \theta^2 - \frac{1}{2} \epsilon^2 \right) \tag{6.4}$$

where θ is the small angle between the magnetic field and the axis of symmetry of the electric field and ϵ characterizes deviations of quadratic order of the cylindrical symmetry of the potential.

The magnetron frequency may be measured very accurately by sweeping a weak sideband drive through the resonance (Fig. 6.1). With drives exciting the axial motion (Sec. 5.3), the amplitude of the axial motion is monitored and the sideband drive is swept. When the drive is resonant, the amplitude of the response decreases [9]. The axial frequency is known exactly as it is driven and the magnetron frequency may be determined very accurately. The difference in the measured ν_m and calculated $\tilde{\nu}_m$ frequencies is -0.8 ± 0.1 Hz which is consistent with a 2% deviation from cylindrical symmetry in the potential.



Figure 6.1: Monitoring the amplitude of the driven axial motion as a weak magnetron cooling sideband drive is swept through resonance.

6.2 Effects of a Non-zero Magnetron Radius

A non-zero magnetron radius shifts the cyclotron frequency if the magnetic field has a quadratic magnetic bottle $(B_2\rho^2/4)$. (Linear gradients in the magnetic field average to zero during one magnetron orbit and thus do not contribute.) The measured magnetic bottle has $B_2 = 71 \text{ ppb/mm}^2$ (Table 9.4). This means that the magnetron radius must be reproducibly cooled to less than 0.1 mm, to perform a 1 ppb measurement. A proton is loaded (Ch. 7) with a magnetron radius of up to two mm which will shift ν_c by 100 ppb (Fig. 6.2). A secondary effect of a large magnetron radius is that the particle samples more perturbations of the trapping potential. For a sufficiently large magnetron radius, an axial signal is not observed (First 5 hours of Fig. 6.2). With a strong and well coupled drive, the magnetron radius can be quickly reduced. Figure 6.3 shows a single cyclotron measurement in which the magnetron motion is first heated to nearly 2 mm and then cooled to within 2 ppb of its original frequency in 20 minutes.



Figure 6.2: Cyclotron frequencies (ν'_c) measured over 10 hours as a weakly coupled cooling drive reduces the magnetron radius from almost 2 mm to less than 0.1 mm.



Figure 6.3: (a) A cyclotron measurement in which at first (i) no side band drive is applied, then (ii) a heating drive increases the magnetron radius to 1.8 mm, and finally a cooling drive decreases the radius (iii) such that the cyclotron frequency is within 2 ppb of the original value. (b) The residuals of a fit to the measurement excluding the heating and first 20 minutes of cooling.

Chapter 7

Loading Particles

To perform the comparison of charge-to-mass ratios of the antiproton and proton, the trap must be loaded with first a single antiproton and then a single proton. Antiprotons of 5.9 MeV are provided by the CERN accelerator complex, but must be slowed to sub-eV energies and cooled into the trap. More than 1000 particles are typically loaded, all but one of which must be removed from the trap to perform the measurement. Protons are loaded internally, and a single proton may be loaded directly. Other positive ions may also be loaded, however, and great care must be taken to assure that the single proton is the only particle in the trap.

7.1 Antiprotons

Antiprotons are produced at the CERN antiproton complex. The details of the production, capture and cooling of antiprotons have been described previously [34, 11, 12, 13] and only an outline will be provided here. Antiprotons, produced as protons strike an iridium target, are collected, cooled and accumulated in the AA/ACOL complex [55] before transfer to the Low Energy Antiproton Ring (LEAR). The an-



Figure 7.1: The CERN antiproton complex.

tiprotons are decelerated from 640 MeV/c momentum to 105 MeV/c (5.9 MeV kinetic energy) in LEAR and then extracted to our experiment where they are slowed in an aluminum degrader (as well as several gas cells for fine tuning the energy loss) [13]. When the degrader is optimally tuned, half the antiprotons annihilate inside the degrader, but the others exit with an energy spectrum which is relatively flat from 0 to 100 keV.

7.1.1 Loading Antiprotons

For trapping, approximately 10^8 antiprotons are delivered in a 200 nsec pulse. They slow in the degrader (the leftmost plate in Fig. 7.2a) while it is biased to $\approx +100$ V so that electrons are not liberated as the antiprotons pass through. As the particles enter the trap (Fig. 7.2b), the central electrodes of the 12 cm long trap are grounded, while the far electrode (the rightmost plate in Fig. 7.2a) is biased to -3 kV. All antiprotons with less than 3 keV of kinetic energy are repelled back towards the degrader by the upper electrode, while those with a higher energy hit the electrode and annihilate. The time for 3 keV antiprotons to traverse the trap and return to the degrader is 300 nsec. Therefore, less than one hundred nsec after the pulse of antiprotons has entered the trap, the degrader potential is quickly raised to -3kV [56] to capture the antiprotons in the trap (Fig. 7.2c). Such high energy antiprotons have been kept in this state for days. If the voltage on the upper electrode is ramped down, antiprotons will leave the trap at voltages reflecting their initial energies (Fig. 7.3).

To further cool the antiprotons from keV energies to the sub-eV energies where the measurement is performed, electrons are loaded into the harmonic region of the trap (central left region of the trap in Fig. 7.2a). These thermalize with the walls of the trap through synchrotron radiation from the cyclotron motion, and by dissipating energy in an amplifier resonant with the electron axial motion. The electrons are loaded from a field emission point (FEP) — a tungsten wire etched to a sharp point — located above the upper high voltage endcap (Fig. 7.2a), which generates a several nA electron beam when biased at 1 kV below neighboring electrodes. The electrons follow magnetic field lines down the trap until they hit the degrader where they produce gas which is then ionized by the electron beam. The secondary electrons from the ionizations inside the trap are captured. When the loaded electrons have been cooled, the usual noise resonance of the electron amplifier (Fig. 7.5a) breaks into two peaks (Fig. 7.5b). From the separation of the two peaks, the number of electrons may be determined [52]. For our trap the relation (for large clouds)



Figure 7.2: A pulse of 5.9 MeV antiprotons approaches the trap (a) and is slowed by the degrader as it enters the 3 kV trap (b). The antiprotons cool as they collide with electrons (c) loaded from the field emission point (FEP). When they have cooled into the harmonic well (d) the high voltages on the end electrodes may be lowered. The electrons are then pulsed out of the trap (e) leaving a trap full of antiprotons (f).



Figure 7.3: 53000 antiprotons dumped from the 3 kV high voltage well after a 100 second storage time. No electrons have been used to cool them into the harmonic well.

is

$$N_e = 6 \times 10^6 \left(\frac{\Delta\nu}{1\mathrm{MHz}}\right)^2,\tag{7.1}$$

where $\Delta\nu$ is the separation between the two peaks. For loading large numbers of antiprotons, a large electron cloud should be used. For clouds greater than 1 MHz, however, the trap can be unstable, resulting in all the antiprotons leaving the trap during the cooling process. The largest clouds that produced reproducible cooling were roughly 1.1 MHz. For loading a single antiproton, however, large numbers of antiprotons (and hence large numbers of electrons) are not needed. Typical cloud sizes used were around 800 kHz, corresponding to $4 \times 10^6 e^-$. Clouds of only a few 100 kHz have been used when the system is working smoothly and only a few antiprotons are desired.

As the antiprotons oscillate in the long trap, they collide with the electrons and lose energy. Over tens of seconds, the antiprotons cool into the harmonic trap with



Figure 7.4: 104000 antiprotons dumped from the harmonic well after stacking ten "shots" from LEAR (b) and the voltage applied to the ring to dump the antiprotons from the harmonic well (b).

the electrons (Fig. 7.2c). The high voltage may then be lowered to zero volts and the few remaining antiprotons that have not cooled into the harmonic region will spill out (Fig. 7.2d). Fractional cooling efficiencies (N_{cooled}/N_{total}) are usually greater than 90% and take roughly 60 seconds to achieve [34].

After the high voltage is ramped down (Fig. 7.2d), the harmonic well is still filled with electrons. With the two high voltage electrodes returned to their initial values (Fig. 7.2b), the trap is ready to load more antiprotons. By injecting another 200 nsec "shot" of antiprotons, they may be accumulated in the trap. Roughly 10 LEAR "shots" have been accumulated in this manner (Fig. 7.4b) with over 100000 antiprotons in the trap at the end of the process.



Figure 7.5: The noise resonance of the electron axial amplifier with an empty trap (a) and again with a large electron cloud used to cool antiprotons (b). The 700 kHz width corresponds to $\approx 3 \times 10^6$ electrons.

7.1.2 Removing Electrons from the Trap

After the high voltage electrodes are ramped to zero, the harmonic well contains roughly 10^6 electrons and 10^4 antiprotons. The electrons must be removed from the trap as they will drastically perturb the measurement. By briefly pulsing open the trap, the light electrons are forced out of the trap leaving solely the 2000 times heavier antiprotons behind. Before the pulse, the electrons are magnetron cooled (Sec. 6.1) to center them along the z axis and the trap voltage is lowered to 1 volt. (Because there is filtering on the AC coupled line that delivers the pulse to the endcap electrode, a pulse from a 17 V well depth would require a much larger pulse than is available from the function generator and would be more difficult to control.) The voltage is allowed to settle for roughly 45 seconds because of the low pass filters on the DC potential lines (Fig. 3.3). A 200 nsec pulse is then applied on the upper endcap using a microcoax cable (Fig. 3.4) which allows high enough frequencies to the ring that the pulse shape is not distorted. This brings the endcap roughly 1 volt more negative than the ring (Fig. 7.2e) and allows the electrons to escape the trap. Because of space charge screening within a large electron cloud, the pulse is fired three times before the voltage is returned to 17 volts. This pulsing process is then repeated as the trap is dipped to remove \bar{p} from the trap as described in the following section, to be sure there are no electrons in the trap.

7.1.3 Reducing the Number of Antiprotons

After pulsing out the electrons, 10^3 to 10^5 antiprotons remain in the trap (Fig. 7.2f), depending upon how many antiprotons were sent from LEAR. To perform a measurement with a single antiproton, all the others must be removed from the trap. Antiprotons with large axial or magnetron orbits are therefore allowed to spill from the trap as the well depth is slowly reduced. The cyclotron motion of the particles is first excited and used to count the number of antiprotons as the voltage is dropped. At relatively large voltages, (1 volt in Fig. 7.6a) there are so many antiprotons that individual particles cannot be resolved. When the potential is dropped further (0.4)volt in Fig. 7.6b), separate resonances from each particle can be resolved. After the voltage has been left low for several minutes, (at 0.25 volt in Fig. 7.6c) the well depth is returned to 18 volts so that ν_z is resonant with the amplifier and the axial motion of the remaining antiprotons may damp. The voltage is then returned to 1 volt and crept back down toward zero. When only one particle remains, the voltage is returned to 18 volts. To speed the process initially, a cyclotron drive may be swept over ν_c' to drive particles out of the trap at 1 volt. The strength of this drive must be adjusted very carefully, however, to drive out some, but not all, of the particles.



Figure 7.6: To remove antiprotons from the trap, their cyclotron motion is excited and the well depth is lowered (a) to 1 V. The voltage is lowered (b) as particles leave the trap. In (c) the trap is at 0.2 V with 4 antiprotons.



Figure 7.7: As the cyclotron energy of a \bar{p} damps, its frequency is shifted downwards from the expected value (solid curve) by a single H⁻ ion, also present in the trap.

7.1.4 Negative Ions

In addition to antiprotons and electrons, negative ions also form in the trap during the loading process. If the process of ejecting electrons and extra antiprotons is performed slowly enough, the extra electron on the negative ions is stripped and the neutral particles leave the trap. However, as the speed of the above process is increased, more negative ions remain. The effects of a single H^- on a cyclotron measurement (Fig. 7.7) is quite dramatic, making an accurate measurement of the cyclotron frequency impossible. However, the H^- opens the possibility of comparing it to the antiproton without the systematics associated with comparing particles of opposite signs of charge (Sec. 10.4).

7.2 Loading Protons

Protons are loaded directly from an internal source, requiring none of the accelerator complex necessary for producing antiprotons. As with loading electrons, an e^- beam from the FEP heats the degrader which liberates gas (mostly H, C, N and O) that is then ionized and captured in the trap.

To study the ions loaded along with the protons, a large current (≈ 50 nA) is fired for 10 minutes to load a large number of ions. During the loading process, the trap depth is tuned slightly below the voltage that brings protons into resonance with the axial detector so that no energy damps through the detector. The protons and other ions (with smaller charge-to-mass ratios) are left with the large axial energies with which they are loaded. Ions are then detected by sweeping the voltage over a broad range and measuring the power in the noise resonance. As the voltage shifts to bring an excited ion species into resonance with the detector, the ions dissipate their energy in the detector and a signal is observed (Fig. 7.8a). In addition to various charge states of carbon, nitrogen and oxygen, the other identifiable ion species are fluorine.

All ions other than protons must be kept out of the trap to perform the measurement. To do this we drive the axial frequencies of all ions except for protons during the time the FEP is firing. A 0 - 10 MHz noise source (from an SRS DS345) is used to generate the noise. To avoid driving protons out of the trap as well as ions, the drive is heavily filtered. A 3 pole low pass elliptic filter has a 3 dB corner frequency between $\nu_z(\text{He}^{++})$ and $\nu_z(p)$, and notches at $\nu_z(p)$ and $2\nu_z(p)$. In series are two very high Q notch filters [57] which attenuate at ν_z and $\nu_z - \nu_m$ of the proton so that the total attenuation at these crucial frequencies is greater than 90 dB. When an ion cloud is loaded with the drive to eliminate ions turned on, an ion scan performed



Figure 7.8: A mass scan of positive ions as well loaded with protons (a) when nothing is done to prevent it. When a broadband drive heats the axial motion of the ions (b), many fewer are loaded.

under the same conditions as discussed above reveals only protons (Fig. 7.8b).

7.2.1 Loading One Proton

To perform a cyclotron measurement, a single proton must be in the trap. If only one proton is loaded directly, the chances of other ions entering the trap along with it is minimized. Thus the technique used for antiprotons (many particles are loaded and then eliminated until only one remains) should not be used for protons. Instead, a single particle must be identified as it is loaded. The cyclotron signal (from an excited proton) is rapidly and easily identified. Our proton loading process, therefore, consists of applying the axial ion cleaning drive, producing a roughly 1 nA FEP current and then alternating between driving the proton cyclotron motion and turning off the drive (as the detector chain is saturated while the drive is on) and looking for a signal from a proton. When a cyclotron line is observed, the FEP and drives are shut off. After damping the p for several minutes to bring the axial motion into equilibrium with the amplifier near 4 K, the trap is "dipped" to approximately 0.4 volts. Unwanted ions which have not been thermalized with an amplifier (as their axial frequencies are much lower than that of the proton) will preferentially leave the trap.

Protons are usually loaded into large axial and magnetron orbits. For orbits whose radius is a significant fraction of the trap radius the frequencies are greatly shifted from their zero energy values. Therefore, to cool a single proton after it has been loaded, a voltage sweep is done with a strong magnetron cooling drive. A sweep of typically 60 mV (two kHz in axial frequency) is initially used. The cyclotron motion is monitored as the sweep is performed. As the voltage is swept, the cyclotron frequency shifts slightly due to the change in ν_z given by

$$\Delta\nu_c' = -\frac{\nu_z^2}{2\nu_c'} \Delta V_0 / V_0. \tag{7.2}$$

It will also shift due to the damping of the cyclotron energy. However, when the proton is resonant with the axial detector (and the magnetron cooling drive), the cyclotron frequency will shift erratically as the changing axial energy shifts ν'_c through the bottle. This provides a resonance condition to observe the axial frequency. When the proton is first loaded, the resonant voltage can be shifted by up to 30 mV from the expected voltage. As repeated sweeps are performed and the axial and magnetron motions are reduced, the resonance shifts to the expected value. At this stage the voltage is left on resonance with the magnetron cooling drive on until the cyclotron frequency stops shifting and no more magnetron cooling is observed.

7.3 Conclusion

Before a measurement of the \bar{p} or p cyclotron frequency may be performed, a single particle is loaded into the trap, cooled to the center, and contaminant ions are removed. This process typically takes several hours. As will be seen in chapter 8, the field drifts by up to several ppb per hour. Thus the faster the trap can be loaded, the less the field will drift and the better the measurement will be.



Figure 7.9: A voltage sweep to center a proton as the cyclotron frequency is monitored. The trap depth is swept with a magnetron cooling drive at full power. When the voltage brings the particle into resonance with the cooling drive, the axial energy increases and the magnetic bottle causes ν'_c to shift. The voltage is measured relative to the expected voltage. As the proton is damped, the resonance will shift to $\Delta V_0 = 0$.
Chapter 8

Magnetic Field Fluctuations

During the measurement of the antiproton and proton cyclotron frequencies, the **B** field and hence ν_c changes. Both changes in the ambient field, external to the apparatus, and drifts internal to the solenoid produce these fluctuations. External changes are substantially reduced by a superconducting coil whose geometry has been carefully chosen to cancel such fluctuations. Time varying fields are also reduced by eddy current screening in the copper dewar. Drifts internal to the apparatus are reduced by stabilizing the pressure of the cryogenic dewars and studied by monitoring residual shifts in pressure and flow rates from these dewars. While these techniques reduce the fluctuations in **B**, rapid switching between protons and antiprotons is still necessary to perform 1 ppb measurements.

8.1 External Fluctuations of the Field

An accelerator hall is necessarily a noisy magnetic environment. Within meters of our experiment, currents in magnets used to produce, store and deliver particles to experiments are constantly adjusted as various species of particles are cycled or the energy of the particles is adjusted. Detector magnets are also changed as different experiments become active and as systematic studies are performed. In addition to magnets, large cranes for assembling various experiments change the field dramatically when they approach. Two fluxgate magnetometers, located two meters from the apparatus monitor changes in the ambient field.

The largest shift observed with the magnetometer is from the final bending magnet in the beamline which delivers antiprotons to our apparatus. Only 1.5 meters below our solenoid, this magnet creates a 230 milligauss (mG) field at the experiment. This magnet is frequently adjusted to steer the beam into the trap. The current is returned to its nominal value while other experiments use the LEAR beam so that the stability and reproducibility of the magnet current is crucial.

During the antiproton production process (Sec. 7.1.1) protons are accelerated to 26 GeV in the Proton Synchrotron (PS) before striking a target to generate the antiprotons. During antiproton production (most of the time LEAR is running), this process is repeated as often as every 2.4 seconds. A brief 40 mG pulse is produced at our apparatus as the protons are accelerated in the PS ring whose closest dipole magnets are 20 meters from our trap.

The LEAR ring, which provides antiprotons for this measurement, receives antiprotons from the PS ring at roughly 200 MeV kinetic energy. The antiprotons are decelerated to 21 MeV for ejection to other experiments which run in parallel with our measurement and to 5.9 MeV for ejection to our trap. One of the four dipole magnets which keep the antiproton beam circulating in LEAR is 10 meters from the trap. The stray field from this magnet changes by up to 20 mG for several minutes every hour as antiprotons are injected and decelerated in LEAR.

Fields are generated by neighboring experiments as well as accelerators. When the current in the detector magnet of the CPLEAR experiment, 30 meters distant,

| Source | ΔB_{ext} | $\Delta B_{int}/B_0$ | Shielding |
|-------------------|-------------------|----------------------|-----------|
| TRAP Beamline | 230 mG | 80 ppb | 48 |
| PS magnets | $37 \mathrm{mG}$ | 6 ppb | 110 |
| Large Crane | $150 \mathrm{mG}$ | 18 ppb | 144 |
| PS200 trap magnet | $83 \mathrm{mG}$ | 10 ppb | 125 |

Table 8.1: Sources of external field fluctuations

is reversed, a few mG shift is observed. A neighboring superconducting magnet whose stray field is quite large produces fields of 80 mG at our apparatus, but is quite stable when operated in persistent mode. Above the experiments of the hall is a large rolling crane which produces a 150 mG field shift when directly overhead.

All of these fluctuations are monitored on the magnetometers and stored along with cyclotron data during measurements. The magnetometer readings are also continually written to a strip chart recorder which is monitored by eye so that large changes in the field can be quickly recognized.

8.2 Self Shielding Solenoid

Fluctuations in the ambient magnetic field of 100 mG correspond to shifts of 2 parts per million in the 5.9 Tesla field from the superconducting solenoid. To perform a 1 ppb measurement, these fluctuations must be reduced by a factor of 2000. Several possibilities exist for removing these fluctuations. Data taking is stopped during large fluctuations in the field such as a crane moving nearby. However, fluctuations from the PS must be eliminated in another way as it is almost always ramping. Magnetic shields cannot screen the external fluctuations as the field is never low enough between the solenoid and the last bending magnet of LEAR to use μ -metal shields. Active Helmholtz coils could also be used in a feedback loop with



Figure 8.1: Fluctuations in the ambient magnetic field from PS and LEAR cycles. A normal LEAR cycle takes 10-15 minutes rather than the 3 minutes shown here. (Figure from [58])

a magnetometer to sense and correct for changes in the external field. While such a device has been constructed [59], the extreme precision with which the external field must be stabilized makes drifts in the power supply driving the coil a significant problem. The solution chosen is to utilize a basic property of a superconducting loop: the magnetic flux through such a loop is constant. By correctly choosing the geometry of a superconducting loop such as those of our solenoid, flux conservation can be used to screen changes in the external field [60].

While the details of the self-shielding solenoid are discussed elsewhere [58, 60, 61] simple arguments suffice to demonstrate the basic principles involved. A closed loop of superconductor conserves flux, $\Phi = \int B \times dA$. If the z component of the field at the center of the coil, $\mathbf{B}_z(0)$, equals the field averaged over the cross sectional area ($\overline{\mathbf{B}_z} = \Phi/A$) then flux conservation will amount to field conservation at the central location. That a solenoid can be constructed with this property can be seen by considering two limiting cases. The field at the center of a single loop of superconductor is less than the field at the edge and hence less than the average value. The reverse is true for a long solenoid as fringing fields reduce the integral of the field over the total volume. Hence, for a solenoid with the proper intermediate aspect ratio, flux conservation is equivalent to field conservation. A real magnet constructed this way would not have all the desired properties of field uniformity. The solenoid used in this experiment is built instead following the standard design, with an additional coil added of the proper aspect ratio to produce the self–shielding property.

The shielding of stray fields would be perfect except that the position of the coil as it contracts when the solenoid is cooled to 4 K is not known exactly. The measured shielding for a uniform external field is 156 for one similar solenoid [58] and 175 for another. Gradients in the fluctuating fields from nearby sources also reduce the shielding by the solenoid. PS fluctuations are screened by a factor of 110 [33] and the solenoid from a neighboring experiment by 125 (Table 8.1). The ratio S of fluctuations in the ambient field at the magnetometer, 2 meters from the trap, to those inside the solenoid is extracted for the bending magnet immediately below the solenoid by monitoring the ambient field, ΔB_{amb} , during a cyclotron measurement (Fig. 8.2). Fitting $\nu(t)$ for the cyclotron measurement and $\Delta B_{amb}(t)$ to

$$\nu(t) = \nu'_c (1 + \frac{\Delta B_{amb}(t)}{S B_0}) - \Delta \nu'_c e^{-t/\tau_c}, \qquad (8.1)$$

where B_0 is the main field of 5.9 T, determines a ratio of S = 45 as well as the usual parameters of the cyclotron measurement (Ch. 4). The large gradients in the field from the bending magnet may reduce the field at the magnetometer from



Figure 8.2: A ν'_c measurement with one proton (a) during which a bending magnet is adjusted to deliver antiprotons to our apparatus and returned to its normal state. After the exponential shift has been removed, ν'_c changes by only 4 ppb (b), while the ambient field, monitored with a magnetometer 1.5 meters from trap, (c) changes by 200 ppb (of 5.85 T).



Figure 8.3: A cyclotron measurement as a large crane moves overhead, stopping in two places (a), and the shift in the ambient field (b) over the same interval allow the determination of a shielding factor of 140 for the field from the crane.

that at the solenoid and thus reduce the apparent shielding factor of the coil. The measured ratio, S, is the calibration needed to determine the effect on a particle of external fields observed with the magnetometer. The ratio measured for shifts from the large overhead crane (Fig. 8.3) is 140. As the crane is farther from the experiment, field differences between the magnetometer and the solenoid should be smaller. While large changes in the ambient field will disturb a measurement, the fluxgate magnetometers are easily sensitive enough to detect fluctuations that will produce 1 ppb shifts inside the solenoid (Fig. 8.2b).

8.3 Screening of Rapidly Varying Fields

After accounting for the self-shielding, the field should shift by no more than 10 ppb as the PS magnets ramp up and down (Fig. 8.1). Eddy current shielding, however, further reduces the strength of a time varying magnetic field as it enters a conductor. The magnitude of a magnetic field H with angular frequency ω is exponentially screened (i.e. $H = H_0 \exp(-z/\delta)$) with a skin depth δ given by¹

$$\delta = \sqrt{\frac{2}{\mu\omega\sigma}},\tag{8.2}$$

where μ is the permeability and σ the conductivity of the material. For room temperature copper, the skin depth at 1 Hz is 7 cm – much thicker than the total copper in the dewars. As high purity copper is cooled to near 4K, however, its conductivity can increase by over a factor of 100, decreasing the skin depth to the level where 1 Hz fluctuations are very effectively screened.

A calculation of the screening factor for a cylindrical geometry is more involved

¹e.g. Eq. 7.77 of [62]

[63] than for the infinite conducting plane discussed above. Utilizing the calculations in [63], a lower bound on the screening of the fundamental frequency of the PS field ramps (T=2.4 sec $\omega = 2.6 sec^{-1}$) is roughly 10 with higher harmonics of the cycle (which contain a significant fraction of the power as the pulses are sharp) screened much more. Therefore, oscillations in the field from the PS which penetrate to the trapping region are less than 1 ppb.

Oscillatory fields which penetrate the copper create sidebands on ν_c rather than shifting its central value. A Fourier component of a time varying field which penetrates to the region occupied by the particles of strength and frequency $B_1 \cos(\omega_1 t)$ will produce a cyclotron frequency

$$\nu_c' = \frac{eB_0}{m} \left(1 + \epsilon \cos(\omega_1 t) \right), \tag{8.3}$$

where $\epsilon = B_1/B_0$. Provided that $\epsilon \omega'_c/\omega_1 < 1$ (which is satisfied, even by the fundamental frequency of the fields from the PS), sidebands will form around the central peak without shifting or broadening it. As the PS magnets ramp from zero to maximum current, however, changing the duty cycle will change the average value of the field, and thus shift ν_c as well as create sidebands. The carefully monitored (Fig. 8.1) PS cycle is rarely changed.

8.4 Internal Field Drifts

Fluctuations internal to the 5.85 Tesla solenoid also change the field. To study these fluctuations, the field is monitored with a single proton (or antiproton) by repeatedly measuring ν_c . Using the same particle for all the measurements eliminates potential systematic effects associated with loading particles as well as reducing the



Figure 8.4: Three days of cyclotron frequency (ν'_c) data measured with one proton.

time between measurements. Typical fluctuations in ν_c over periods in which the external, ambient field is stable show smooth drifts of less than 3 ppb/hour within a 50 ppb range over more than a week (Fig. 8.4).

To maintain this low level of field drift, the gas pressure above the dewars is carefully regulated. The cryogenic system comprises four dewars. Separate helium dewars for the trap and the solenoid allow the the trap to be brought to room temperature with the solenoid left at high field. Each helium dewar has a complementary nitrogen dewar which lessens the heat load on the helium. An electronic system (Fig. 8.5 is a block diagram) regulates the pressure of the dewars and monitors the pressures and flows in the system. When the measurement was performed, the two helium dewars and the magnet nitrogen dewar were regulated, while the trap nitrogen dewar was vented directly to atmosphere. Since that time, a larger valve has been added on the magnet nitrogen regulator allowing regulation of the two nitrogen dewars in parallel.

Differential pressure transducers monitor both magnet dewars. While the measurement described here was performed, the trap helium dewar was regulated with a simple absolute pressure sensor which has less absolute sensitivity than a differential sensor, but was easier to construct. (It has since been replaced with a differential sensor.) A cavity at the desired gas pressure is the reference for the differential sensors. The cavity is both leak tight and temperature stabilized so that its pressure remains constant. (A bypass used to set the reference pressure is not shown in the figure.) Electronic valves then regulate the flow of helium and nitrogen gas out of the magnet dewars to maintain the same pressures in the dewars as the reference cavity.

The partial pressure of a gas above a boiling liquid determines the boiling temperature. Pressure fluctuations can therefore change the temperature (and hence the magnetization) of materials near the trap which are heat sunk to the dewar. For example, the paramagnetic MACOR spacers surrounding the trapping region shift the magnetic field by approximately 1×10^{-5} of the total field [33, 64]. As the magnetization of paramagnetic materials, B_M , is inversely proportional to temperature, it changes very rapidly as the temperature approaches zero. The field then depends on the pressure, P as

$$\Delta B_M = -\frac{B_M}{T} \frac{\partial T}{\partial \mathbf{P}} \Delta \mathbf{P}.$$
(8.4)

Near one atmosphere, the change in the boiling temperature of helium with pressure is 1.5 mK/torr [65] and hence the change in field as a fraction of total field is given



Figure 8.5: Block Diagram of cryogenic regulation, recovery and monitoring system.



Figure 8.6: Schematic diagram of second order lock loops regulating the helium gas pressure (a) and the temperature (b) of the pressure reference cavity. (Thanks to Lisa Lapidus and Daphna Enzer for design and construction of this pressure regulation system.)

by

$$\frac{\Delta B_M/B_0}{\Delta P} = 3 \times 10^{-8} \text{ Torr}^{-1}.$$
(8.5)

The pressure of the trap helium dewar should, therefore, be regulated to a fraction of a Torr to keep field fluctuations below 1 ppb.

The magnet and trap dewars are in fact regulated to better than 100 mTorr at the exhaust of the dewars. While large changes in ambient temperature or pressure can cause the regulation system to fail, leading to large changes in the field, continuous monitoring of the regulation system and ambient variables allows us to exclude data under such circumstances.

To study the actual dependence of magnetic field on the pressure of the dewars, cyclotron frequencies are measured as a function of pressure. Sudden changes in the pressure can lead to large changes in the flow of gas from the dewar and correspondingly large changes in the field. Therefore, the pressure must be changed very slowly to measure a dependence of field upon pressure without effects from changing flow rates. An increase in pressure of 8 Torr/hour was created by closing the exhaust to the dewar which cools the solenoid coils and allowing it to pressurize (Fig. 8.7a). After five hours, the flow was restarted and the pressure held constant at this new value. Even with this extremely slow change in the pressure, a two hour time lag was observed between the stabilization of the pressure and that of the field. As both the pressure change and $\Delta\nu'_c$ are linear in time, the slopes determine a correlation (Fig. 8.7b) of 1.5 ppb/Torr which is in agreement with other measures of the pressure dependence of the magnet dewar [66].

Gas cooling is also used to reduce the amount of helium used by the apparatus. Structures whose temperatures are between those of liquid helium and room temperature are cooled by the gas boiling off of the dewar as well as by conduction from the dewar. If the pressure of the gas is changed suddenly, the flow rate will change as a new equilibrium is established and this can change the amount of gas cooling above the dewar and hence the temperature distribution. While differential contraction of materials around 4K is typically quite small, the support structure which is gas cooled can be considerably warmer and thus changes in differential contraction may play a more significant role. While the precise mechanism by which **B** depends upon the flow rate is not well understood, a large correlation is observed (Fig. 8.8). Fluctuations of up to 10% in the total flow rate of 0.2 mg/s (or 8 l/h of gas) from the solenoid helium dewar are observed with a corresponding shift in **B** of 7 ppb/%. As can be seen in the figure, these changes happen smoothly over many hours.

There are several causes of the fluctuations in the helium flow rate observed in the magnet dewar. Changes in ambient temperature alter the heat load on the dewar. For conduction of heat, the fractional change in power, P, dissipated in the dewar as the ambient temperature, T varies is

$$\frac{\Delta \mathsf{P}}{\mathsf{P}} = \frac{\Delta T}{T}.\tag{8.6}$$

Thus the heat load from conduction processes should change by a few percent for typical temperature fluctuations. Radiative heating scales as T^4 , leading to fractional changes in power four times bigger than that given for conductive losses, possibly reaching the 10% level needed to explain the data in figure 8.8b.

Small changes in the dewar pressure can also lead to changing flow rates with a constant heat load on the dewar [67, 68]. Changes in the internal energy of the liquid ΔU_l are given by

$$\Delta U_l = C_v m_l \Delta T \tag{8.7}$$



Figure 8.7: The **B** field (measured with a particle) and the pressure above the dewar of helium as the dewar pressurizes (a) and the correlation of 1.5 ppb/Torr between field and pressure (b) in equilibrium for slow changes.

where m_l is the mass of the 50 l of helium in the dewar, ΔT the change in temperature and C_v the specific heat. The change in temperature of the boiling helium may be expressed in terms of the pressure of the gas phase as $\Delta T/\Delta P = 1.5$ mK/torr near 1 atmosphere, leading to a dependence of the internal energy of the dewar on the pressure of 22 Joules/Torr.

The energy Q removed from the bath by the boiling helium is

$$\frac{dQ}{dt} = \Delta_{vap} \frac{dm_f}{dt} \tag{8.8}$$

where m_f is the mass of helium and Δ_{vap} the heat of vaporization of helium. The typical flow rate of 8 l/h of gas, corresponds to 40 mW of cooling by the dewar. This energy may also be stored by increasing the internal energy of the dewar, however. A 2 mTorr/s increase in pressure will absorb this energy, bringing the flow rate to zero. Smaller drifts in the pressure will affect the total flow rate out of the dewar to a lesser degree.

To keep field drifts from this mechanism below 1 ppb/hour, the flow must be regulated to 0.1%/hour. Pressure drifts must be less than 7 mTorr/hour and the ambient temperature should be regulated to a fraction of a degree. Alternately, the flow could be stabilized directly by adjusting the heat load on the dewar (using a resistive heater) in response to the measured flow. While attempts to regulate the flow (as well as the pressure) might be used at a later date, the flow is only monitored at the present time.



Figure 8.8: ν_c measured over three days (a) and the fractional variations in magnet helium flow rate (b) about a 0.2 mg/s average. The correlation between ν_c and the gas flow rate (c) is 75 ppb/(l/h)

8.5 Conclusion

Shielding the trap from changes in the external magnetic field and stabilizing the pressures of the cryogenic dewars stabilizes the field in the trapping region. Monitoring the ambient field and the pressures and flow of the cryostats allows a determination of when the field is stable. While more heroic efforts in the future may further reduce the fluctuations, 1 ppb measurements may presently be performed (as will be discussed in chapter 10) by switching rapidly enough between antiprotons and protons to measure the ppb/hour drift in the magnetic field.

Chapter 9

Magnetic Gradients

Comparing the charge-to-mass ratios of antiprotons and protons with their cyclotron frequencies relies on the assumption that the two particles are in the same magnetic field. As the charge of \bar{p} and p have opposite signs, they require externally applied trapping potentials of opposite sign. The potential experienced by the particle may not be perfectly reversed by inverting the applied voltage due to small uncontrolled voltages such as the patch effect on the inner surfaces of the trap electrodes. During a mass measurement the \bar{p} and p thus reside at slightly different locations, as if an unchanging offset potential was applied to trap electrodes to either side of the particle. If the magnetic field were uniform, the two species would still see the same magnetic field. However, an inhomogeneity in the magnetic field leads to different ν_c without a difference in the charge-to-mass ratios.

9.1 The Magnetic Field

When the 5.85 Tesla superconducting solenoid was energized, the currents in 9 superconducting shim coils were adjusted to minimize magnetic inhomogeneities

using an acetone NMR probe. This 1 cm³ probe was placed at the center of the the solenoid to measure the field as the magnet was energized before the trap was put in place. While the line width obtained from the NMR probe was 50 ppb, magnetic inhomogeneities in both the probe and the trap itself made the gradients observed by a \bar{p} or p substantially larger. This was first observed when a smaller NMR probe was placed in the magnet and the line width increased to 500 ppb despite a 100 times smaller volume. The magnetic field was, therefore, reshimmed with the trap in place, using both a proton and antiproton as a probe.

9.1.1 Moving a Particle

To measure the gradients with an antiproton or proton, the particle must be moved from the center of the trap. A differential potential applied across the two (otherwise grounded) endcaps (Fig. 9.1a) moves particles in the z direction. Voltages $(+V_A/2)$ and $-V_A/2$ are applied to the endcap electrodes (Sec. 3.2.1 and Fig. 3.3). A battery is used to reduce noise and to eliminate any possible ground loops between power supplies which could be created by using an additional solid state power supply. A single current through a balanced set of 1 M Ω resistors provides equal and opposite voltages on the electrodes. In response to this applied voltage, the equilibrium position of a particle along the \hat{z} axis of the trap shifts by (Sec. 2.3)

$$\Delta z = -\frac{c_1}{C_2} \frac{d^2}{2z_0} \frac{V_A}{V_0}.$$
(9.1)

Applying the calculated values of the trap coefficients into the above equation gives an expected shift in position of 77 μ m/V.

Asymmetric voltages are applied across opposing segments of the quad split ring (Fig. 9.1b) to move a particle in a radial direction. The two opposing electrodes on



Figure 9.1: Asymmetric voltage applied to the endcaps (a) to move particles axially in the trap and the asymmetric component of the voltages applied to the ring (b) to move particles in the x direction. (Similar voltages on the other quadrants move the particle along y.) The standard trapping potential V_0 applied to the ring must be superposed upon these asymmetric potentials (see text). The details of the the wiring used to apply these voltages may be found in Fig. 3.3 and 3.5

Table 9.1: Measured dependence of ΔV_0 on V_A ($\Delta V_0 = \alpha V_A^2$) from a \bar{p} and p in the two orthogonal radial directions (with uncertainty in the last digit in parenthesis) may be compared to a calculated value of 17.4 mV/V².

| species | direction | curvature (α) |
|-----------|-----------|--------------------------|
| p | x | $18(1) \text{ mV/V}^2$ |
| p | y | $12(1) \text{ mV/V}^2$ |
| \bar{p} | x | $16.9(1) \text{ mV/V}^2$ |
| \bar{p} | y | $17.3(1) \text{ mV/V}^2$ |

the axis in which the particle is moved are at potentials $V_0 + V_A/2$ and $V_0 - V_A/2$ while the other two electrodes are left at V_0 . The voltages are applied with batteries and resistive dividers as in the axial case. The complexity of the ring wiring (Fig. 3.5), however, requires the use of two sets of batteries the y direction. The potentials produced when such voltages are applied were discussed in chapter 2. The calculated shift in equilibrium position when such a potential is applied (Eq. 2.14) is 112 μ m/V.

The well depth at the new equilibrium position is less than that at the trap center, shifting ν_z by many linewidths of the axial detector. As a result, neither the axial nor magnetron motions are damped when ν_z is not resonant with the detector. Unchecked by damping forces, the amplitudes of these motions may grow causing shifts in ν_c through anharmonicity (Sec. 5.4) and the magnetic bottle (Sec. 6.2). Therefore, as the particle is moved, ν_z is kept fixed and resonant with the detector by adjusting the the trapping voltage V_0 . Combining the shift in ν_z as the position is shifted (Eq. 2.8), with $\Delta \nu_z / \nu_z = \Delta V_0 / 2V_0$, the change in ν_z for a small change in the trap depth, gives an expected change in trapping potential of [31]

$$\frac{\Delta V_0}{V_0} = -\frac{3}{2} \left(\frac{d}{z_0}\right)^4 \frac{c_1 c_3}{(C_2)^2} \left(\frac{V_A}{V_0}\right)^2 \tag{9.2}$$

to keep ν_z constant (Eq. 9.2 is for the \hat{z} direction) may be determined. Comparing



Figure 9.2: Change in V_0 needed to keep ω_z constant when a particle is moved off-axis. The squares are p points and the circles are \bar{p} .

the measured and calculated shifts in ΔV_0 provides a first test of the calculation of the asymmetric potential. The measured and calculated parameters (Table 9.1) are in good agreement. A small linear term in the curve of figure 9.2 is due to slight imbalance in the resistors used to apply V_A .

9.2 Measuring the Field Difference

Because the magnetic field drifts at up to several ppb per hour (Ch. 8), the cyclotron frequency can change by more than 1 ppb during a single measurement of ν_c . In the several days needed to measure the gradients in three orthogonal directions, the field can drift by more than 10 ppb. To minimize the effects from these drifts, the field difference $\Delta \nu_c(\mathbf{r}) = \nu_c(\mathbf{r}) - \nu_c(0)$ is measured rather than the total field $\nu_c(\mathbf{r})$. The difference is determined during a single 20 minute cyclotron measurement (Fig. 9.3) further minimizing the effects of field drifts. In a gradient measurement, the particle's equilibrium position starts at the center of the trap (region *i* of Fig. 9.3) is then moved off the center (region *ii*) and finally returned to the center (region *iii*). The change in cyclotron frequency, ν'_c , when the particle is moved from 0 to \mathbf{r} , $\Delta \nu_{\mathbf{r}}$, is then determined along with the usual parameters (ν'_c , $\Delta \nu'_c$ and τ) when the data is fit (solid line in Fig. 9.3) to

$$\nu(t) = \nu'_c - \Delta \nu'_c e^{-t/\tau} + \begin{cases} \Delta \nu_{\mathbf{r}} & \text{particle at } \mathbf{r} \\ 0 & \text{particle at center} \end{cases}$$
(9.3)

The measured frequency shift, $\Delta \nu_{\mathbf{r}}$, can only be directly interpreted as the change in ν_c and hence field if ν_z is unchanged. Therefore, the trapping voltage is changed to keep ν_z constant (Eq. 9.2) as the particle is moved off-center. Adjusting ΔV_0 so that $\Delta \nu_z < 1$ Hz, insures that $\Delta \nu'_c = \Delta \nu_c$ to 0.1 ppb from the invariance theorem (Eq. 2.2). Therefore, before a gradient measurement is performed, the particle is moved to \mathbf{r} and V_0 adjusted to ensure that $\Delta \nu_z$ is sufficiently small. While a particle is at the trap center in a gradient measurement, V_0 is left at its normal value (regions *i* and *iii*). In region *ii*, the voltage is shifted to keep ν_z constant. When $\Delta \nu_z = 0$, the change in ν'_c , $\Delta \nu_{\mathbf{r}}$ is also the change in ν_c and hence proportional to the change in magnetic field.

By moving the particle in three orthogonal directions (x, y and z), the linear gradient of B was measured. After the trap had been shimmed only with the NMR probe, gradients of 10 to 30 ppb/volt (Table 9.2) were observed. As offset potential differences between the two endcaps or segments of the ring of up to 0.1 V



Figure 9.3: In a gradient measurement, the equilibrium position of the particle starts at the center (i). At t = 5 min, the particle is moved up in the trap, 0.5 mm (ii). At t = 10 min the particle is returned to trap center (iii) and the cyclotron measurement is finished.

| direction | gradient | |
|-----------|--------------|----------------------|
| x | 12 ppb/volt | 7 mG/mm |
| y | 27 ppb/volt | $16 \mathrm{~mG/mm}$ |
| z | 24 ppb/volt | $19 \mathrm{~mG/mm}$ |

Table 9.2: Magnetic field gradients measured with \bar{p} after NMR shimming.

were expected, gradients of 20 ppb/volt could produce 4 ppb systematic differences between the \bar{p} and p cyclotron frequencies. (A 0.1 volt offset shifts ν_c of a p by 2 ppb and ν_c of a \bar{p} by 2 ppb in the other direction.) To perform a 1 ppb measurement, the gradients had to be further reduced.

9.3 Shimming

Three of the superconducting shims in the solenoid generate magnetic fields with linear gradients in three orthogonal directions. The **X**, **Y** and **Z** shims produce magnetic fields with constant gradients in three linear directions. By adjusting these three shims, the magnetic gradient at trap center may be eliminated leaving only a second order curvature as the deviation in field. The calibration of these coils was specified by the manufacturer as 0.72 gauss/cm/amp [69] — changing the current by one amp changes the slope of the field in **X**, **Y** or **Z** by 0.72 gauss/cm without changing the field at the center of the solenoid. Deviations from ideal behavior in the shims include slight nonlinearities which lead to mixing between the three shims and an offset between the trap center and shim center causes changes of several ppm in ν_c when a shim is changed by an amp. The **Z** direction corresponds to the z coordinate of the axial motion. The **X** and **Y** directions, however are rotated from the x and y directions in which a particle may be moved, radially.

Table 9.3: Strengths (in mGauss/volt/amp) of \mathbf{X} and \mathbf{Y} superconducting shims in the x and y trap directions as determined from Fig. 9.4.

| trap | $^{\rm sh}$ | im |
|------|--------------|--------------|
| axis | \mathbf{X} | \mathbf{Y} |
| x | -4.26 | -5.00 |
| y | -5.05 | 4.26 |

As the accuracy of the shim calibration, our conversion from applied voltage to distance and the angle between the **X** shim axis and the x trap axis were all uncertain, the shims had to be calibrated before they could be adjusted to reduce the gradients in the trap. Thus, the x and y gradients were measured for various **X** and **Y** shim settings (Fig. 9.4). At various shim settings (on the x axis of the graphs), two components of the gradient of the magnetic field are shown. Each point is determined by averaging $\Delta \nu_c/V_A$ at $\pm V_A$. The slopes of each of these plots is listed in table 9.3.

The angle between the x trap axis and the X shim is therefore $\theta = 230^{\circ}$ and the measured strength of both the X and Y shims is

$$\frac{\partial (\Delta B / \Delta V)}{\partial I} = 6.6 \text{ mGauss/Volt/Amp}, \qquad (9.4)$$

The conversion from volts to distance in the radial direction (Eq. 2.14) of 0.1 mm/volt leads to 66 mG/mm/Amp, 10% smaller than the nominal value specified by the manufacturer. After several iterations of shimming (Fig. 9.5), the gradients were reduced by an order of magnitude (Table 9.4). The x and z directions have very small gradients, although the curvature is quite visible. The radial y direction, however, has a much larger gradient, due in part due to our lack of understanding of the nonorthogonality of the shims — that adjusting the \mathbf{Z} shim slightly changed the



Figure 9.4: Calibration of ${\bf X}$ and ${\bf Y}$ superconducting shims from gradient measurements

Table 9.4: Gradients and curvatures measured with a \bar{p} after the final shimming. The fractional change in the field, $\Delta B/B_0 = b_1 V_A + b_2 V_A^2$ for each of three orthogonal directions (x, y and z) where B_0 is the 5.85 Tesla full field,

| direction | gradient (b_1) | curvature (b_2) |
|-----------|------------------|-------------------------------|
| z | 0.0 ppb/V | $+0.4 \text{ ppb/V}^2$ |
| x | -0.2 ppb/V | $-0.7 \text{ ppb}/\text{V}^2$ |
| y | -2.4 ppb/V | $-0.1 \text{ ppb}/\text{V}^2$ |

gradient in the **X** direction for example. With more iterations of adjusting the shims and remeasuring the gradients, the 2.5 ppb/volt gradient in the y direction could be further reduced. The several week period for each iteration led us to perform the charge-to-mass ratio comparison of the antiproton and proton without further shimming. The systematic error associated with this gradient will be discussed later (Sec. 9.6).

9.4 Magnetic Bottle

The "magnetic bottle" is the leading axially symmetric component of the inhomogeneous magnetic field. This distortion of the homogeneous field of the ideal Penning trap, makes the frequencies ν_z and ν'_c shift as the energies E_c and E_z are increased. The \hat{z} component of the field of a bottle is symmetric about the z axis and links the curvature of the magnetic field in the z direction to the average of those in the x and y directions according to

$$\Delta B = B_2 \left(z^2 - \frac{x^2 + y^2}{2} \right). \tag{9.5}$$

(There are also terms proportional to $x^2 - y^2$ and xy in a multipole expansion of the field which break rotational symmetry, but do not affect the average radial



Figure 9.5: Magnetic field map with a \bar{p} after shimming in the (a) axial, (b) x and (c) y directions.

curvature.) Comparing the curvature in the the two radial directions (Table 9.4) to the z curvature we see that the average radial curvature is $-0.4 \text{ ppb}/\text{V}^2$, in agreement with the value for the z direction.

This inhomogeneous field is referred to as the bottle because it changes the axial well depth, shifting ν_z by $\Delta \nu_z$ in proportion to E_c , the cyclotron energy (Eq. 10.18 of [9]):

$$\frac{\Delta\nu_z}{\nu_z} = \frac{B_2}{B} \frac{E_c}{2M\omega_m\omega_c'}.$$
(9.6)

This coupling has been used extensively to measure g - 2 of the electron [70]. The bottle in such experiments was typically of order 150 G/cm², created by adding ferromagnetic material to the trap. In our apparatus, the observed bottle is 300 times smaller (Table 9.4) at only 0.5 G/cm². Our bottle is also small in the sense that the shift it causes in ν'_c , proportional to E_c [9],

$$\frac{\Delta\nu_c'}{\nu_c'} = -\left(\frac{\omega_z}{\omega_c'}\right)^2 \frac{B_2}{B} \frac{E_c}{2M\omega_m\omega_c'},\tag{9.7}$$

is 40 times smaller than the equivalent shift from relativity (Eq. 4.3).

An alternate technique can be used to measure B_2 during a single cyclotron decay. Replacing E_c in Eq. 9.6 with the relativistic shift in the cyclotron frequency (Eq. 4.3) yields an expression for B_2 in terms of the shifts in the axial and cyclotron frequencies caused by the same cyclotron energy

$$\frac{B_2}{B} = -\frac{\Delta\nu_z}{\Delta\nu'_c} \frac{\omega'_c \omega_z}{c^2}.$$
(9.8)

When the constants are evaluated, $B_2 = -3.73 \,\Delta \nu_z /\Delta \nu'_c \text{ ppm/cm}^2$. To measure $\Delta \nu_z /\Delta \nu'_c$ the axial frequency must be measured at different cyclotron energies. A typical axial measurement (Ch. 5) made by driving and detecting the axial frequency

directly takes a significant fraction of an 8 minute cyclotron damping time. Care must be taken to ensure that corresponding pairs of ν_z and ν'_c are determined at the same time. Instead, we drive the axial motion and detect the resulting shifts in the cyclotron frequency due to the magnetic bottle which are proportional to the axial energy E_z .

In a single cyclotron measurement (Fig. 9.6a) ν_z may be measured at different cyclotron energies by applying several axial drives (five in Fig. 9.6) with which the axial motion will resonate as E_c decreases. As E_z increases while ν_z is resonant with the drive, ν'_c shifts, causing the "blip" seen in the figure. By plotting the cyclotron frequencies where the axial frequency resonates with the axial drive frequencies (Fig. 9.6b), $\Delta \nu_z / \Delta \nu'_c$ and hence the bottle may be determined. The slope in figure 9.6b is 2.00 ± 0.04 Hz/Hz, corresponding to a bottle of 7.5(1) ppm/cm² or 0.44(1) Gauss/cm² with uncertainties in the last digits.

A second check of the conversion factor between asymmetric voltage applied to move the particle and the actual distance moved can be performed by comparing the bottle determined by moving a particle in the trap with $\Delta \nu_z / \Delta \nu'_c$, also based on the magnetic bottle, but independent of asymmetric potentials. Comparing the 0.46 ± 0.01 ppb/V² axial curvature to the 7.5 ± 0.1 ppm/cm² bottle measured above, leads to a conversion factor of $110 \pm 10 \ \mu m/V$ in both radial directions, which is once again consistent with the calculated value of $112 \ \mu m/V$ (Sec. 2.3). Using the curvature in the axial direction, one finds a calibration of $80 \pm 10 \ \mu m/V$, in good agreement with the calculated value of $77 \ \mu m/V$.



Figure 9.6: E_c shifts ν_z and E_z shifts ν'_c through the magnetic bottle. A cyclotron measurement with axial drives at five frequencies (a) shows bumps (marked with triangles) where ν_z of the *p* shifts into resonance with the drives. From the slope of $\Delta\nu'_c$ vs. $\Delta\nu_z$ (b), the size of the magnetic bottle may be determined.

9.5 The $\bar{p} - p$ Separation

The difference in cyclotron frequencies caused by the slight variation in magnetic field across the trap is the product of the linear gradient (Table 9.4) and the distance between the equilibrium location of the \bar{p} and p. Stray potentials in the trap which don't change sign when the sign under reversal of the trapping voltage cause the equilibrium position to shift as the trap voltage is changed. Several mechanisms could conceivably produce such potentials. Thermocouple effects between copper wire and constantan wire which transmit the voltage to the electrodes shift the voltage compared to that at the power supply. The wiring is symmetric for all the electrodes, however, and thus the voltage shifts should cancel to first order. Stray potentials are also created on the electrode surfaces themselves. Commonly referred to as patch effects, they can be caused by insulating oxides charging up on an otherwise conducting surface and even by crystalline grain boundaries. Under ideal circumstances, they have been measured as less than 1 mV [71], which would shift the particle's position by an insignificant 0.1 μ m.

Surface potentials on the electrodes may be made much worse, however, if an insulating layer remaining on an electrode surface is charged by electrons. After cooling the antiprotons, the electrons are dumped from the trap toward either the degrader or the upper high voltage endcap (Fig. 7.2) where they will not produce significant electric fields inside the harmonic trap. Electrons deposited on the electrodes of the harmonic trap (if an electrode is incorrectly biased to allow this during an electron dump, for example) can cause more substantial shifts if they hit insulating oxides.

The voltage produced by electrons on the ring may be estimated using a crude capacitive model. Charge deposited on an insulating surface above a conductor will produce a voltage $\Delta V = q/C$ where q is the total charge of the electrons and C is the capacitance between the surface charged with electrons and the underlying conductor. A large electron cloud used for antiproton cooling (Fig. 7.5a) contains roughly 3×10^6 electrons and has a radius of approximately 2 mm. If the electrons are spread uniformly over the 0.7 cm^2 ring, and the insulating layer is taken to have a thickness of 1 μ m, $C \approx 100$ pF and $\Delta V = 0.5$ mV. This small potential only shifts the trap depth. The equilibrium position is unaffected as the extra charge is symmetric across the xy plane (the ring is centered there) and about the z axis. If the electrons are instead deposited in a 1 mm radius circle (the size of the cloud in the trap) on the ring, the capacitance will be only several pF. Combined with the total charge, this creates a maximum offset potential of 0.1 V which breaks rotation symmetry, shifts the equilibrium position in the plane, and causes an antiproton and proton to sample different magnetic fields.

An experimental estimate of a possible offset voltage may be determined from shifts in the voltage needed to bring the axial motion in resonance with the amplifier. When electrons were dumped in uncontrolled ways, the voltage applied to the electrodes to keep ν_z constant changed by up to 200 mV. (Typically, the shifts were 50mV or less.) While there were 5 of these voltage jumps from the time the apparatus was first cooled to cryogenic temperatures until the measurement was completed, the total offset potential was within ±100 mV. After the electron dumping procedure was improved to assure electrons were always dumped to a surface far from the trap, these shifts dropped to 10 mV or less.

This measured offset is symmetric with respect to both the xy plane and the z axis, shifting the trapping potential but not the particle position. Offsets that shift the particle position must be asymmetric and should be of the same order or smaller than the symmetric component as the charge should distribute itself uniformly about
the electrodes. Thus the symmetric potential provides a conservative limit of 0.2 volts for the offset potentials which shift the positions of the antiproton relative to the proton.

9.6 An Estimate of the Systematic Error

Combining the observed gradients with the inferred limits on asymmetric offset potentials allows an estimate of the maximum field differences between \bar{p} and p and hence the dominant systematic error in the measurement. The gradients in the xand z directions are 10 times smaller than the 2.4 ppb/volt (1.4 mG/mm) gradient in the y direction. The y component of the separation thus determines the error. A 0.2 V potential would shift a proton 20 μ m towards the electrode upon which the charge has accumulated and the antiproton the same distance away from the electrode. The corresponding field difference is 56 mG or 1 ppb of the full field. Therefore, we cannot exclude measured cyclotron frequencies of the antiproton and proton differing by as much as 1 ppb, even if the charge–to–mass ratios are equal. We shall see in the next chapter that this systematic dominates the uncertainty of the comparison of proton and antiproton charge-to-mass ratios.

Chapter 10

Extracting the Difference Frequency

After determining ν_c for both an antiproton and proton, their charge-to-mass ratios may be compared. Were the magnetic field constant in time, ν_c of each particle could be determined once and the two frequencies compared. Because the magnetic field changes in time (Ch. 8), different frequencies are measured even if the chargeto-mass ratios are the same. To correct for the drift in the magnetic field, a proton is first loaded and its cyclotron frequency measured several times (Fig. 10.1), revealing the drift in the field as well as its current value. Then an antiproton is loaded and its cyclotron frequency is measured several times. Finally, a second proton is loaded and similarly observed. The proton cyclotron frequency is then interpolated for the time the antiproton is measured and this is compared to the antiproton frequency.

The cyclotron frequency ν_c is determined in each measurement from measurements of ν'_c and ν_z . Because ν_m need only be determined to an accuracy of a few percent (Ch. 6), it is measured only occasionally. Uncertainties in ν_c are determined from errors in the three measured frequencies: ν'_c , ν_z and ν_m (Chs. 4, 5 and 6) using



Figure 10.1: Measured ν_c for one p (squares), one \bar{p} (circles) and then a second p (squares). The solid line is the fitted function (Eq. 10.1) while the dashed lines are the polynomial representing the field drift with and without the addition of the \bar{p} -p frequency difference, $\Delta \nu_c$.

the invariance theorem (Eq. 2.2)

$$(\nu_c)^2 = (\nu'_c)^2 + (\nu_z)^2 + (\nu_m)^2.$$

Contributions to the uncertainty in ν_c come from uncertainties in the zero energy frequency of ν'_c and the linewidths of ν_z and ν_m . Typically, the total uncertainty in a measurement of ν_c is 0.4 ppb (Table 10.1).

10.1 Fitting the Magnetic Field Fluctuations

The magnetic field drifts by several ppb/hour when the regulation system is functioning and no external fluctuations change the field (Ch. 8). Many days of studying the variations in the magnetic field using a single trapped particle as a magnetome-

| Measured | Uncertainty | Со | ntribution to | |
|---------------------------------|--------------|--------------------|------------------------|--|
| frequency | | unc | uncertainty in ν_c | |
| | | (Hz) | (ppb) | |
| $\nu_c' \approx 89 \text{ MHz}$ | 0.02 Hz | 0.02 Hz | 0.2 ppb | |
| $\nu_z \approx 954 \text{ kHz}$ | 2 Hz | $0.02~\mathrm{Hz}$ | $0.2 \mathrm{~ppb}$ | |
| $\nu_m \approx 5 \text{ kHz}$ | 500 Hz | $0.03~\mathrm{Hz}$ | $0.3~{ m ppb}$ | |
| | total error: | 0.04 Hz | 0.4 ppb | |

Table 10.1: Typical errors in determining the free space cyclotron frequency from the three measured frequencies. Contributions to the uncertainties in ν_c are derived from the invariance theorem (Eq. 2.2).

ter show smooth variations in the field. The field typically fluctuates on a daily period with a local minimum in the field in the early evening (Figs. 8.4 and 10.5). Superimposed on this correlation are shifts lasting several hours as the cryogenic dewars repressurize after liquid fills which happen several times a week as well as other long term drifts in the field. During a single 30 minute measurement of ν'_c , the field may drift by up to one ppb. Over the six to ten hours of a full measurement changes of 30 ppb are possible. Because ν_c cannot be measured during the time required to load a single proton or antiproton (Ch. 7) and shrink its magnetron orbit (Sec. 6.1), this process must be performed quickly enough that the fit may be performed. In practice our measurements should be performed in 12 hours or less to reliably interpolate the magnetic field.

For such data sets, second to fourth order polynomials fit the data (Fig. 10.2a) with residuals (Fig. 10.2b) of 1 ppb and differences between the fits well below 1 ppb. Examining figure 10.2a, the second order polynomial does not follow the drift in the field as closely as the third order polynomial. The fourth order model fits a turning point to the data near t = 0 and is thus perhaps of too high order. Third order polynomials are therefore used in the analysis presented here. The whole analysis

was also done with a second order polynomial, changing the results by less than one sigma.

When antiproton and proton data are compared, a polynomial is used to fit the field drift and a frequency offset between the antiproton and proton is allowed. The measurement sets are fit to

$$\nu_c = \sum_{i=0}^n a_i t^i + \begin{cases} 0 & \text{for } p \\ \Delta \nu_c & \text{for } \bar{p} \end{cases},$$
(10.1)

where n = 3 for the analysis here (Table 10.2) and the parameter $\Delta \nu_c = \nu_c(\bar{p}) - \nu_c(p)$ is added to the \bar{p} data points but not the p points. Figure 10.1 is one of these data sets in which the proton cyclotron frequencies are shown as squares and the antiprotons frequencies as circles. The solid line in the figure is the fitted function (Eq. 10.1) and the dotted lines indicate the interpolated cyclotron frequencies of the antiproton and proton. The 1 ppb vertical separation of the two curves is the measured frequency difference $\Delta \nu_c = 1.0 \pm 1.2$ ppb between the antiproton and proton.

10.2 Multi-curve Fitting

In the analysis above, each cyclotron measurement was fit to extract ν'_c (Eq. 4.7) assuming a constant magnetic field, and then a drift in the field extracted from changes in ν'_c . The field, however, changes continuously during the cyclotron measurements. Therefore, the cyclotron frequency drift should be included in fitting the measurement. This may be done by fitting all the measurements simultaneously with

$$\nu_c(t) = \Delta \nu_j e^{-t/\tau_j} + \sum_{i=0}^M a_i t^i + \begin{cases} 0 & \text{for } p \\ \Delta \nu_c & \text{for } \bar{p} \end{cases}$$
(10.2)



Figure 10.2: Fit (a) of 12 hours of typical cyclotron measurements of one proton to quadratic cubic and quartic polynomials and (b) residuals of the fit.

As in the previous fits the drift in the field is parameterized as a polynomial with a potential frequency difference between antiprotons and protons parameterized by $\Delta\nu_c$. Rather than fitting only the zero energy cyclotron frequencies, ν'_c , all the individual cyclotron frequencies, measured as the energy damps out of the cyclotron motion are fit to this drift. In Eq. 10.2, j labels each cyclotron measurement. A separate excitation size is allowed for each measurement, but rather than also having independent zero energy frequencies, they are constrained to follow the polynomial (and offset, $\Delta\nu_c$). The time constants (τ_j) can either be varied independently or forced to be the same ($\tau_j = \tau$). Note that to extract $\Delta\nu_c$ from this fit, each measured $\nu'_c(t)$ must be converted to $\nu_c(t)$ using the invariance theorem before the fit is applied.

While this technique is in principle superior to fitting the cyclotron measurements independently and then fitting the results together, we have chosen not to use it. Field drifts and more importantly antiproton – proton frequency differences determined by the two techniques agree to within one sigma. Due to the large number of parameters in these nonlinear multi-curve fits, however, initial estimates of the parameters must be chosen carefully so that a fitting routine may find the true minimum. The residuals from such fits (Fig. 10.3) also have large systematic scatter, which is not well understood. Therefore, the simpler two step fitting process was used.

10.3 Results

Over a span of ten days, seven $p-\bar{p}-p$ switches were performed. However, one data set was eliminated because the time needed to load a single antiproton, free of electrons and negative ions, was over six hours. In this time, during which ν_c cannot be measured, the field drifted so much that the comparison was impossible. Another



Figure 10.3: Cyclotron endpoints from a sample measurement set (a) showing both the standard fit to endpoints (solid) and a fit to all the data simultaneously (dashed). The residuals of the simultaneous fit (b) show much larger scatter than typical individual measurements.

measurement set was lost when a large change in the ambient temperature caused a pressure regulator to fail and change the magnetic field such that it could not be fit (Fig. 10.4). Finally, in one measurement, a single H⁻ ion, loaded along with the antiprotons, remained in the trap during the measurement period. This ion strongly perturbed the cyclotron measurement (Fig. 7.7), forcing the elimination of a third data set. The four remaining data sets (Fig. 10.5) were fit as described above (Sec. 10.1). Averaging the four measurements gives a frequency difference

$$\frac{\nu_c(\bar{p}) - \nu_c(p)}{\nu_c} = 1.5 \pm 0.4 \text{ ppb.}$$
(10.3)

The systematic error from the uncertainty in the relative separation of the two particles (Ch. 9) must also be included. The maximum potential separation between the equilibrium positions of antiprotons and protons of 50 μ m (Sec. 9.5), coupled with the 0.02 ppb/ μ m gradient in the magnetic field (Sec. 9.3) leads to a 1 ppb uncertainty in the magnetic field between the positions of the particles. Thus, the charge-to-mass ratio of the antiproton and proton including both the statistical and systematic error is

$$\frac{e_{\bar{p}}}{M_{\bar{p}}} / \frac{e_p}{M_p} = 1.000\ 000\ 001\ 5\ (11),\tag{10.4}$$

with the uncertainty in the last digits in parentheses. In the usual notation of mass spectroscopy (assuming that the charges of the two particles are of equal magnitude), the mass ratio is

$$M(\bar{p})/M(p) = 0.999\,999\,998\,5\,(11),\tag{10.5}$$

again with the uncertainty in the last digits in parentheses.¹

¹The charge-to-mass reported in Ref. [1] is correct. Unfortunately, the "8" in the mass ratio (Eq. 10.5) was misprinted as a "9".



Figure 10.4: Sudden changes in ambient temperature of hall (a) as well as direct sunlight, can lead the regulator controlling the temperature of the pressure reference cavity (b) to fail thereby rapidly changing the magnetic field and hence ν_c (c).



Figure 10.5: The four $p-\bar{p}-p$ comparisons used in this measurement. Circles represent protons and triangles represent antiprotons. The field drift is fit to a cubic polynomial and an offset between \bar{p} and p frequencies is allowed. (Cyclotron frequencies are measured relative to 89251540 Hz.)

| | $\Delta \nu_c$ | σ_{ν} |
|---------------------------|----------------|----------------|
| | 2.0 | 2.0 |
| | 1.0 | 1.2 |
| | 1.6 | 1.3 |
| | 2.4 | 1.9 |
| $\overline{\Delta \nu_c}$ | 1.5 | 0.4 |

Table 10.2: Fitted cyclotron frequency differences for four comparisons in ppb.



Figure 10.6: Summary of $\nu_c(\bar{p}) - \nu_c(p)$ frequency differences. The dashed line is the average frequency difference of 1.5 ppb.

10.4 Future Improvements

The residual linear magnetic gradient coupled with the separation of the equilibrium positions of the antiproton and proton (Sec. 9.5) limit the accuracy of this comparison. Further reducing the gradient by performing additional cycles of adjusting the shims and measuring the gradients (Ch. 9) would reduce this systematic error. A more elegant solution was provided by the discovery that H⁻ ions are created in the trapping region when antiprotons are loaded (Sec. 7.1.4). Because the charge-tomass ratios of the \bar{p} and H⁻ are equal to with parts in 10³ and have the same sign, the trapping potentials used in measuring the two particles are almost the same and hence the effect of offset potentials on the equilibrium positions will be reduced by orders of magnitude. Thus the existing shimming will be more than sufficient to perform a 10 times more accurate measurement. The mass ratio of the electron and proton [72, 14] and the H⁻ binding energy [73] are sufficiently well known so as to not contribute additional error to a deduced antiproton to proton comparison.

An additional benefit of using an H⁻ ion instead of a proton is that it can be trapped at the same time as the \bar{p} . The trap may be loaded with \bar{p} and H⁻ and particles removed until only one \bar{p} and one H⁻ remain. Then the cyclotron frequency of one species may be measured while the other is placed in a cyclotron orbit sufficiently large that it does not disturb the measurement of the first particle (Fig. 10.7). This allows comparisons of antiprotons and H⁻ to be performed without having to load new particles repeatedly, reducing the time between measurements and thus improving the statistical errors in the measurement associated with field drifts as well as the systematic errors.

At accuracies below 10^{-9} , additional techniques may be required to reduce the drifts in the field (Ch. 8). Either more data must be taken to average away the drifts



Figure 10.7: An H⁻ cyclotron measurement (hollow circles) in which its energy damps from 240 eV towards zero. The energy in the cyclotron motion of the \bar{p} (solid circles) damps from 3.3 keV to 1.9 keV during the measurement period. Because the tuned circuit detecting the cyclotron frequency is resonant with the H⁻, its time constant is much faster (10 minutes) than that of the \bar{p} (35 minutes).

in the field, or the measurement may need to be done when the ambient magnetic field is more stable. Because the \bar{p} and H⁻ can be held in the trap simultaneously, they can be loaded immediately before the accelerators are turned off for an extended period. During this period the fluctuations in the ambient field are smaller.

10.5 Conclusions

A one ppb (10^{-9}) comparison of the charge-to-mass ratios of the antiproton and proton has been completed. A single antiproton cooled to energies as low as 4.2 K is measured using high Q amplifiers including an axial tuned circuit constructed from type II superconductors. Measuring the cyclotron frequencies to fractional resolutions up to 2×10^{-10} leads to a very clean demonstration of the "relativistic mass shift", observed as the cyclotron frequency shifts in proportion to its energy. This comparison of the charge-to-mass ratios of the antiproton and proton provides one of the most accurate tests of the CPT invariance, a fundamental theorem of quantum field theory. In the future, comparison of H⁻ and antiprotons should allow at least another order of magnitude improvement in this comparison.

Bibliography

- G. Gabrielse, D. Phillips, W. Quint, H. Kalinowsky, G. Rouleau, and W. Jhe, Phys. Rev. Lett. 74, 3544 (1995).
- [2] R. Carosi and et al. Phys. Lett. **B237**, 303 (1990).
- [3] R. V. Dyck, Jr., P. Schwinberg, and H. Dehmelt, Phys. Rev. Lett. 59, 26 (1987).
- [4] O. Chamberlain, E. Segre, C. Wiegand, and T. Ypsilantis, Phys. Rev. 100, 947 (1955).
- [5] A. Bamberger *et al.*, Phys. Lett. **33B**, 233 (1970).
- [6] E. Hu, Y. Asano, M. Chen, S. Cheng, G. Dugan, L. Lidofsky, W. Patton, C. Wu, V. Hughes, and D. Lu, Nucl. Phys. A254, 403 (1975).
- [7] P. Roberson *et al.*, Phys. Rev. C **16**, 1945 (1977).
- [8] B. Roberts, Phys. Rev. D 17, 358 (1978).
- [9] L. Brown and G. Gabrielse, Rev. Mod. Phys. 58, 233 (1986).
- [10] F. DiFilippo, V. Natarajan, K. Boyce, and D. Pritchard, Phys. Rev. Lett. 73, 1481 (1994).

- [11] G. Gabrielse, X. Fei, K. Helmerson, S. Rolston, R. Tjoelker, T. Trainor, H. Kalinowsky, J. Haas, and W. Kells, Phys. Rev. Lett. 57, 2504 (1986).
- [12] G. Gabrielse, X. Fei, L. Orozco, R. Tjoelker, J. Haas, H. Kalinowsky, T. Trainor, and W. Kells, Phys. Rev. Lett. 63, 1360 (1989).
- [13] G. Gabrielse, X. Fei, L. Orozco, S. Rolston, R. Tjoelker, T. Trainor, J. Haas,
 H. Kalinowsky, and W. Kells, Phys. Rev. A 40, 481 (1989).
- [14] G. Gabrielse, X. Fei, L. Orozco, R. Tjoelker, J. Haas, H. Kalinowsky, T. Trainor, and W. Kells, Phys. Rev. Lett. 65, 1317 (1990).
- [15] E. D. Commins, Am. J. Phys. **61**, 778 (1993).
- [16] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
- [17] L. D. Landau, Nucl. Phys. 3, 127 (1957).
- [18] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. 13, 138 (1964).
- [19] Particle Data Group, Phys. Rev. D 50, 1228 (1994).
- [20] G. Lüders, Ann. Phys. 2, 1 (1957).
- [21] J. Bell, Proc. Roy. Soc. A231, 479 (1955).
- [22] J. Wess, Hyper. Int. 44, 3 (1988).
- [23] R. Streater and A. Wightman, PCT, Spin and Statistics, and All That (Benjaman, New York, 1964).
- [24] J. Ellis, N. E. Mavaromatos, and D. V. Nanopoulos, Phys. Lett. B293, 142 (1992).

- [25] E. Adelberger, B. Heckel, C. Stubbs, and Y. Su, Phys. Rev. Lett. 66, 850 (1991).
- [26] R. J. Hughes and M. H.Holzscheiter, Phys. Rev. Lett. 66, 854 (1991).
- [27] Beverini and et al., Technical Report No. CERN/PSCC/86-2/P-94, CERN (unpublished).
- [28] T. Darling, F. Rossi, G. Opat, and G. Moorhead, Rev. Mod. Phys. 64, 237 (1992).
- [29] G. Gabrielse, S. Rolston, L. Haarsma, and W. Kells, Phys. Lett. A129, 38 (1988).
- [30] L. Brown and G. Gabrielse, Phys. Rev. A 25, 2423 (1982).
- [31] G. Gabrielse, L. Haarsma, and S. Rolston, Intl. J. of Mass Spec. and Ion Phys.
 88, 319 (1989), ibid. 93, 121 (1989).
- [32] G. Gabrielse, Phys. Rev. A 27, 2277 (1983).
- [33] R. L. Tjoelker, Ph.D. thesis, Harvard University, 1990.
- [34] X. Fei, Ph.D. thesis, Harvard University, 1990.
- [35] M. Sirkis and N. Holonyak, Am. J. Phys. **34**, 943 (1966).
- [36] W. Maxalpine and R. Schildknecht, Proceedings of the IRE 2099 (1959).
- [37] F. R. Ficket, IEEE Trans. Magn. 24, 1156 (1988).
- [38] B. Lengeler, Cryogenics 439 (1974).
- [39] R. Kirschman, Cryogenics **25**, 115 (1985).

- [40] A. T. Lee, Rev. Sci. Instrum. **60**, 3315 (1989).
- [41] A. T. Lee, Rev. Sci. Instrum. **64**, 2373 (1993).
- [42] M. G. Richards, A. R. Andrews, C. P. Lusher, and J. Schratter, Rev. Sci. Instrum. 57, 404 (1986).
- [43] D. Stefanovitch, G. Epdtein, P. Puget, R. Knoll, R. Picault, and Y. Carpentier, Cryogenics 32, 403 (1992).
- [44] D. V. Camin, G. Pessina, E. Previtali, and G. Ranucci, Cryogenics 29, 857 (1989).
- [45] R. M. Weisskoff, G. P. Lafyatis, K. R. Boyce, E. A. Cornell, R. W. Flanagan, Jr., and D. Pritchard, J. Appl. Phys. 63, 4599 (1988).
- [46] S. R. Jefferts, T. Heavner, P. Hayes, and G. H. Dunn, Rev. Sci. Instrum. 64, 737 (1993).
- [47] F. Ayela, J. Bret, and J. Chaussy, Cryogenics **32**, 35 (1992).
- [48] in CERN accelerator school: superconductivity in particle accelerators, edited by S. Turner (unpublished, Geneva, 1988).
- [49] W. H. Hartwig, Proc. IEEE **61**, 58 (1973).
- [50] T. Ogasawara, Y. Takahashi, K. Kanbara, Y. Kubota, K. Yasohama, and K. Yasukochi, Cryogenics 736 (1979).
- [51] W. Press, B. Flannery, S. Teukolsky, and W. Vetterling, NUMERICAL RE-CIPIES in C (Cambridge University Press, New York, 1988).
- [52] D. J. Wineland and H. G. Dehmelt, J. Appl. Phys. 46, 919 (1975).

- [53] L. D. Landau and E. M. Lifshitz, *Mechanics (Third Edition)* (Pergamon Press, New York, 1976).
- [54] D. Wineland and H. Dehmelt, Intl. J. of Mass Spec. and Ion Phys. 16, 338 (1975), ibid. 19 251E.
- [55] S. van der Meer, Rev. Mod. Phys. 689 (1985).
- [56] X. Fei, R. Davisson, and G. Gabrielse, Rev. Sci. Instrum. 58, 2197 (1987).
- [57] F. L. Moore, Ph.D. thesis, University of Washington, 1989.
- [58] G. Gabrielse, J. Tan, L. Orozco, S. Rolston, C. Tseng, and R. Tjoelker, J. of Mag. Res. 91, 564 (1991).
- [59] J. Gröbner, Master's thesis, Innsbruck, 1991.
- [60] G. Gabrielse and J. Tan, J. Appl. Phys. **63**, 5143 (1988).
- [61] J. N. Tan, Ph.D. thesis, Harvard University, 1992.
- [62] J. D. Jackson, Classical Electrodynamics, 2nd edition (John Wiley and Sons, Inc., New York, 1975).
- [63] E. S. Meyer, I. F. Silvera, and B. L. Brandt, Rev. Sci. Instrum. 60, 2964 (1989).
- [64] J. Haas, Ph.D. thesis, Joannes Gutenberg–Universität Mainz, 1990.
- [65] A Physicist's Desk Reference, edited by H. L. Anderson (American Institute Of Physics, New York, 1989).
- [66] D. Enzer, private communication.
- [67] Y. S. Cha, R. C. Niemann, and J. R. Hull, Cryogenics **33**, 675 (1993).

- [68] Y. S. Cha, R. C. Niemann, and J. R. Hull, Cryogenics 34, 99 (1994).
- [69] J. Carolan, private communication.
- [70] R. S. Van Dyck Jr., P. B. Schwinberg, and H. G. Dehmelt, Phys. Rev. Lett. 59, 26 (1987).
- [71] J. Camp, T. W. Darling, and R. E. Brown, J. Appl. Phys. 69, 7126 (1991).
- [72] R. S. Van Dyck Jr., F. L. Moore, D. L. Farnham, and P. B. Schwinberg, Phys. Rev. A 40, 6308 (1989).
- [73] Wineberger, Phys. Rev. A 43, 6104 (1991).